The Directional Coupler

A lossless, reciprocal, matched 4-port coupler will have a scattering matrix of the form:

\[
\begin{bmatrix}
0 & \alpha & j\beta & 0 \\
\alpha & 0 & 0 & j\beta \\
j\beta & 0 & 0 & \alpha \\
0 & j\beta & \alpha & 0
\end{bmatrix}
\]

This ideal coupler is completely characterized by the coupling coefficient \( c \), where we find:

\[
\begin{bmatrix}
0 & \sqrt{1-c^2} & jc & 0 \\
\sqrt{1-c^2} & 0 & 0 & jc \\
jc & 0 & 0 & \sqrt{1-c^2} \\
0 & jc & \sqrt{1-c^2} & 0
\end{bmatrix}
\]

In other words:

\[
\beta = c \quad \text{and} \quad \alpha = \sqrt{1-\beta^2} = \sqrt{1-c^2}
\]

Additionally, for a directional coupler, the coupling coefficient \( c \) will be less than \( 1/\sqrt{2} \) always. Therefore, we find that:

\[
0 \leq c \leq \frac{1}{\sqrt{2}} \quad \text{and} \quad \frac{1}{\sqrt{2}} \leq \sqrt{1-c^2} \leq 1
\]
Let's see what this means in terms of the physical behavior of a directional coupler. First, consider the case where some signal is incident on port 1, with power $P_1^+$. If all other ports are matched, we find that the power flowing out of port 1 is:

$$R_1^- = |S_{11}|^2 P_1^+ = 0$$

while the power out of port 2 is:

$$P_2^- = |S_{21}|^2 P_1^+ = (1 - c^2) P_1^+$$

and the power out of port 3 is:

$$P_3^- = |S_{31}|^2 P_1^+ = c^2 P_1^+$$

Finally, we find there is no power flowing out of port 4:

$$P_4^- = |S_{41}|^2 P_1^- = 0$$

In the terminology of the directional coupler, we say that port 1 is the input port, port 2 is the through port, port 3 is the coupled port, and port 4 is the isolation port.
Note however, that any of the coupler ports can be an input, with a different through, coupled and isolation port for each case.

For example, if a signal is incident on port 2, while all other ports are matched, we find that:

\[
\begin{align*}
\text{through} & : 1 \quad c^2 \quad P_2 \\
\text{coupled} & : c^2 P_2
\end{align*}
\]

Thus, from the scattering matrix of a directional coupler, we can form the following table:

<table>
<thead>
<tr>
<th>Input</th>
<th>Through</th>
<th>Coupled</th>
<th>Isolation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Port 1</td>
<td>Port 2</td>
<td>Port 3</td>
<td>Port 4</td>
</tr>
<tr>
<td>Port 2</td>
<td>Port 1</td>
<td>Port 4</td>
<td>Port 3</td>
</tr>
<tr>
<td>Port 3</td>
<td>Port 4</td>
<td>Port 1</td>
<td>Port 2</td>
</tr>
<tr>
<td>Port 4</td>
<td>Port 3</td>
<td>Port 2</td>
<td>Port 1</td>
</tr>
</tbody>
</table>

Typically, the coupling coefficients for a directional coupler are in the range of approximately:

\[0.25 \quad c^2 \quad 0.0001\]
As a result, we find that $\sqrt{1 - c^2} \approx 1$. What this means is that the power out of the through port is just slightly smaller (typically) than the power incident on the input port.

Likewise, the power out of the coupling port is typically a small fraction of the power incident on the input port.

A directional coupler is often used for sampling a small portion of the signal power. For example, we might measure the output power of the coupled port (e.g., $P_3^-$) and then we can determine the amount of signal power flowing through the device (e.g., $P_1^+ = P_3^- / c^2$)

Unfortunately, the ideal directional coupler cannot be built! For example, the input match is never perfect, so that the diagonal elements of the scattering matrix, although very small, are not zero.
Likewise, the isolation port is never perfectly isolated, so that the values $S_{41}$, $S_{32}$, $S_{23}$ and $S_{14}$ are also non-zero—some small amount of power leaks out!

As a result, the through port will be slightly less than the value $\sqrt{1-c^2}$. The scattering matrix for a non-ideal coupler would therefore be:

$$
\mathbf{S} = \begin{bmatrix}
S_{11} & S_{21} & jc & S_{41} \\
S_{21} & S_{11} & S_{41} & jc \\
jc & S_{41} & S_{11} & S_{21} \\
S_{41} & jc & S_{21} & S_{11}
\end{bmatrix}
$$

From this scattering matrix, we can extract some important parameters about directional couplers:

**Coupling $C$**

The coupling value is the ratio of the coupled output power to the input power, in dB:

$$
C = 10 \log_{10} \left( \frac{P_1^+}{P_3^-} \right) = -10 \log_{10} \left( c^2 \right)
$$

This is the primary specification of a directional coupler!
Directivity $D$

The directivity is the ratio of the power out of the coupling port to the power out of the isolation port, in dB. This value indicates how effective the device is in "directing" the coupled energy into the correct port. The higher the directivity, the better.

$$D = 10\log_{10} \frac{P_3^-}{P_4^-} = 10\log_{10} \left( \frac{c^2}{|S_{41}|^2} \right)$$

Isolation $I$

Isolation is the ratio of the input power to the power out of the isolation port, in dB. This value indicates how “isolated” the isolation port actually is. The higher the isolation, the better.

$$I = 10\log_{10} \frac{P_4^+}{P_4^-} = -10\log_{10} \left( |S_{41}|^2 \right)$$

Mainline Loss $ML$

The mainline loss is the ratio of the input power to the power out of the through port, in dB. It indicates how much power the signal loses as it travels from the input to the through port.

$$ML = 10\log_{10} \frac{P_1^+}{P_2^-} = -10\log_{10} \left( |S_{21}|^2 \right)$$
**Coupling Loss ML**

The coupling loss indicates the portion of the mainline loss that is due to coupling some of the input power into the coupling port. This loss is **unavoidable**.

\[ CL = -10 \log_{10} \left( 1 - c^2 \right) \]

**Insertion Loss IL**

The coupling loss indicates the portion of the mainline loss that is **not** due to coupling some of the input power into the coupling port. This loss is **avoidable**, and thus the **smaller** the insertion loss, the better.

\[ IL = ML - CL \]