D. Matching Networks

Q: *Yikes! The signal source is generally a Thevenin's equivalent of the **output** of some **useful device**, while the load impedance is generally the **input** impedance of some other **useful device**. I do **not** want to—nor typically can I—**change** these devices or **alter** their characteristics.*

*Must I then just **accept** the fact that I will achieve **suboptimum** power transfer?*

A: NOPE! **All** of the available power of the source can be delivered to the load—if we properly construct a **matching network**.

HO: Matching Networks

Q: *But in microwave circuits, a source and load are connected by a transmission line. Can we implement matching networks in transmission line circuits?*

A: *HO: Matching Networks and Transmission Lines*

Q: *Matching networks seem almost too good to be true; can we really design and construct them to provide a perfect match?*

A: It is relatively easy to provide a near perfect match at precisely **one frequency**!

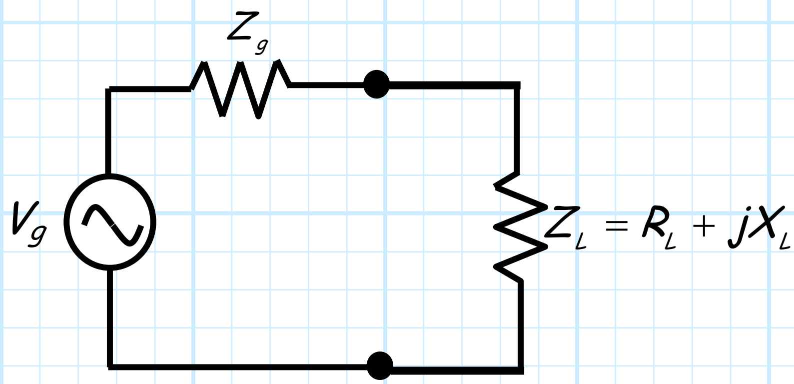
But, since lossless matching networks are made entirely of **reactive** elements (not to mention the reactive components of the source and load impedance), we find that changing the signal frequency will typically "**mismatch**" our circuit!

Thus a difficult challenge for any microwave component designer is to provide a **wideband** match to a transmission line with characteristic impedance Z_0.

→ **All** microwave components thus have a **finite** operating **bandwidth**!

Matching Networks

Consider again the problem where a **passive load** is attached to an **active source**:



The load will **absorb power**—power that is **delivered** to it by the **source**.

$$
\begin{aligned}
P_L &= \frac{1}{2} Re\left\{V_L I_L^*\right\} \\
&= \frac{1}{2} Re\left\{\left(V_g \frac{Z_L}{Z_g + Z_L}\right)\left(\frac{V_g}{Z_g + Z_L}\right)^*\right\} \\
&= \frac{1}{2}\left|V_g\right|^2 \frac{Re\left\{Z_L\right\}}{\left|Z_g + Z_L\right|^2} \\
&= \frac{1}{2}\left|V_g\right|^2 \frac{R_L}{\left|Z_g + Z_L\right|^2}
\end{aligned}
$$

Recall that the power delivered to the load will be **maximized** (for a given V_g and Z_g) **if** the load impedance is equal to the **complex conjugate** of the source impedance ($Z_L = Z_g^*$).

We call this maximum power the **available power** P_{avl} **of the source**—it is, after all, the **largest** amount of power that the source can **ever** deliver!

$$
\begin{aligned}
P_L^{max} &\doteq P_{avl} \\
&= \frac{1}{2}\left|V_g\right|^2 \frac{R_g}{\left|Z_g + Z_g^*\right|^2} \\
&= \frac{1}{2}\left|V_g\right|^2 \frac{R_g}{\left|2R_g\right|^2} \\
&= \frac{\left|V_g\right|^2}{8\,R_g}
\end{aligned}
$$

* Note the available power of the **source** is dependent on **source** parameters **only** (i.e., V_g and R_g). This makes sense! Do **you** see why?

* Thus, we can say that to "take full advantage" of all the available power of the source, we must to make the load impedance the complex conjugate of the source impedance.

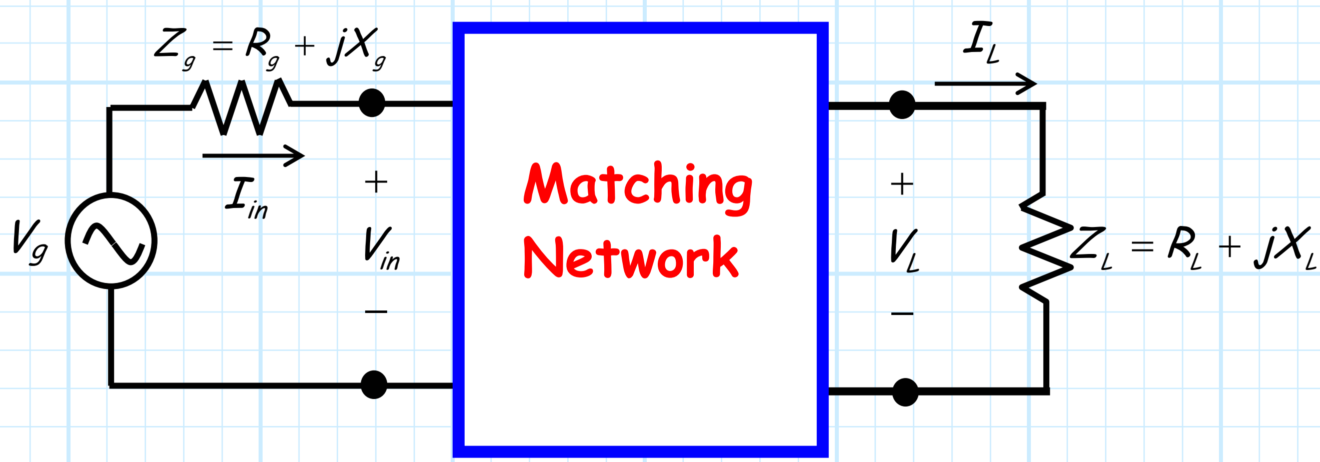
* Otherwise, the power delivered to the load will be less than power made available by the source! In other "words":

$$P_L \leq P_{avl}$$

Q: *But, you said that the load impedance typically models the input impedance of some useful device. We* ***don't*** *typically get to "select" or adjust this impedance—it* ***is*** *what it* ***is****. Must we then simply* ***accept*** *the fact that the delivered power will be less than the available power?*

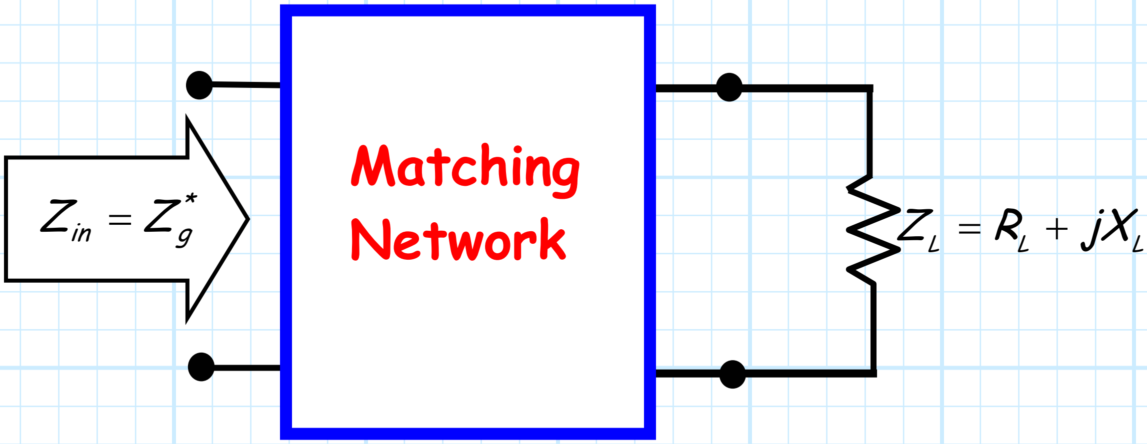
A: NO! We can in fact modify our circuit such that all available source power is delivered to the load—**without** in any way **altering** the impedance value of that **load**!

To accomplish this, we must insert a **matching network** between the source and the load:



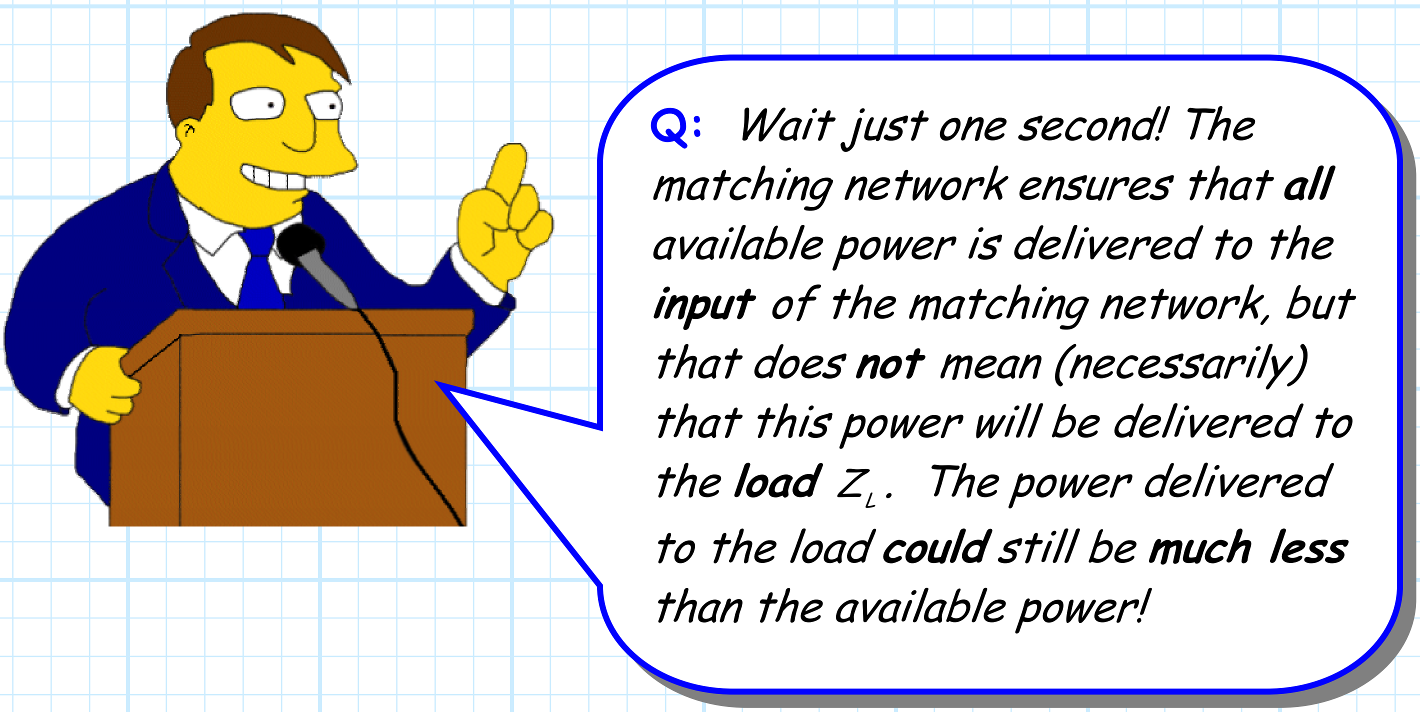
The sole purpose of this matching network is to "**transform**" the load impedance into an input impedance that **is conjugate matched** to the source! I.E.:

$$Z_{in} = \frac{V_{in}}{I_{in}} = Z_g^*$$



Because of this, **all** available source power is delivered to the input of the matching network (i.e., delivered to Z_{in}):

$$P_{in} = P_{avl}$$



A: True! To ensure that the **available power** delivered to the input of the matching network is **entirely** delivered to the **load**, we must construct our matching network such that it **cannot absorb any power**—the matching network must be **lossless**!

We must construct our matching network entirely with **reactive elements**!

Examples of reactive elements include inductors, capacitors, transformers, as well as lengths of **lossless transmission lines**.

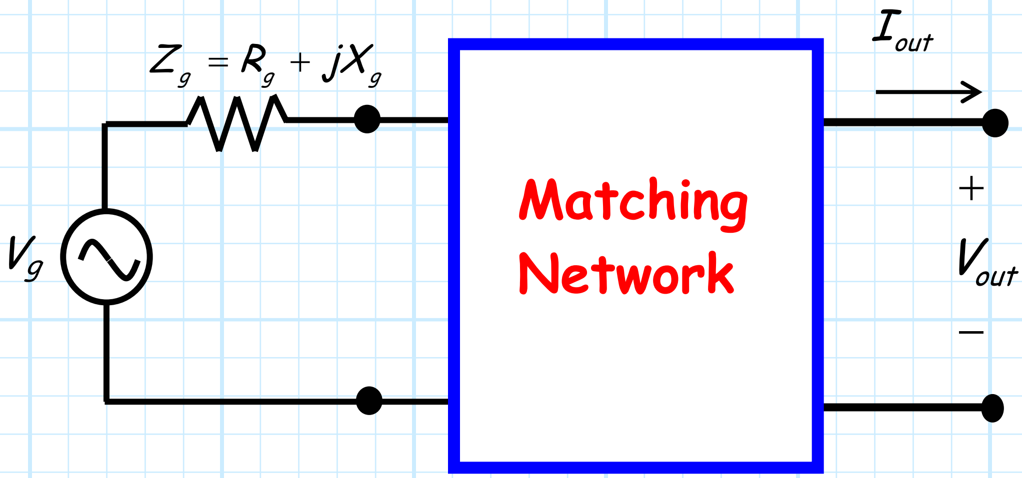
Thus, constructing a proper lossless matching network will lead to the **happy** condition where:

$$P_L = P_{in} = P_{avl}$$

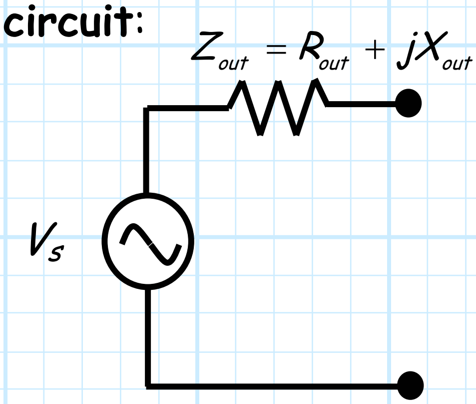
* Note that the design and construction of this lossless network will depend on **both** the value of source impedance Z_g and load impedance Z_L.

* However, the matching network does **not physically alter** the values of either of these two quantities—the source and load are left physically unchanged!

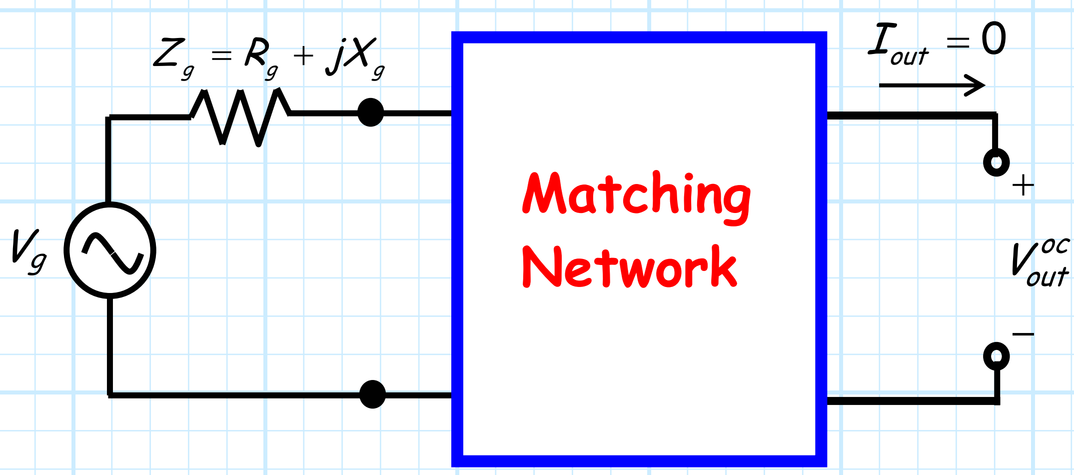
Now, let's consider the matching network from a **different perspective**. Instead of defining it in terms of its **input impedance** when attached the **load**, let's describe it in terms of its **output impedance** when attached to the **source**:



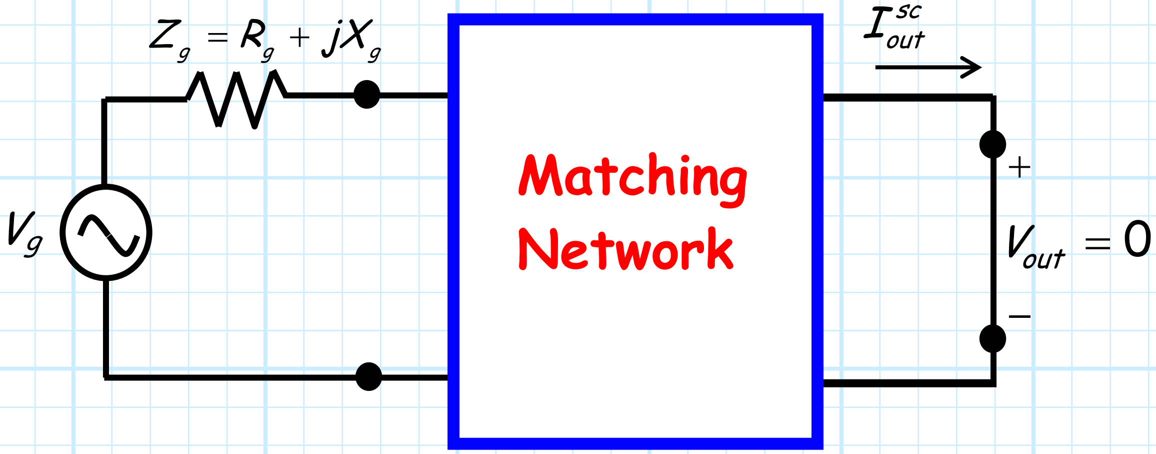
This "new" source (i.e., the original source with the matching network attached) can be expressed in terms of its **Thevenin's equivalent circuit**:



This equivalent circuit can be determined by first evaluating (or measuring) the **open-circuit** output **voltage** V_{out}^{oc} :



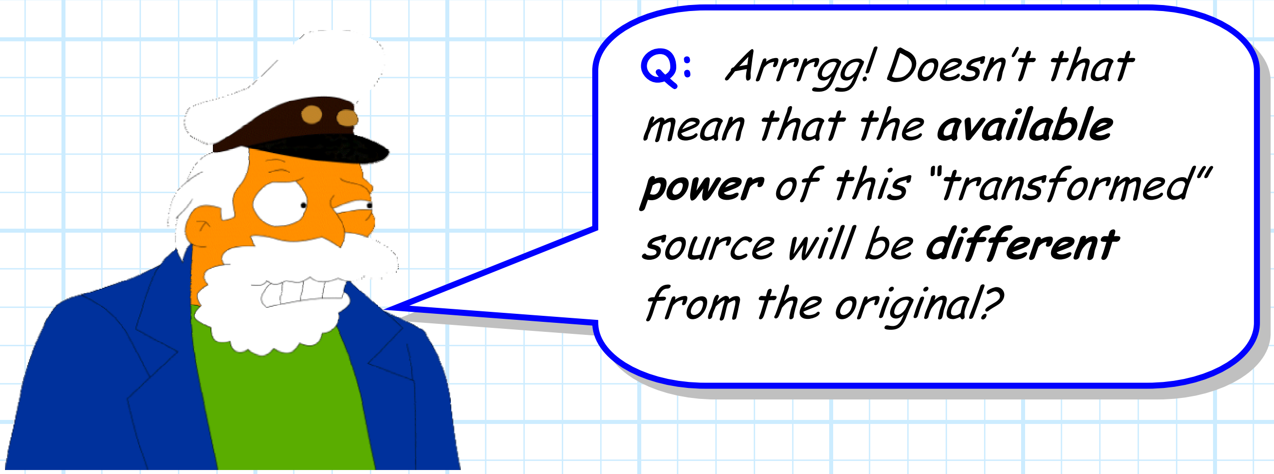
And likewise evaluating (or measuring) the **short-circuit** output **current** I_{out}^{sc}:



From these two values (V_{out}^{oc} and I_{out}^{sc}) we can determine the **Thevenin's equivalent source**:

$$V_s = V_{out}^{oc} \qquad\qquad Z_{out} = \frac{V_{out}^{oc}}{I_{out}^{sc}}$$

Note that in general that $V_s \neq V_g$ and $Z_{out} \neq Z_g$—the matching network "transforms" **both** the values of both the impedance **and** the voltage source.



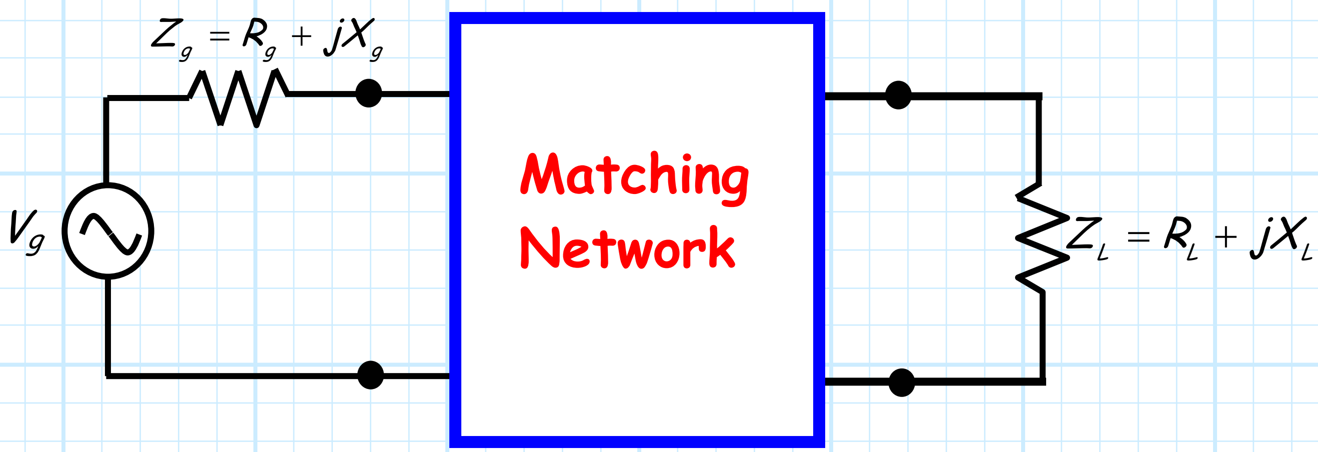
A: Nope. **If** the matching network is **lossless,** the available power of this equivalent source is **identical** to the available power of the original source—the lossless matching network does **not** alter the available power!

Now, for a **properly** designed, **lossless** matching network, it turns out that (as **you** might have expected!) the output impedance Z_{out} is equal to the complex conjugate of the load impedance. I.E.:

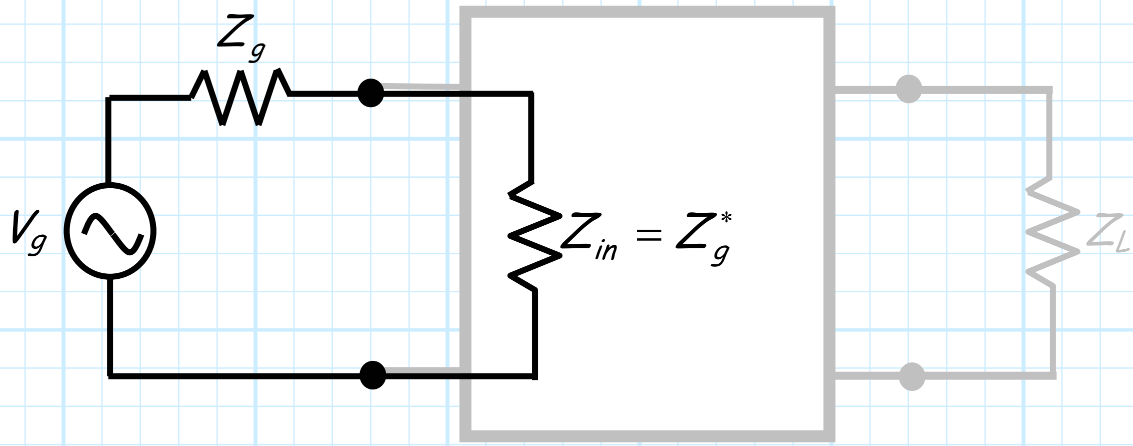
$$Z_{out} = Z_L^*$$

→ The source and load are again matched!

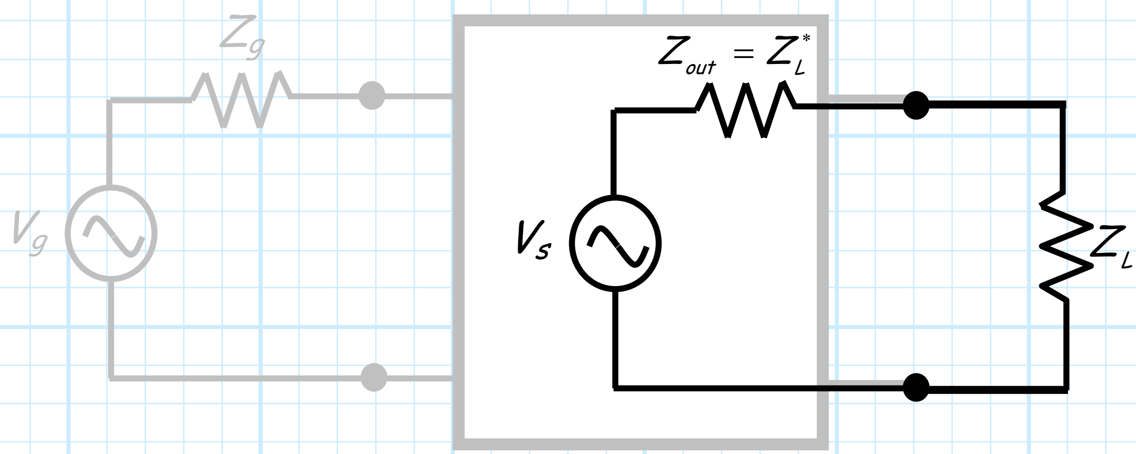
Thus, we can look at the matching network in two equivalent ways:



1. As a network attached to a load, one that "transforms" its impedance to Z_{in}—a value matched to the source impedance Z_g:



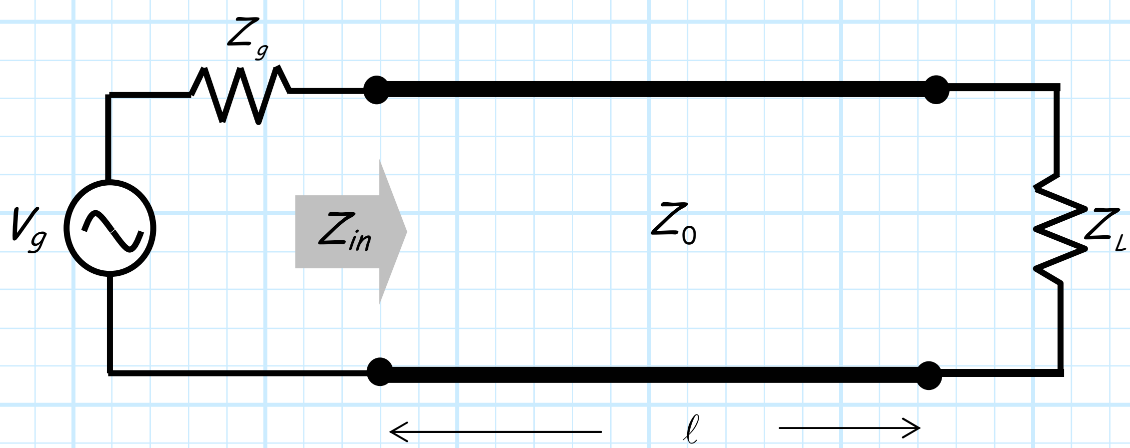
2. Or, as network attached to a source, one that "transforms" its impedance to Z_{out} —a value matched to the load impedance Z_L:



Either way, the source and load impedance are conjugate matched—**all** the available power is delivered to the load!

Matching Networks and Transmission Lines

Recall that a primary purpose of a transmission line is to allow the transfer of **power** from a source to a load.



Q: *So, say we directly connect an **arbitrary** source to an **arbitrary** load via a length of transmission line. Will the power delivered to the load be equal to the **available power** of the source?*

A: Not likely! Remember we determined earlier that the efficacy of power transfer depends on:

1. the source impedance Z_g.

2. load impedance Z_L.

3. the transmission line characteristic impedance Z_0.

4. the transmission line length ℓ.

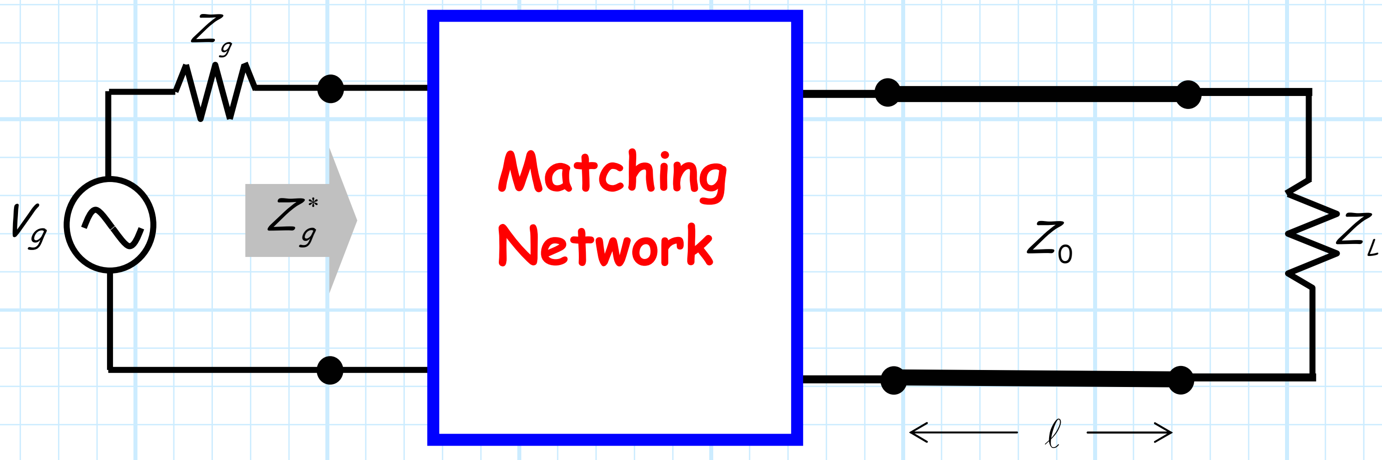
Recall that **maximum** power transfer occurred only when these four parameters resulted in the **input impedance** of the transmission line being equal to the **complex conjugate** of the **source impedance** (i.e., $Z_{in}^* = Z_g$).

It is of course **unlikely** that the very **specific** conditions of a **conjugate match** will occur if we simply connect a length of transmission line between an **arbitrary** source and load, and thus the power delivered to the load will generally be **less** than the **available power** of the source.

Q: *Is there any way to use a **matching network** to fix this problem? Can the power delivered to the load be increased to **equal** the available power of the source if there is a transmission line connecting them?*

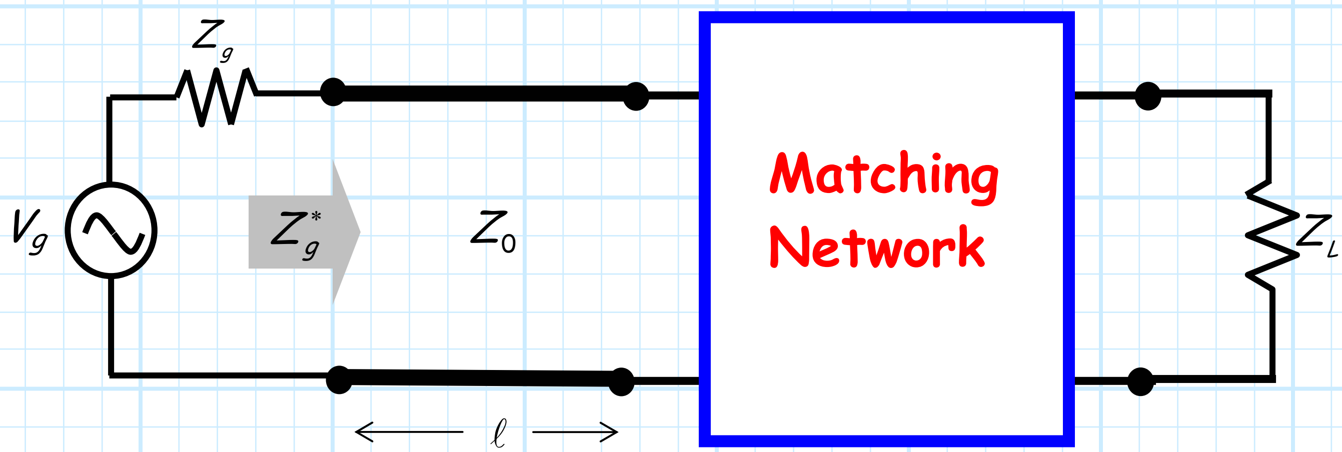
A: There sure is! We **can** likewise construct a matching network for the case where the source and load are connected by a **transmission line**.

For example, we can construct a network to transform the **input impedance** of the transmission line into the complex conjugate of **the source impedance**:

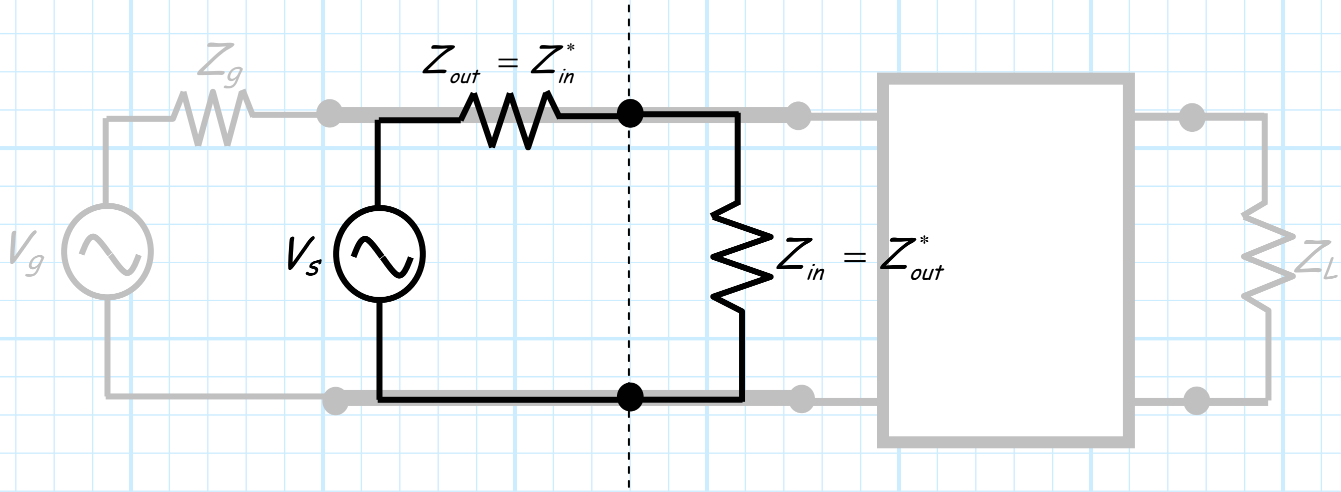


Q: *But, do we* ***have*** *to place the matching network between the source and the transmission line?*

A: Nope! We could **also** place a (different) matching network between the transmission line and the load.



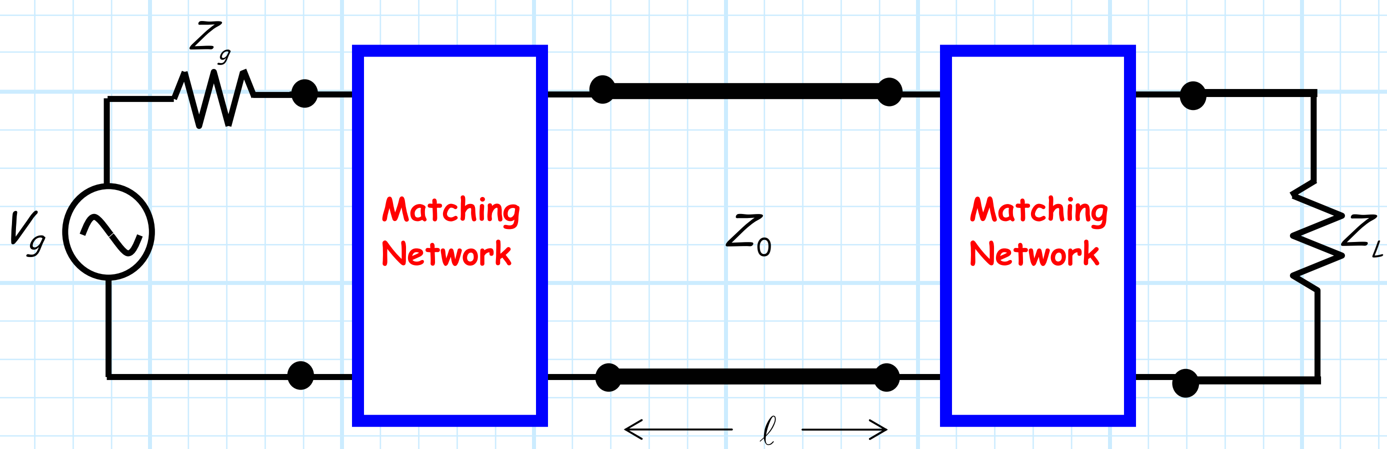
In either case, we find that at **any** and **all** points along this matched circuit, the output impedance of the equivalent **source** (i.e., looking left) will be equal to the **complex conjugate** of the **input** impedance (i.e., looking right).



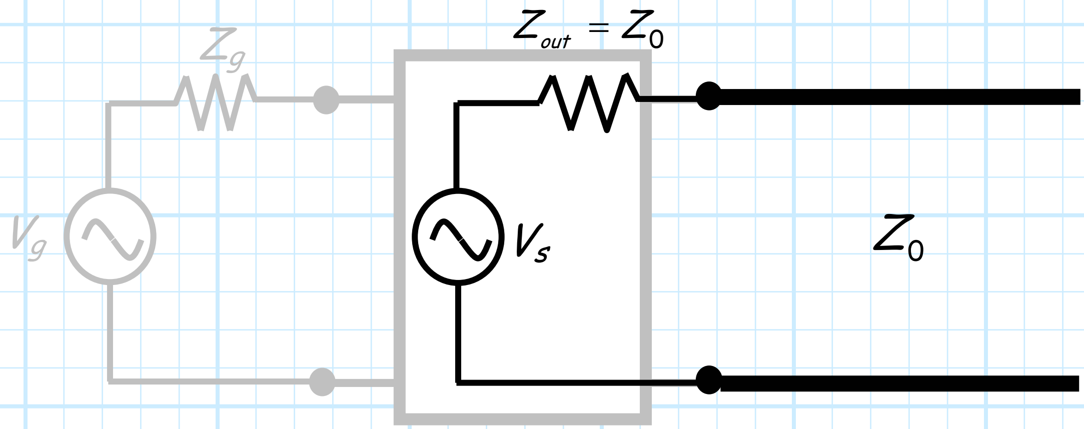
Q: *So **which** method should we chose? Do engineers typically place the matching network between the source and the transmission line, **or** place it between the transmission line and the load?*

A: Actually, the typical solution is to do **both**!

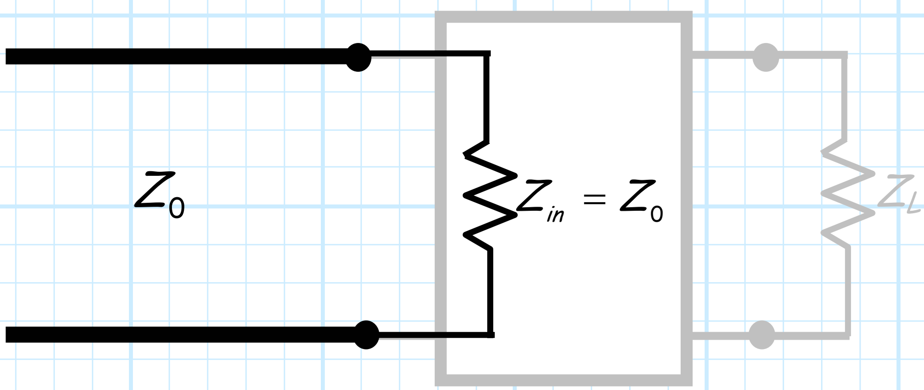
We find that often there is a matching network between the a source and the transmission line, **and** between the line and the load.



The first network matches the **source** to the **transmission line**—in other words, it transforms the **output impedance** of the equivalent source to a value numerically equal to **characteristic impedance** Z_0:



The second network matches the **load** to the **transmission line**—in other words it transforms the **load impedance** to a value numerically equal to **characteristic impedance** Z_0:



Q: *Yikes! Why would we want to build **two** separate matching networks, instead of just **one**?*

A: By using two separate matching networks, we can **decouple** the design problem. Recall again that the design of a **single** matching network solution would depend on four separate parameters:

1. the source impedance Z_g.

2. load impedance Z_L.

3. the transmission line characteristic impedance Z_0.

4. the transmission line length ℓ.

Alternatively, the design of the network matching the **source** and **transmission line** depends on **only**:

1. the load impedance Z_g.

2. the transmission line characteristic impedance Z_0.

Whereas, the design of the network matching the **load** and **transmission line** depends on **only**:

1. the source impedance Z_L.

2. the transmission line characteristic impedance Z_0.

Note that **neither** design depends on the transmission line **length** ℓ!

Q: *How is that possible?*

A: Remember the case where $Z_g = Z_0 = Z_L$. For that **special** case, we found that a conjugate match was the result—**regardless** of the transmission line length.

Thus, by matching the source to line impedance Z_0 **and** likewise matching the load to the line impedance, a conjugate match is **assured**—but the **length** of the transmission line does **not** matter!

In fact, the typically problem for microwave engineers is to match a load (e.g., device input impedance) to a **standard** transmission line impedance (typically $Z_0 = 50\Omega$); **or** to independently match a source (e.g., device output impedance) to a **standard** line impedance.

A **conjugate match** is thus obtained by connecting the two with a transmission line of **any length**!