Intermodulation Distortion

The 1 dB compression curve shows that amplifiers are only **approximately** linear.

Actually, this should be **obvious**, as amplifiers are constructed with transistors—**non-linear** devices!

So, instead of the **ideal** case:

$$v_{out} = A_v\, v_{in}$$

An actual amplifier behavior is more like:

$$v_{out} = A_v\, v_{in} + B\, v_{in}^2 + C\, v_{in}^3 + \cdots$$

I.E., a **Taylor Series** representation of a **non-linear** function:

$$v_{out} = f(v_{in})$$

Generally speaking, B, C, D, etc. are very small compared to the voltage gain A_v, therefore if v_{in} is small, we can **truncate** the Taylor Series and **approximate** amplifier behavior as:

$$v_{out} \approx A_v\, v_{in}$$

BUT! As v_{in} gets large, the values v_{in}^2 and v_{in}^3 will get really large!

In this case the terms $B\,v_{in}^2$ and $C\,v_{in}^3$ will become significant, resulting in **Intermodulation Distortion.**

Say the input to the amplifier is sinusoidal, with magnitude *a*:

$$v_{in} = a\cos\omega t$$

Using our knowledge of trigonometry, we can determine the result of the second term of the output Taylor series:

$$\begin{aligned} B\,v_{in}^2 &= B\,a^2\cos^2\omega t \\ &= \frac{B\,a^2}{2} + \frac{B\,a^2}{2}\cos 2\omega t \end{aligned}$$

We have created a **harmonic** of the input signal!

In other words, the input signal is at a frequency ω, while the output includes a signal at **twice** that frequency (2ω).

We call this signal a **second** order product, as it is a result of **squaring** the input signal.

Note we also have a **cubed** term in the output signal equation:

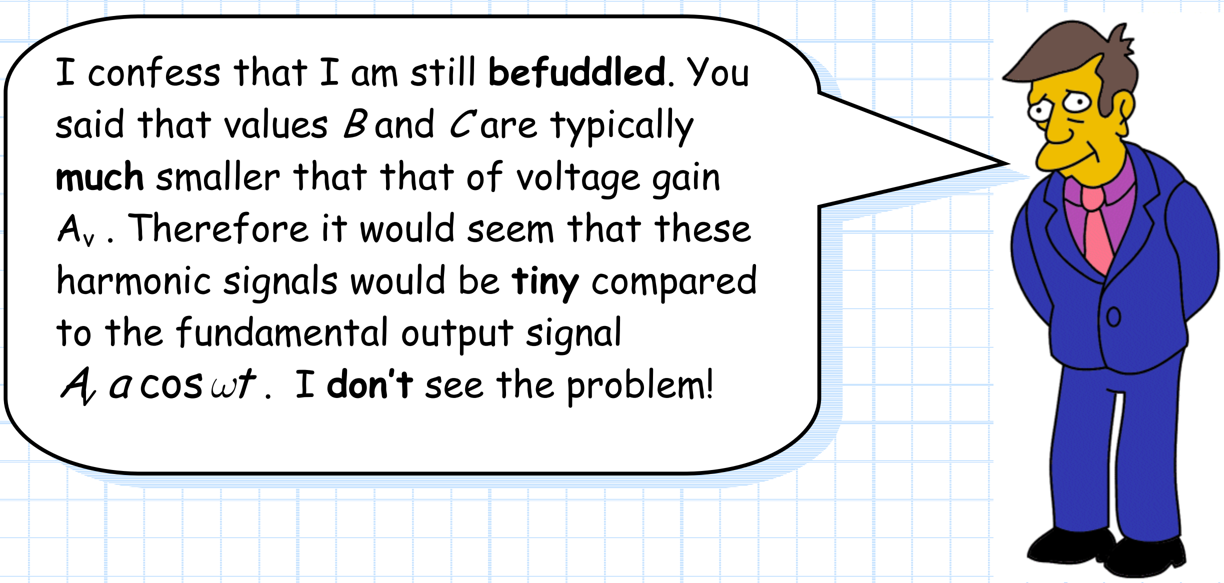
$$v_{out} = A_v\, v_{in} + B\, v_{in}^2 + C\, v_{in}^3 + \cdots$$

Using a trig identity, we find that:

$$\begin{aligned} C\, v_{in}^3 &= C\, a^3 \cos^3 \omega t \\ &= \frac{C\, a^3}{2} \cos \omega t + \frac{C\, a^3}{4} \cos 3\omega t \end{aligned}$$

Now we have produced a **second harmonic** (i.e., 3ω)!

As you might expect, we call this harmonic signal a **third-order** product (since it's produced from v_{in}^3).



To understand why intermodulation distortion can be a problem in amplifiers, we need to consider the **power** of the output signals.

We know that the power of a sinusoidal signal is proportional to its **magnitude squared**. Thus, we find that the power of each output signal is related to the input signal power as:

$$\text{1rst-order output power} \doteq P_1^{out} = A^2 P_{in} = G\, P_{in}$$

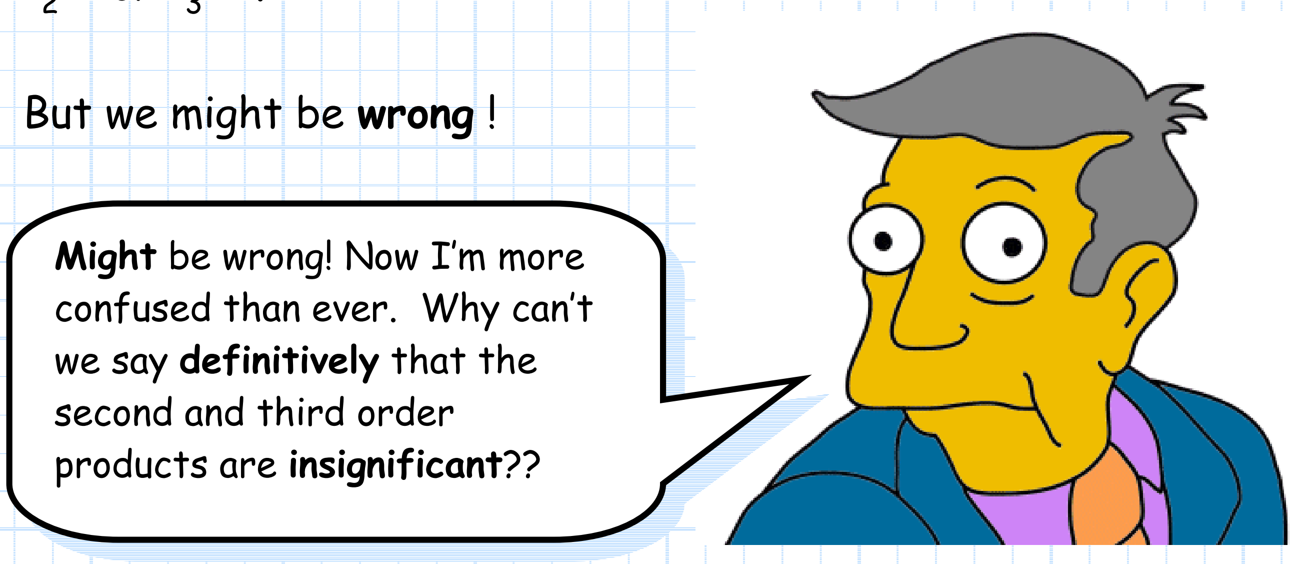
$$\text{2nd-order output power} \doteq P_2^{out} = \frac{B^2}{4} P_{in}^2 = G_2\, P_{in}^2$$

$$\text{3rd-order output power} \doteq P_3^{out} = \frac{C^2}{16} P_{in}^3 = G_3\, P_{in}^3$$

where we have obviously **defined** $G_2 \doteq B^2/4$ and $G_3 \doteq C^2/16$.

We know that typically, G_2 and G_3 are much **smaller** than G. Thus, we are **tempted** to say that P_1^{out} is much **larger** than P_2^{out} or P_3^{out}.

But we might be **wrong** !



Look **closely** at the expressions for the output power of the first, second, and third order products:

$$
\begin{aligned}
P_1^{out} &= G\ P_{in} \\
P_2^{out} &= G_2\ P_{in}^{2} \\
P_3^{out} &= G_3\ P_{in}^{3}
\end{aligned}
$$

This **first** order output power is of course **directly** proportional to the input power. However, the **second** order output power is proportional to the input power **squared**, while the **third** order output is proportional to the input power **cubed**!

Thus we find that if the input power is **small**, the second and third order products **are** insignificant. But, as the input power **increases**, the second and third order products get **big** in a hurry!

For example, if we **double** the input power, the **first** order signal will of course likewise **double**. However, the **second** order power will **quadruple**, while the **third** order power will increase **8 times**.

For **large** input powers, the second and third order products can in fact be **almost as large** as the first order signal!

Perhaps this can be most easily seen by expressing the above equations in **decibels**:

$$P_1^{out}(dBm) = G(dB) + P_{in}(dBm)$$
$$P_2^{out}(dBm) = G_2(dB) + 2\left[P_{in}(dBm)\right]$$
$$P_3^{out}(dBm) = G_3(dB) + 3\left[P_{in}(dBm)\right]$$

where we have used the fact that $\log x^n = n \log x$.

Note the value $2\left[P_{in}(dBm)\right]$ does **not** mean the value $2P_{in}$ expressed in decibels. The value $2\left[P_{in}(dBm)\right]$ is fact the value of P_{in} expressed in decibels—**times two**!

For **example**, if $P_{in}(dBm) = -30\,dBm$, then $2\left[P_{in}(dBm)\right] = -60\,dBm$. Likewise, if $P_{in}(dBm) = 20\,dBm$, then $2\left[P_{in}(dBm)\right] = 40\,dBm$.

What this means is that for every **1dB** increase in **input** power P_{in} the fundamental (**first-order**) signal will increase **1dB**; the **second-order** power will increase **2dB**; and the **third-order** power will increase **3dB**.

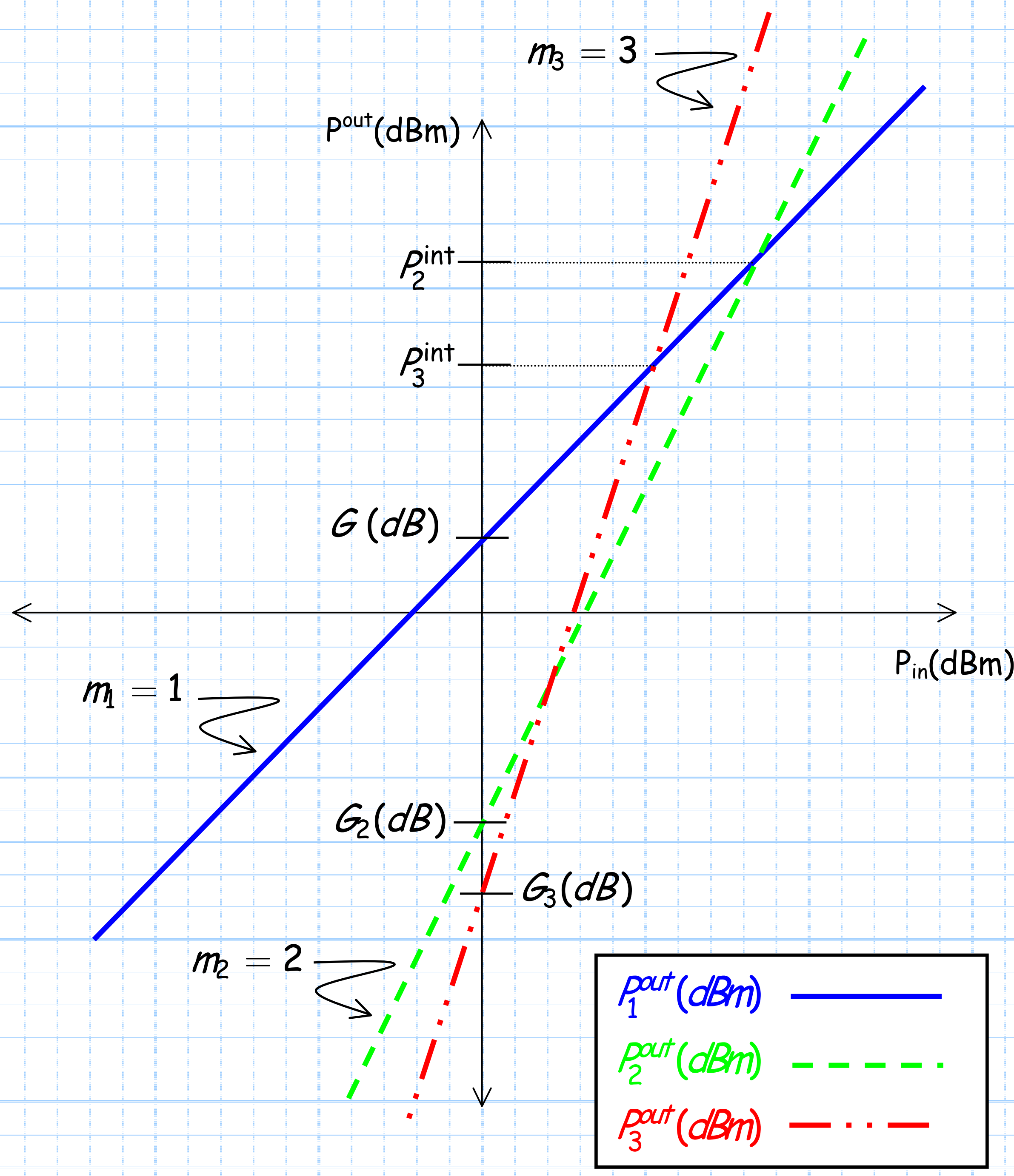
This is evident when we look at the three power equations (in decibels), as each is an equation of a **line** (i.e., $y = mx + b$).

For example, the equation:

$$P_3^{out}(dBm) = 3\left[P_{in}(dBm)\right] + G_3(dB)$$
$$y = m\,x + b$$

describes a line with **slope** $m = 3$ and "y **intercept**" $b = G_3(dB)$ (where $x = P_{in}(dBm)$ and $y = P^{out}(dBm)$).

Plotting each of the three equations for a typical amplifier, we would get something that looks like this:



Note that for $P_{in}(dBm) < 0\,dBm$ (the left side of the plot), the second and third-order products are small compared to the fundamental (first-order) signal.

However, when the input power increases **beyond** 0 dBm (the right side of the plot), the second and third order products rapidly **catch up**! In fact, they will (theoretically) become **equal** to the first order product at some large input power.

The point at which each higher order product **equals** the first-order signal is defined as the **intercept point**. Thus, we define the **second order intercept** point as the output power **when**:

$$P_2^{out} = P_1^{out} \doteq P_2^{int} \quad Second\text{ - }order\ intercept\ power$$

Likewise, the **third order intercept** point is defined as the third-order output power **when**:

$$P_3^{out} = P_1^{out} \doteq P_3^{int} \quad Third\text{ - }order\ intercept\ power$$

Using a little algebra **you** can show that:

$$P_2^{int} = \frac{G^2}{G_2} \qquad \text{and} \qquad P_3^{int} = \sqrt{\frac{G^3}{G_3}}$$

or, expressed in **decibels**:

$$P_2^{int}(dBm) = 2\,G(dB) - G_2(dB)$$

$$P_3^{int}(dBm) = \frac{3\,G(dB) - G_3(dB)}{2}$$

* Radio engineers specify the intermodulation distortion performance of a specific amplifier in terms of the **intercept points**, rather than values G_2 and G_3.

* Generally, only the **third-order** intercept point is provided by amplifier manufactures (we'll see in a moment why).

* **Typical** values of P_3^{int} for a **small-signal** amplifier range from +20 dBm to +50 dBm

* Note that as G_2 and G_3 **decrease**, the intercept points **increase**.

Therefore, the **higher** the intercept point of an amplifier, the better the amplifier !

One other **important point**: the **intercept points** for most amplifiers are much **larger** than the **compression point**! I.E.,:

$$P^{\text{int}} > P_{1dB}$$

In other words the intercept points are "theoretical", in that we can **never**, in fact, increase the input power to the point that the higher order signals are **equal** to the fundamental signal power.

All signals, including the higher order signals, have a **maximum** limit that is determined by the amplifier **power supply**.