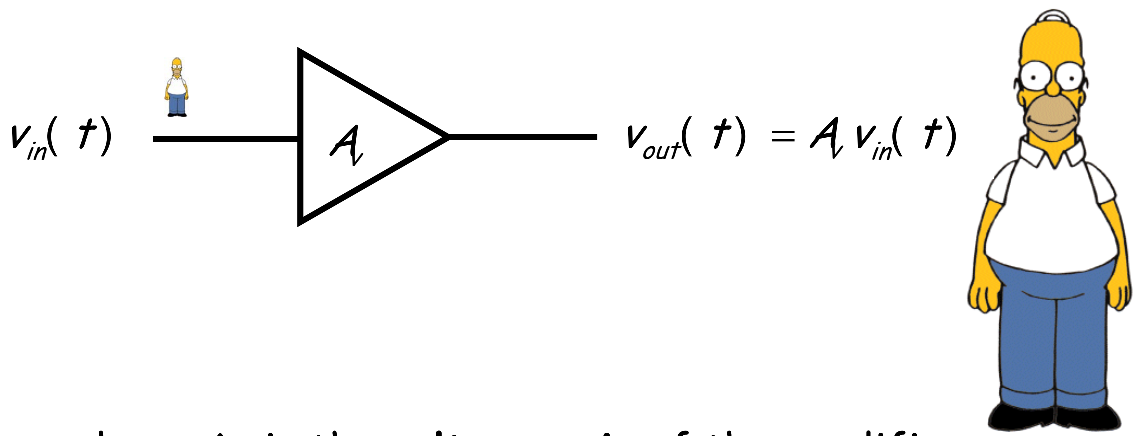
Amplifiers

An **ideal** amplifier takes an input signal and reproduces it **exactly** at its output, only with a **larger** magnitude!



where A_v is the **voltage gain** of the amplifier.

Now, let's express this result using our **linear circuit theory** !

Recall, the output $v_{out}(t)$ of a linear device can be determined by **convolving** its input $v_{in}(t)$ with the device **impulse response** $g(t)$:

$$v_{out}(t) = \int_{-\infty}^{t} g(t-t')\,v_{in}(t')\,dt'$$

The impulse response for the **ideal** amplifier would therefore be:

$$g(t) = A_v\,\delta(t)$$

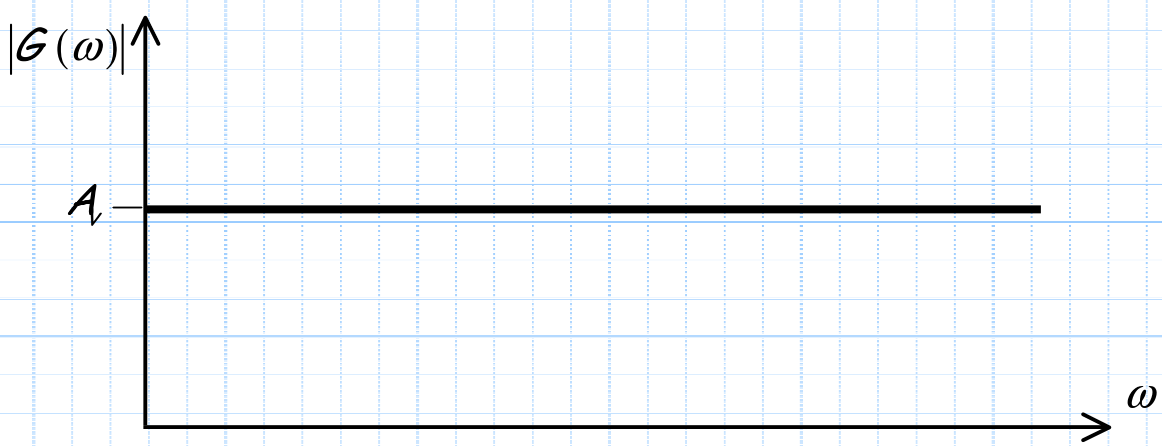
so that:

$$\begin{aligned} v_{out}(t) &= \int_{-\infty}^{t} g(t-t')\, v_{in}(t')\, dt' \\ &= \int_{-\infty}^{t} A_v\, \delta(t-t')\, v_{in}(t')\, dt' \\ &= A_v\, v_{in}(t) \end{aligned}$$

We can alternatively represent the ideal amplifier response in the **frequency domain**, by taking the **Fourier Transform** of the impulse response:

$$\begin{aligned} G(\omega) &= \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt \\ &= \int_{-\infty}^{\infty} A_v\, \delta(t)\, e^{-j\omega t}\, dt \\ &= A_v + j0 \end{aligned}$$

This result, although simple, has an interesting interpretation. It means that the amplifier exhibits of magnitude gain of A_v for sinusoidal signals of **any** and **all** frequencies!



Moreover, the ideal amplifier does not alter the **relative phase** of the sinusoidal signal (i.e., no phase shift).

In other words, if:

$$v_{in}(t) = \cos(\omega t)$$

then at the output of the ideal amplifier we shall see:

$$\begin{aligned} v_{out}(t) &= |G(\omega)| \cos(\omega t + \angle G(\omega)) \\ &= A_v \cos(\omega t) \end{aligned}$$

BUT, there is one **big** problem with an ideal amplifier:

→ They are **impossible** to build !!

Q: *Why is that ??*

A: Two reasons:

a) An ideal amplifier has **infinite** bandwidth.

b) An ideal amplifier has **zero** delay.

Not gonna happen !

Let's look at this **first** problem first. The ideal amplifier impulse response $g(t) = A_v \, \delta(t)$ means that the signal at the output occurs **instantaneously** with the signal at the input. This of course **cannot** happen, as it takes some small, but non-zero amount of **time** for the signal to propagate through the amplifier. A more **realizable** amplifier impulse response is:

$$g(t) = A_v\, \delta(t-\tau)$$

resulting in an amplifier output of:

$$\begin{aligned} v_{out}(t) &= \int_{-\infty}^{t} g(t-t')\, v_{in}(t')dt' \\ &= \int_{-\infty}^{t} A_v\, \delta(t-\tau-t')\, v_{in}(t')dt' \\ &= A_v\, v_{in}(t-\tau) \end{aligned}$$

In other words, the output is both an amplified and **delayed** version of the input.

* Note the delay does not **distort** the signal, as the output has the same form as the input.

* Moreover, the delay for electronic devices such as amplifiers is **very small** in comparison to human time scales (i.e., $\tau \ll 1$ second).

* Therefore, propagation delay τ is **not** considered a **problem** for most amplifier applications.

BUT, the delay better be a **constant** with **frequency** (otherwise, signal **distortion** results)!

Let's examine what this delay means in the **frequency domain**.

Evaluating the Fourier Transform of this **modified** impulse response gives:

$$\begin{aligned} G(\omega) &= \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt \\ &= \int_{-\infty}^{\infty} A\, \delta(t-\tau)\, e^{-j\omega t}\, dt \\ &= A\cos(\omega\tau) + j A \sin(\omega\tau) \\ &= A\, e^{j\omega\tau} \end{aligned}$$

We see that, as with the ideal amplifier, the magnitude $|G(\omega)| = A$. However, the relative **phase** is now:

$$\angle G(\omega) = \omega\tau$$

As a result, if $v_{in}(t) = \cos(\omega t)$, the output signal will be:

$$\begin{aligned} v_{out}(t) &= |G(\omega)| \cos(\omega t - \angle G(\omega)) \\ &= A \cos(\omega t - \omega\tau) \end{aligned}$$

In other words, the output signal of a **real** amplifier is **phase shifted** with respect to the input.

Now, let's examine the **second problem** with the ideal amplifier. This problem is best discussed in the **frequency** domain.

We discovered that the **ideal** amplifier has a frequency response of $|G(\omega)| = A_v$. Note this means that the amplifier gain is **A_v** for **all** frequencies $0 < \omega < \infty$ (D.C. to daylight !).

The bandwidth of the ideal amplifier is therefore **infinite** !

* Since every electronic device will exhibit **some** amount of inductance, capacitance, and resistance, every device will have a **finite** bandwidth.

* In other words, there will be frequencies where the device does **not work** !

* From the standpoint of an amplifier, "not working" means $A_v < 1$ (i.e., **no gain**).

* Microwave/RF amplifiers will therefore have **finite** bandwidths.

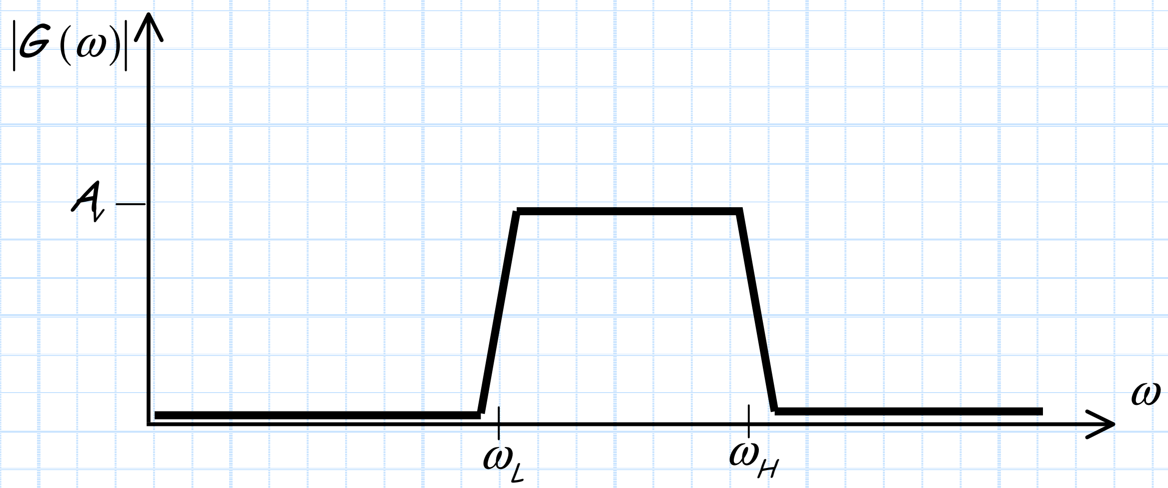
There is a range of frequencies ω between ω_L and ω_H where the gain will (approximately) be A_v. For frequencies outside this range, the gain will typically be small (i.e. $A_v \approx 0$):

$$|G(w)| = \begin{cases} \approx A_v & \omega_L < \omega < \omega_H \\ \approx 0 & \omega < \omega_L, \omega > \omega_H \end{cases}$$

The **width** of this frequency range is called the amplifier **bandwidth**:

$$Bandwidth \doteq \omega_H - \omega_L \quad \text{(radians/sec)}$$

$$\doteq f_L - f_H \quad \text{(cycles/sec)}$$



One result of having a **finite bandwidth** is that the amplifier impulse response is **not** an impulse function !

$$g(t) = \int_{-\infty}^{\infty} G(\omega)\, e^{+j\omega t} dt \neq A_v\, \delta(t-\tau)$$