dB, dBm, dBw

dB, or decibel, is a function that operates on a **unitless** parameter:

$$dB \doteq 10 \log_{10}(x)$$

where x is <u>unitless</u>!

Q: *Unitless! What good is that ! ?*

A: **Many** values are unitless, for example **ratios** and **coefficients**.

For instance, **gain** is a unitless value!

E.G., amplifier gain is the ratio of the output power to the input power:

$$\frac{P_{out}}{P_{in}} = G$$

$\therefore$ Gain in dB = $10 \log_{10} G$

Wait a minute! We've seen statements such as:

…. the output <u>power</u> is 5 dBw ….
or
…. the input <u>power</u> is 17 dBm ….

Power is **not** unitless (but Watt is the unit ?) !!

True! But look at how power is expressed; not in dB, but in **dBm** or **dBw**.

Q: What does dB<u>m</u> or dB<u>w</u> refer to ??

A: Its sort of a trick !

Say we have some power *P*. We can express *P* in dBm or dBw as:

$$dBw = 10 \log_{10}\left(P / 1\,W \right)$$

$$dBm = 10 \log_{10}\left(P / 1\,mW \right)$$

Therefore dBw and dBm express P **relative** to 1 Watt and 1 mWatt, respectively !

Note: The argument of the log_{10} function is a **ratio** (i.e., unitless).

For example,

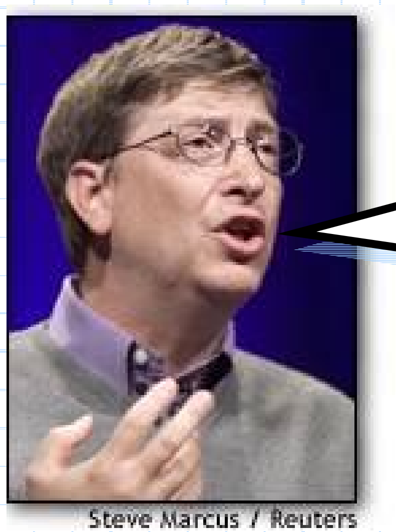
$$\left\{ \begin{array}{c} 20\ dBm \ \ means\ 100 \times 1mW = 100\ mW \\ and \\ 3\ dBw \ \ means\ 2 \times 1W = 2W \end{array} \right\}$$

Make sure you are careful when doing math with decibels!

Standard dB Values

Note that $10 \log_{10}(10) = 10$.

Therefore an amplifier with a gain $G = 10$ is said to have a gain of 10 dB.



Yes of course; then a **20 dB** gain amplifier has **G=20** and a **30 dB** gain amp has **G=30**. I comprehend all!

NO! Do not make this mistake!

Note that:

$$\boxed{\begin{aligned} 10\log_{10}\left[10^{n}\right] &= n\,10\log_{10}\left[10\right] \\ &= 10n \end{aligned}}$$

In other words, G =100 = 10^2 is equal to 20 dB, while 30 dB indicates G = 1000 = 10^3.

Likewise 100 mW is denoted as 20 dBm, and 1000 Watts is denoted as 30 dBW.

Note also that 0.001 mW = 10^{-3} mW is denoted as –30 dBm.

Another important relationship to keep in mind when using decibels is $10\log_{10}\left[2\right] \approx 3.0$. This means that:

$$\boxed{\begin{aligned} 10\log_{10}\left[2^{n}\right] &= n\,10\log_{10}\left[2\right] \\ &\simeq 3n \end{aligned}}$$

As a result, a 15 dB gain amplifier has G = 2^5 = 32, or 0.125 = 2^{-3} mW is denoted as –9 dBm.

Multiplicative Products and Decibels

Other logarithmic relationship that we will find useful are:

$$10\log_{10}[x\,y] = 10\log_{10}[x] + 10\log_{10}[y]$$

and its close cousin:

$$10\log_{10}\left[\frac{x}{y}\right] = 10\log_{10}[x] - 10\log_{10}[y]$$

Thus the relationship P_{out} = G P_{in} is written in decibels as:

$$P_{out} \quad G\,P_{in}$$

$$\frac{P_{out}}{1mW} \quad \frac{G\,P_{in}}{1mW}$$

$$10\log_{10} \frac{P_{out}}{1mW} \quad 10\log_{10} \frac{G\,P_{in}}{1mW}$$

$$10\log_{10} \frac{P_{out}}{1mW} \quad 10\log_{10} G \quad 10\log_{10} \frac{P_{in}}{1mW}$$

$$P_{out}(dBm) \quad G(dB) \quad P_{out}(dBm)$$

It is evident that "deebees" are **not** a unit! The units of the result can be found by **multiplying** the units of each term in a **summation** of decibel values.

But of course **I** am typically and impressively correct in stating that, for example:

6 dBm + 10 dBm = 16 dBm

NO! Never do this either! Logarithms are very helpful in expressing **products** or **ratios** of parameters, but they are **not** much help in expressing sums and differences!

$$10\log_{10}[x+y] = ????$$

So, if you wish to add **power** denoted as 6dBm to **power** denoted as 10 dBm, you must first **convert** back to non-decibel parameters.

10 dBm = 10 mW and 6 dBm = 4 mW

Thus the power of the sum of these two is 10 + 4 = 14 mW. Expressed in dBm, 14 mW is 11.46 dBm ($\neq$ 16 dBm).

We can mathematically add 6 dBm and 10 dBm, but we must understand what result means (nothing useful!).

$$
\begin{aligned}
6\,\text{dBm} + 10\text{ dBm} &= 10\log_{10}\left[\frac{4mW}{1mW}\right] + 10\log_{10}\left[\frac{10mW}{1mW}\right] \\
&= 10\log_{10}\left[\frac{40\ mW^2}{1mW^2}\right] \\
&= 16\text{ dB relative to 1 mW}^2
\end{aligned}
$$

Thus, mathematically speaking, 6 dBm + 10dBm implies a multiplication of power, resulting in a value with units of **Watts squared** !

A few more tidbits about decibels:

1. 1 is 0 dB

2. 0 is $-\infty$ dB

3. 5 is 7 dB (can you show why?)