The Lossless Transmission Line

If a transmission line is lossless (i.e., \(R=G=0\)), the transmission line equations are significantly simplified!

**Characteristic Impedance**

\[
Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \\
= \frac{j\omega L}{\sqrt{j\omega C}} \\
= \sqrt{\frac{L}{C}}
\]

Note the characteristic impedance of a lossless transmission line is purely real (i.e., \(\text{Im}(Z_0) = 0\))!

**Propagation Constant**

\[
\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \\
= \sqrt{(j\omega L)(j\omega C)} \\
= \sqrt{-\omega^2 LC} \\
= j\omega\sqrt{LC}
\]

The wave propagation constant is purely imaginary!
In other words, for a lossless transmission line:

$$\alpha = 0 \quad \text{and} \quad \beta = \omega \sqrt{LC}$$

**Voltage and Current**

The complex functions describing the magnitude and phase of the voltage/current at every location $z$ along a transmission line are for a lossless line are:

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}$$

**Line Impedance**

The complex function describing the impedance at every point along a lossless transmission line is:

$$Z(z) = \frac{V(z)}{I(z)} = Z_0 \cdot \frac{V_0^+ e^{-j\beta z} + \frac{V_0^- e^{j\beta z}}{V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z}}}$$

**Wavelength and Phase Velocity**

We can now explicitly write the wavelength and propagation velocity of the two transmission line waves in terms of transmission line parameters $L$ and $C$: 
\[ \lambda = \frac{2\pi}{\beta} = \frac{1}{f\sqrt{LC}} \]

\[ \nu_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \]

**Q:** Oh please, continue wasting my valuable time. We both know that a **perfectly lossless transmission line** is a physical **impossibility.**

**A:** True! However, a **low-loss** line is possible. If \( R \ll \omega L \) and \( G \ll \omega C \), we find that the lossless transmission line equations are excellent approximations!

Unless otherwise indicated, we will use the lossless equations to approximate the behavior of a **low-loss** transmission line.

The lone exception is when determining the attenuation of a **long** transmission line. For that case we will use the approximation:

\[ \alpha \approx \frac{1}{2} \left( \frac{R}{Z_0} + GZ_0 \right) \]

where:

\[ Z_0 = \sqrt{\frac{L}{C}} \]