Filter Design Worksheet

Q: Given the order of a Butterworth or Chebychev bandpass filter, what will my stop band attenuation be? Or stated another way, what should the order of my filter be, in order to achieve a desired amount of attenuation ($-10\log_{10} T(\omega)$)?

A: Consult the normalized attenuation charts (They’re in your book)!

For example, the normalized attenuation chart for a Butterworth filter is:
While the normalized attenuation chart for a Chebyshev with 0.5 dB of passband ripple is:

And the normalized attenuation chart for a Chebyshev with 3.0 dB of passband ripple is:
**Q:** Great, how the heck do I use these??

**A:** The variable $\alpha$ is a **normalized** frequency variable. The plots show attenuation versus frequency for a filter of order $n$.

Say we have a **bandpass filter**, whose (3 dB) passband extends from $f_1$ to $f_2$ ($f_2 > f_1$). The bandwidth of this filter would therefore be $f_2 - f_1$.

Using these values, we can define a **normalized frequency** $\alpha$ as:

$$\alpha = \frac{1}{\Delta} \left( \frac{f_1 - f_2}{f_0} \right) - 1$$

where:

$$f_0 = \sqrt{f_1 f_2}$$

$$\Delta = \frac{f_2 - f_1}{f_0}$$

Thus, given a frequency $f$, we can calculate a value $\alpha$.

* It turns out that all frequencies $f$ **outside** the pass band of the filter will have **positive** values of $\alpha$, while frequencies **within** the pass band will result in **negative** values of $\alpha$.

* Accordingly, if $f = f_1$ or $f = f_2$, the value of $\alpha$ will be **zero** (try it!).
* As a result, the attenuation charts give answers for positive values of $\alpha$ only, corresponding to frequencies in the stop band.

* In other words, the attenuation charts provide information about the stop band attenuation only. Note as $\alpha$ gets larger, the attenuation for all filter orders increases.

* This makes sense, as an increasing $\alpha$ corresponds to a frequency $f$ either greater than $f_2$ and increasing, or a frequency $f$ less than $f_1$ and decreasing.
For example, consider a Butterworth bandpass filter whose passband extends from 1 GHz to 4 GHz.

Therefore, $f_1 = 1$ GHz and $f_2 = 4$ GHz, resulting in $f_0 = 2$ GHz and $\Delta = 1.5$ GHz.

**Q1:** By how much is a 500 MHz signal attenuated if the filter has order $n=6$?

For $f = 0.5$ GHz:

$$\alpha = \left| \frac{1}{\Delta} \left( \frac{f}{f_0} - \frac{f_0}{f} \right) \right| - 1$$

$$= \left| \frac{1}{1.5} \left( \frac{0.5}{2.0} - \frac{2.0}{0.5} \right) \right| - 1$$

$$= 1.5$$

It appears from the attenuation chart that this filter attenuates a 500 MHz signal approximately 50 dB.

**Q2:** What should the filter order $n$ be, if we need to attenuate signals at 6.6 GHz by at least 40 dB?

For $f = 8$ GHz:

$$\alpha = \left| \frac{1}{\Delta} \left( \frac{f}{f_0} - \frac{f_0}{f} \right) \right| - 1$$

$$= \left| \frac{1}{1.5} \left( \frac{6.6}{2.0} - \frac{2.0}{6.6} \right) \right| - 1$$

$$= 1.0$$
Again from the chart, we find at $\alpha = 1.0$, a filter with order $n = 7$ (or higher) will attenuate a 6.6 GHz signal by more than 40 dB.

Now you too can determine filter attenuation and/or order. I hope you’ve been paying attention!!