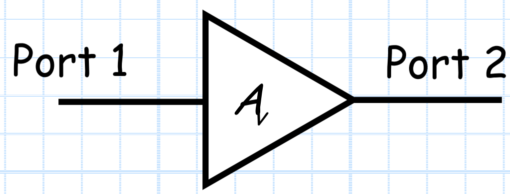
Amplifier Gain

Note that an amplifier is a **two-port** device.



As a result, we can describe an amplifier with a 2 x 2 **scattering matrix**:

$$\bar{\bar{\mathbf{S}}}(\omega) = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

Q: What is the scattering matrix of an **ideal** amplifier??

A: Let's start with $\mathbf{S}_{11}$.

Recall that $|S_{11}|^2 = |\Gamma_1|^2$. Since any non-zero reflection coefficient will result in return loss, we **ideally** would like the input (port 1) impedance to be matched:

$$\Gamma_1 = S_{11} = 0$$

Similarly, we desire the output impedance to be **matched** to Z_0, so that maximum **power transfer** occurs:

$$\Gamma_2 = S_{22} = 0$$

Now, let's look at $S_{21}(\omega)$. We know that:

$$P_2^- = |S_{21}|^2 P_1^+$$

or, stated **another** way:

$$P_{out} = |S_{21}|^2 P_{in}$$

Therefore, we can **define** the amplifier **power gain** as:

$$G \doteq |S_{21}|^2$$

For an **ideal** amplifier, $S_{21} = A_v$, or $G = A_v^2$. More generally, an amplifier will likewise have some delay, such that:

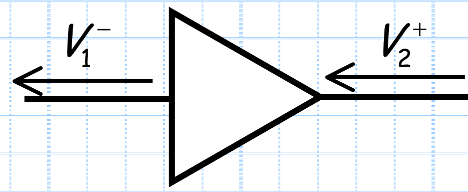
$$S_{21} = A_v e^{j\omega\tau + \phi_0}$$

When radio engineers speak of amplifier **gain**, they almost always are speaking of **power gain G**. Recall for this gain is valid **only** within some specified amplifier bandwidth.

Additionally, radio engineers almost always speak of amplifier gain in **decibels** (dB):

$$\text{Gain in dB} = 10 \log_{10} G$$

Finally, let's consider S_{12}. This scattering parameter relates the wave into port 2 (the output) to the wave out of port 1 (the input).



Q: Are amplifiers **reciprocal** devices? In other words, is $S_{12} = S_{21}$??

A: No! An amplifier is strictly a **directional** device; there is a specific input, and a specific output—it does **not** work in reverse!

Ideally then, $S_{12} = 0$. Any other value can just cause problems!

Typically though, S_{12} is small, but not zero. We define the value:

$$\text{reverse isolation} \doteq -10\log_{10}\left|S_{12}\right|^2$$

Therefore, the larger the reverse isolation, the better!

Summarizing, we find that the scattering matrix of the ideal amplifier is:

$$\bar{\bar{\mathbf{S}}}_{ideal} = \begin{bmatrix} 0 & 0 \\ \mathcal{A} & 0 \end{bmatrix}$$

Sort of like an **isolator** with gain!

Whereas the scattering matrix of an **actual** amplifier would be more of the form:

$$\bar{\bar{\mathbf{S}}}(\omega) = \begin{bmatrix} \Gamma_1 & \varepsilon \\ \mathcal{A} e^{j(\omega\tau+\phi_0)} & \Gamma_2 \end{bmatrix}$$

for frequencies within its bandwidth.

The value ε represents specifies the reverse isolation of the amplifier, a complex value of **small** magnitude.

Likewise, the reflection coefficients Γ are **very small** for good amplifiers.