Two-Tone Intermodulation

Q: It doesn't seem to me that this **dad-gum** intermodulation distortion is really that much of a problem.

I mean, the first and second **harmonics** will likely be well **outside** the amplifier bandwidth, right?

A: True, the **harmonics** produced by intermodulation distortion typically are **not** a problem in radio system design. There is a problem, however, that is **much worse** than **harmonic** distortion!

This problem is called **two-tone** intermodulation distortion.

Say the input to an amplifier consists of two signals at dissimilar frequencies:

$$v_{in} = a\cos\omega_1 t + a\cos\omega_2 t$$

Here we will assume that both frequencies ω_1 and ω_2 are within the **bandwidth** of the amplifier, but are **not** equal to each other $(\omega_1 = \omega_2)$.

This of course is a much more **realistic** case, as typically there will be **multiple** signals at the input to an amplifier!

For example, the two signals considered here could represent two **FM radio stations**, operating at frequencies within the FM band (i.e., $88.1 \text{ MHz} \leq f_1 \leq 108.1 \text{ MHz}$ and $88.1 \text{ MHz} \leq f_2 \leq 108.1 \text{ MHz}$).

My point exactly! Intermodulation distortion will produce those **dog-gone** second-order products:

$$\frac{a^2}{2}\cos 2\omega_1 t \quad \text{and} \quad \frac{a^2}{2}\cos 2\omega_2 t$$

and **gul-durn** third order products:

$$\frac{a^3}{4}\cos 3\omega_1 t \quad \text{and} \quad \frac{a^3}{4}\cos 3\omega_2 t$$

but these harmonic signals will lie well **outside** the FM band!

True! Again, the **harmonic** signals are **not** the problem. The problem occurs when the **two input** signals combine together to form **additional** second and third order products.

Recall an amplifier output is accurately described as:

$$v_{out} = A_v\, v_{in} + B\, v_{in}^2 + C\, v_{in}^3 + \cdots$$

Consider first the **second-order** term if **two** signals are at the input to the amplifier:

$$\begin{aligned} v_2^{out} &= B\, v_{in}^2 \\ &= B\left(a\cos\omega_1 t + a\cos\omega_2 t\right)^2 \\ &= B\left(a^2\cos^2\omega_1 t + 2a^2\cos\omega_1 t\ \cos\omega_2 t + a^2\cos^2\omega_2 t\right) \end{aligned}$$

Note the first and third terms of the above expression are **precisely** the same as the terms we examined on the previous handout. They result in **harmonic** signals at frequencies $2\omega_1$ and $2\omega_2$, respectively.

The **middle** term, however, is something **new**. Note **it** involves the product of $\cos\omega_1 t$ and $\cos\omega_2 t$. Again using our knowledge of **trigonometry**, we find:

$$2a^2\cos\omega_1 t\ \cos\omega_2 t = a^2\cos(\omega_2-\omega_1)t + a^2\cos(\omega_2+\omega_1)t$$

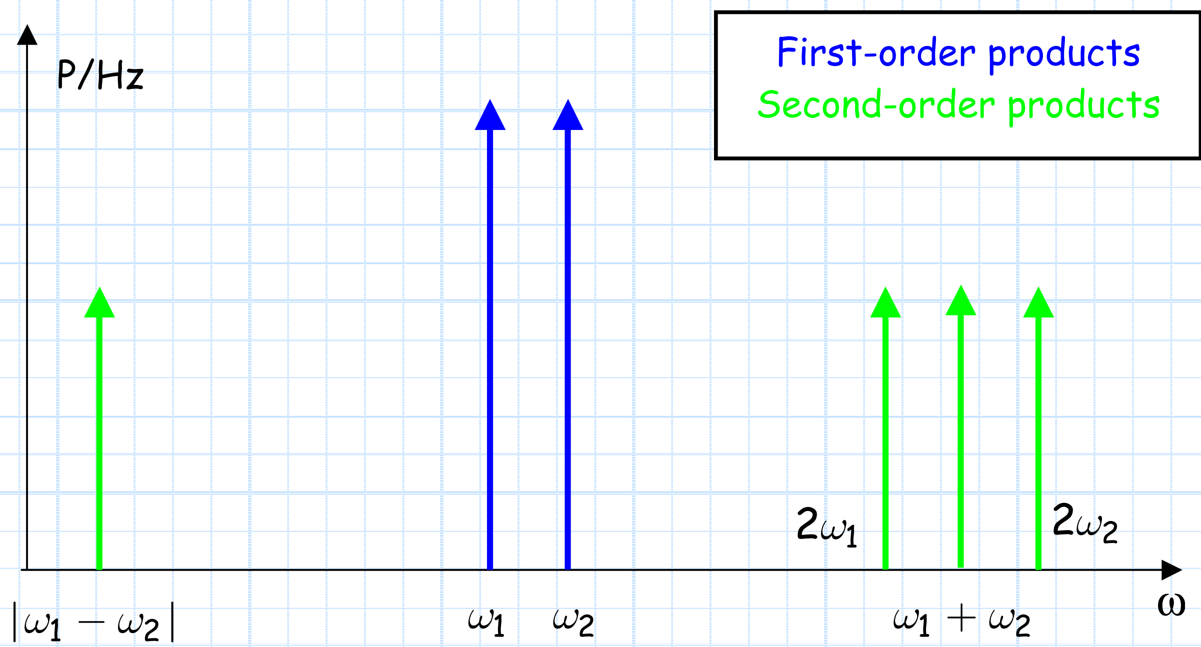
Note that since $\cos(-x) = \cos x$, we can **equivalently** write this as:

$$2a^2\cos\omega_1 t\ \cos\omega_2 t = a^2\cos(\omega_1-\omega_2)t + a^2\cos(\omega_1+\omega_2)t$$

Either way, the result is obvious—we produce **two new signals**!

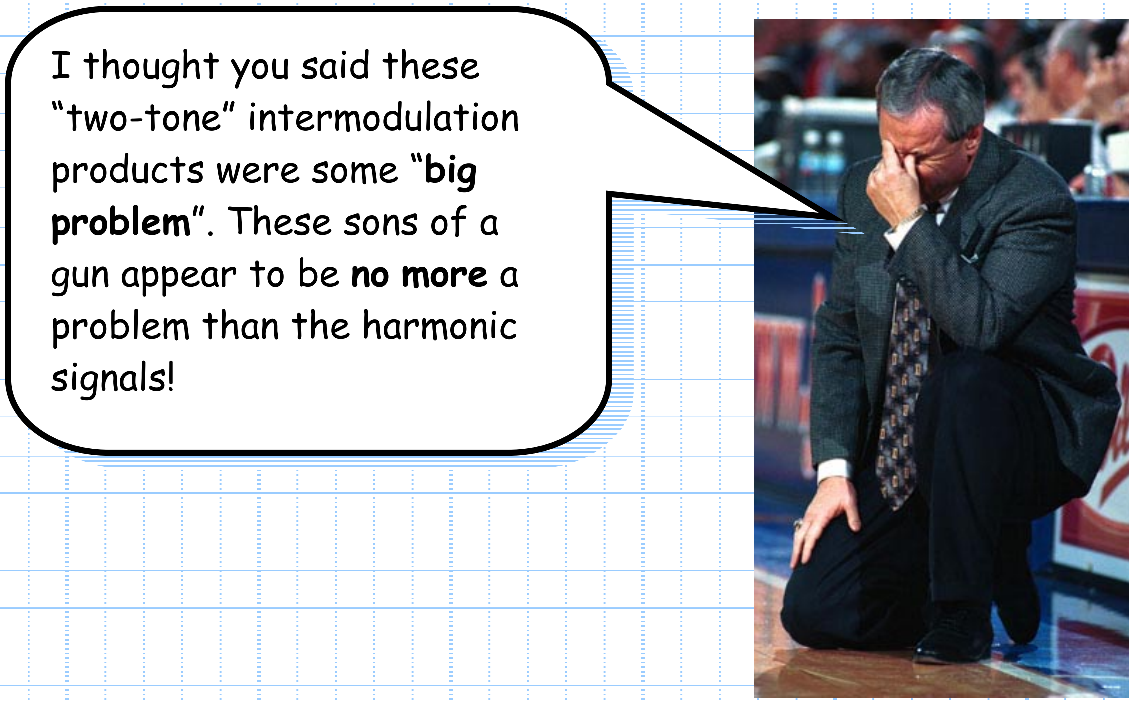
These new **second-order** signals oscillate at frequencies $(\omega_1 + \omega_2)$ and $|\omega_1 - \omega_2|$.

Thus, if we looked at the **frequency spectrum** (i.e., signal power as a function of frequency) of an amplifier **output** when two sinusoids are at the input, we would see something like this:



Note that the new terms have a frequency that is either much **higher** than both ω_1 and ω_2 (i.e., $(\omega_1 + \omega_2)$), or much **lower** than both ω_1 and ω_2 (i.e., $|\omega_1 - \omega_2|$).

Either way, these new signals will typically be **outside** the amplifier bandwidth!



This observation is indeed correct for **second**-order, two-tone intermodulation products. But, we have **yet** to examine the **third**-order terms! I.E.,

$$
\begin{aligned}
v_3^{out} &= C\, v_{in}^3 \\
&= C\left(a\cos\omega_1 t + a\cos\omega_2 t\right)^3
\end{aligned}
$$

If we multiply this all out, and again apply our trig knowledge, we find that a **bunch** of new **third-order** signals are created.

Among these signals, of course, are the second **harmonics** $\cos 3\omega_1 t$ and $\cos 3\omega_2 t$. Additionally, however, we get these **new** signals:

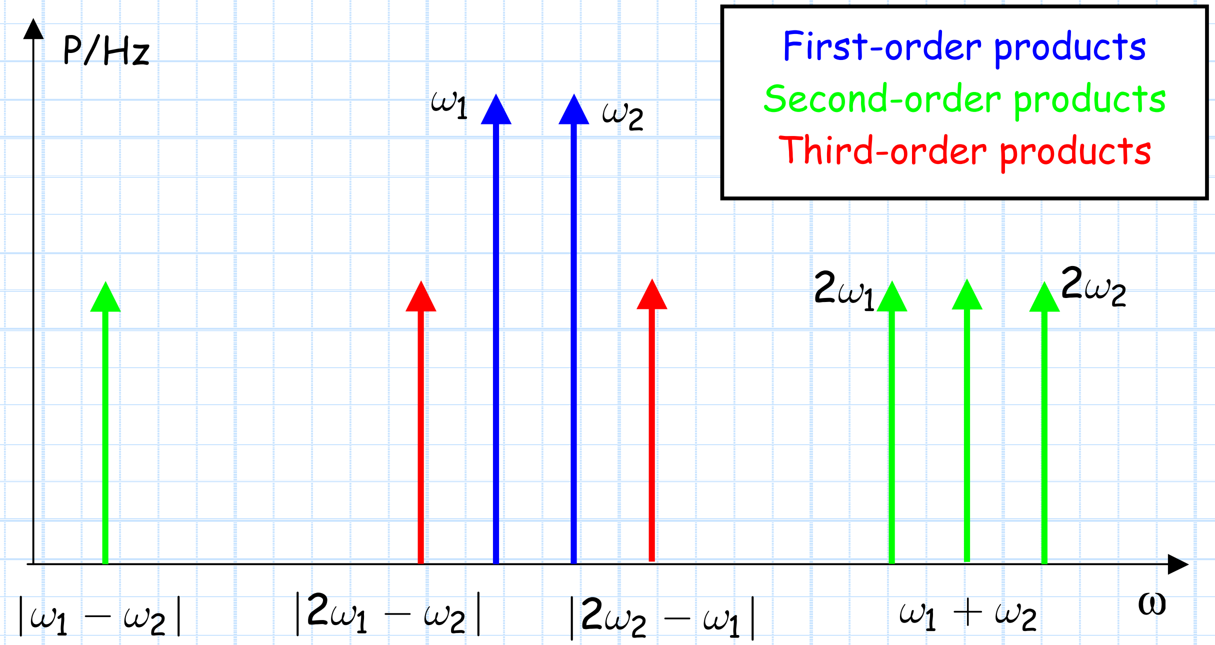
$$
\cos(2\omega_2 - \omega_1)t \quad \text{and} \quad \cos(2\omega_1 - \omega_2)t
$$

Note since $\cos(-x) = \cos x$, we can **equivalently** write these terms as:

$$\cos(\omega_1 - 2\omega_2)t \quad \text{and} \quad \cos(\omega_2 - 2\omega_1)t$$

Either way, it is apparent that the **third-order** products include signals at frequencies $|\omega_1 - 2\omega_2|$ and $|\omega_2 - 2\omega_1|$.

Now lets look at the output spectrum with **these new** third-order products included:



Now **you** should see the problem! **These** third-order products are very **close** in frequency to ω_1 and ω_2. They will likely lie **within** the bandwidth of the amplifier!

For example, if f_1=100 MHz and f_2=101 MHz, then $2f_2-f_1$=102 MHz and $2f_1-f_2$= 99 MHz. All frequencies are **well** within the FM radio bandwidth!

Thus, these **third-order**, **two-tone** intermodulation products are the **most significant** distortion terms.

This is why we are most concerned with the **third-order intercept point** of an amplifier!