The Directional Coupler

A lossless, reciprocal, matched 4-port coupler will have a scattering matrix of the form:

$$\bar{\bar{\mathbf{S}}} = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

This ideal coupler is **completely** characterized by the **coupling coefficient** c, where we find:

$$\bar{\bar{\mathbf{S}}} = \begin{bmatrix} 0 & \sqrt{1-c^2} & jc & 0 \\ \sqrt{1-c^2} & 0 & 0 & jc \\ jc & 0 & 0 & \sqrt{1-c^2} \\ 0 & jc & \sqrt{1-c^2} & 0 \end{bmatrix}$$

In other words:

$$\beta = c \qquad \text{and} \qquad \alpha = \sqrt{1-\beta^2} = \sqrt{1-c^2}$$

Additionally, for a directional coupler, the coupling coefficient c will be less than $1/\sqrt{2}$ **always**. Therefore, we find that:

$$0 \leq c \leq \frac{1}{\sqrt{2}} \qquad \text{and} \qquad \frac{1}{\sqrt{2}} \leq \sqrt{1-c^2} \leq 1$$

Lets see what this means in terms of the **physical behavior** of a directional coupler. First, consider the case where some signal is incident on **port 1**, with power P_1^+.

If all other ports are matched, we find that the power flowing out of **port 1** is:

$$P_1^- = |S_{11}|^2 P_1^+ = 0$$

while the power out of **port 2** is:

$$P_2^- = |S_{21}|^2 P_1^+ = \left(1 - c^2\right) P_1^+$$

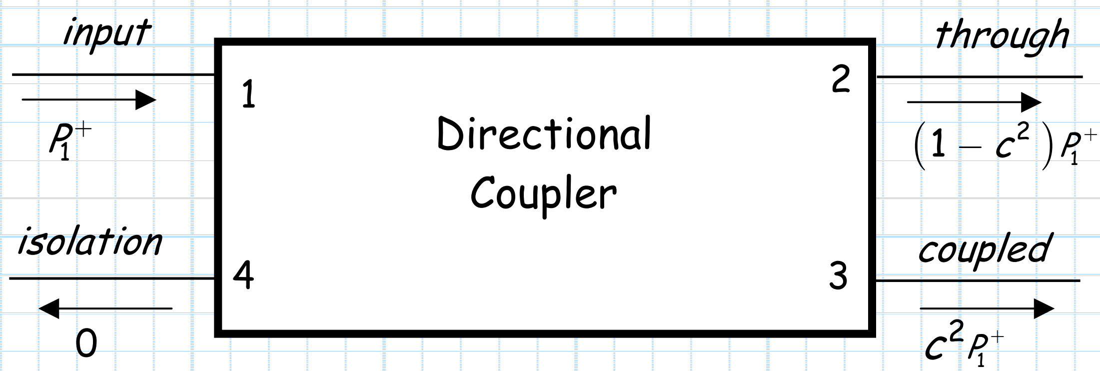
and the power out of **port 3** is:

$$P_3^- = |S_{31}|^2 P_1^+ = c^2 P_1^+$$

Finally, we find there is **no power** flowing out of **port 4**:

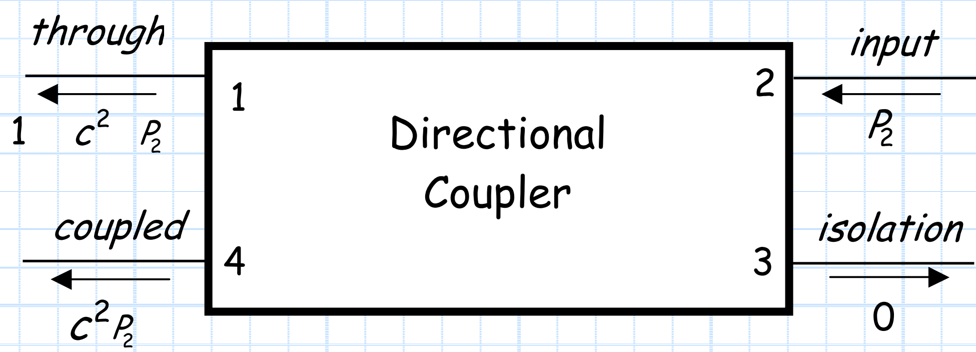
$$P_4 \qquad |S_{41}|^2 P_1 \qquad 0$$

In the terminology of the directional coupler, we say that port 1 is the **input** port, port 2 is the **through** port, port 3 is the **coupled** port, and port 4 is the **isolation** port.



Note however, that **any** of the coupler ports can be an input, with a different through, coupled and isolation port for each case.

For example, **if** a signal is incident on **port 2**, while all other ports are matched, we find that:



Thus, from the scattering matrix of a directional coupler, we can form the following table:

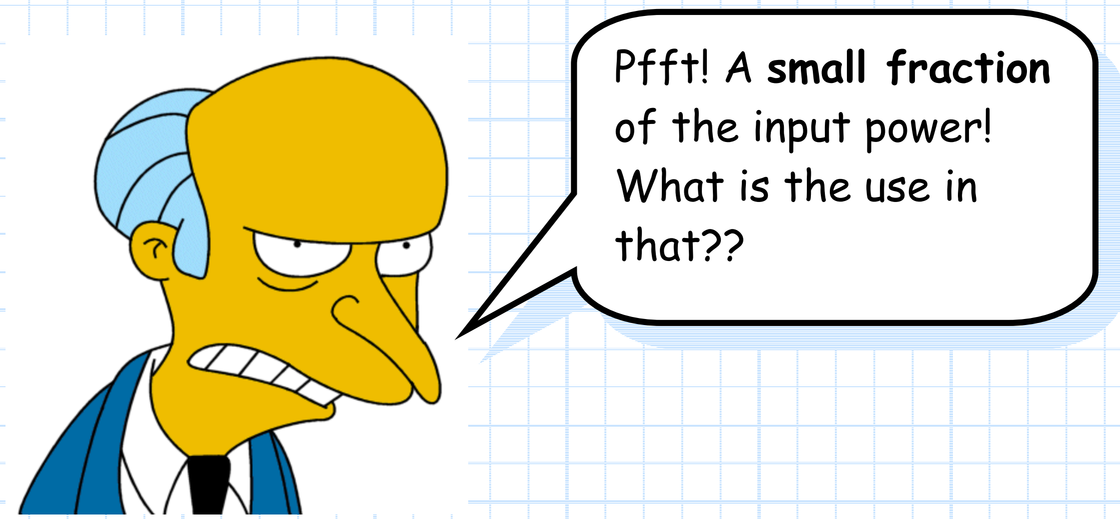
Input	*Through*	*Coupled*	*Isolation*
Port 1	Port 2	Port 3	Port 4
Port 2	Port 1	Port 4	Port 3
Port 3	Port 4	Port 1	Port 2
Port 4	Port 3	Port 2	Port 1

Typically, the coupling coefficients for a directional coupler are in the range of approximately:

$$0.25 \qquad c^2 \qquad 0.0001$$

As a result, we find that $\sqrt{1-c^2} \approx 1$. What this means is that the power out of the **through** port is just **slightly smaller** (typically) than the power incident on the input port.

Likewise, the power out of the **coupling** port is typically a **small fraction** of the power incident on the input port.



A directional coupler is often used for **sampling** a small portion of the signal power. For example, we might **measure** the output power of the **coupled** port (e.g., P_3^-) and then we can determine the amount of signal power flowing through the device (e.g., $P_1^+ = P_3^- / c^2$)

Unfortunately, the **ideal** directional coupler **cannot** be built! For example, the input match is never **perfect**, so that the diagonal elements of the scattering matrix, although **very small**, are not zero.

Likewise, the isolation port is never **perfectly** isolated, so that the values S_{41}, S_{32}, S_{23} and S_{14} are also non-zero—some **small** amount of power leaks out!

As a result, the through port will be **slightly less** than the value $\sqrt{1-c^2}$. The scattering matrix for a non-ideal coupler would therefore be:

$$\bar{\bar{\mathbf{S}}} = \begin{bmatrix} S_{11} & S_{21} & jc & S_{41} \\ S_{21} & S_{11} & S_{41} & jc \\ jc & S_{41} & S_{11} & S_{21} \\ S_{41} & jc & S_{21} & S_{11} \end{bmatrix}$$

From this scattering matrix, we can extract **some important parameters** about directional couplers:

<u>Coupling C</u>

The coupling value is the ratio of the coupled output power to the input power, in dB:

$$C = 10\log_{10}\left[\frac{P_1^+}{P_3^-}\right] = -10\log_{10}\left[c^2\right]$$

This is the **primary** specification of a directional coupler!

Directivity D

The directivity is the ratio of the power out of the coupling port to the power out of the isolation port, in dB. This value indicates how effective the device is in "**directing**" the coupled energy into the correct port. The **higher** the directivity, the better.

$$D = 10\log_{10}\left[\frac{P_3^-}{P_4^-}\right] = 10\log_{10}\left[\frac{c^2}{|S_{41}|^2}\right]$$

Isolation I

Isolation is the ratio of the input power to the power out of the isolation port, in dB. This value indicates how "isolated" the isolation port actually is. The **higher** the isolation, the better.

$$I = 10\log_{10}\left[\frac{P_1^+}{P_4^-}\right] = -10\log_{10}\left[|S_{41}|^2\right]$$

Mainline Loss ML

The mainline loss is the ratio of the input power to the power out of the through port, in dB. It indicates how much power the signal **loses** as it travels from the input to the through port.

$$ML = 10\log_{10}\left[\frac{P_1^+}{P_2^-}\right] = -10\log_{10}\left[|S_{21}|^2\right]$$

Coupling Loss ML

The coupling loss indicates the portion of the mainline loss that is due to coupling some of the input power into the coupling port. This loss is **unavoidable**.

$$CL = -10\log_{10}\left[1 - c^2\right]$$

Insertion Loss IL

The coupling loss indicates the portion of the mainline loss that is **not** due to coupling some of the input power into the coupling port. This loss **is** avoidable, and thus the **smaller** the insertion loss, the better.

$$IL = ML - CL$$