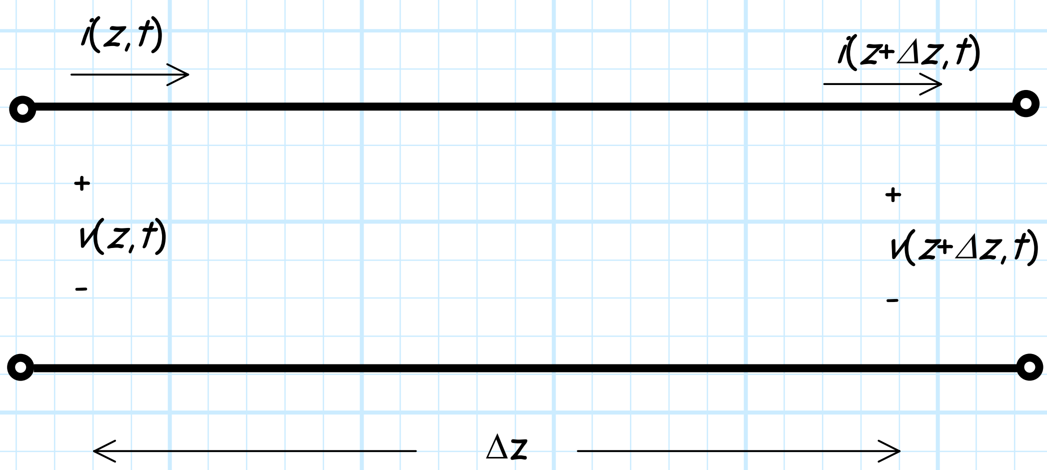
The Telegrapher Equations

Consider a section of "wire":



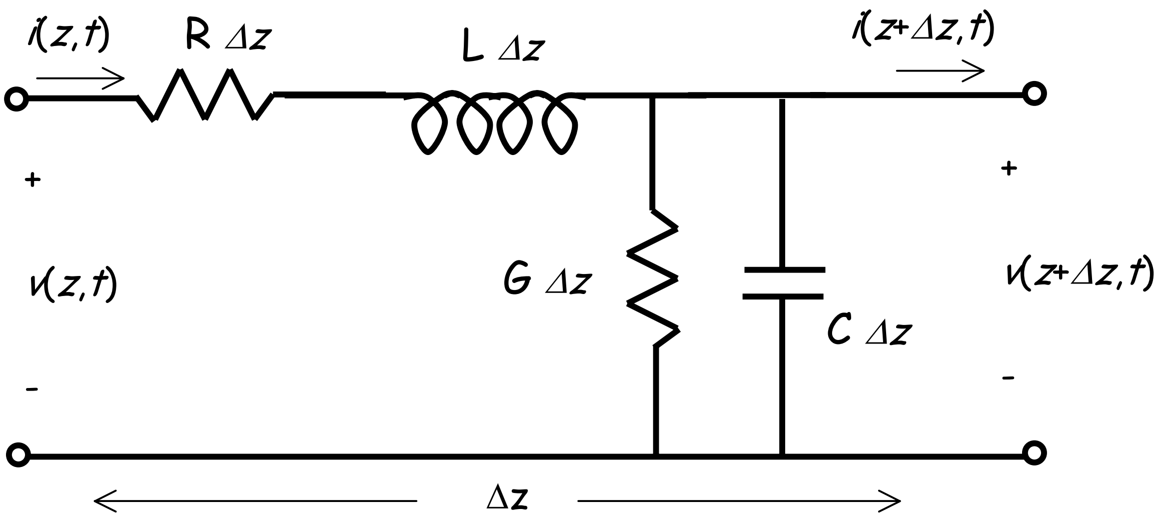
Q: *Huh ?! Current i and voltage v are a function of* ***position*** *z ??* *Shouldn't* $i(z,t)=i(z+\Delta z,t)$ *and* $v(z,t)=v(z+\Delta z,t)$ *?*

A: NO ! Because a wire is never a **perfect** conductor.

A "wire" will have:

1) Inductance
2) Resistance
3) Capacitance
4) Conductance

i.e.,



Where:

R = resistance/unit length
L = inductance/unit length
C = capacitance/unit length
G = conductance/unit length

∴ resistance of wire length Δz is $R\Delta z$.

Using KVL, we find:

$$v(z+\Delta z,t)-v(z,t)=-R\Delta z\, i(z,t)-L\Delta z\frac{\partial i(z,t)}{\partial t}$$

and from KCL:

$$i(z+\Delta z,t)-i(z,t)=-G\Delta z\, v(z,t)-C\Delta z\frac{\partial v(z,t)}{\partial t}$$

Dividing the first equation by Δz, and then taking the limit as $\Delta z \rightarrow 0$:

$$\lim_{\Delta z \rightarrow 0} \frac{v(z+\Delta z,t)-v(z,t)}{\Delta z} = -R\, i(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

which, by definition of the derivative, becomes:

$$\frac{\partial v(z,t)}{\partial z} = -R\, i(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

Similarly, the KCL equation becomes:

$$\frac{\partial i(z,t)}{\partial z} = -G\, v(z,t) - C\frac{\partial v(z,t)}{\partial t}$$

If $v(z,t)$ and $i(z,t)$ have the form:

$$v(z,t) = \mathrm{Re}\left\{V(z)e^{j\omega t}\right\} \quad \text{and} \quad i(z,t) = \mathrm{Re}\left\{I(z)e^{j\omega t}\right\}$$

then these equations become:

$$\boxed{\begin{aligned} \frac{\partial V(z)}{\partial z} &= -(R+j\omega L)I(z) \\ \frac{\partial I(z)}{\partial z} &= -(G+j\omega C)V(z) \end{aligned}}$$

These equations are known as the **telegrapher's equations** !

* The functions *I(z)* and *V(z)* are **complex,** where the **magnitude** and **phase** of the complex functions describe the **magnitude** and **phase** of the sinusoidal time function $e^{j\omega t}$.

* Thus, *I(z)* and *V(z)* describe the current and voltage along the transmission line, as a function as position *z*.

* **Remember**, not just **any** function *I(z)* and *V(z)* can exist on a transmission line, but rather **only** those functions that satisfy the **telegraphers equations**.

Our task, therefore, is to **solve** the telegrapher equations and find **all** solutions $I(z)$ and $V(z)$!