B. Amplifiers

We will find that the signal power collected by a receiver antenna is often **ridiculously small** (e.g., less than one **trillionth** of a Watt!)

To accurately recover the information impressed on this signal, we must **increase** the signal power a whole bunch—**without** modifying or distorting the signal in any way.

We accomplish this with a RF/microwave amplifier—one of the few **active** components we will study.

But first, a few comments about the **decibel**!

HO: dB, dBm, dBw

HO: Amplifiers

Q: *By how much will an amplifier increase signal power?*

A: HO: Amplifier Gain

Q: *Can we increase this signal power an unlimited amount?*

A: NO! At some point we are limited by conservation of energy!

HO: Amplifier Output Power

Q: *So, just how precisely does an amplifier reproduce a signal at its output?*

A: HO: Intermodulation Distortion

Q: *Is intermodulation distortion really that big of a problem?*

A: It can be if there are **multiple** signals at the amplifier input!

HO: Two-Tone Intermodulation Distortion

Every good radio engineer knows and understands that parameters of the amplifier **spec sheet**!

HO: The Amplifier Spec Sheet

dB, dBm, dBw

Decibel (dB), is a specific function that operates on a **unitless** parameter:

$$dB \doteq 10 \log_{10}(x)$$

where x is unitless!

Q: A unitless parameter! What good is that ! ?

A: **Many** values are unitless, such as **ratios** and **coefficients**.

For example, amplifier **gain** is a unitless value!

E.G., amplifier gain is the ratio of the output power to the input power:

$$\frac{P_{out}}{P_{in}} = G$$

$$\therefore \text{ Gain in dB } = 10 \log_{10} G \doteq G(dB)$$

Q: *Wait a minute! I've seen statements such as:*

.... the output **power** is 5 dBw ....
or
.... the input **power** is 17 dBm ....

*Of course, Power is **not** a unitless parameter!?!*

A: True! But look at how power is expressed; not in dB, but in **dBm** or **dBw**.

Q: *What the heck does dB**m** or dB**w** refer to ??*

A: It's sort of a **trick** !

Say we have some power P. Now say we divide this value P by one **1 Watt**. The result is a **unitless** value that expresses the value of P in **relation** to 1.0 Watt of power.

For **example**, if $P = 2500\, mW$, then $P/1W = 2.5$. This simply means that power P is 2.5 times larger than one Watt!

Since the value $P/1W$ is **unitless**, we can express this value in **decibels**!

Specifically, we define this operation as:

$$P(dBw) \doteq 10 \log_{10}\left(\frac{P}{1\,W}\right)$$

For example, $P = 100$ Watts can alternatively be expressed as $P(dBw) = +20\,dBw$. Likewise, $P = 1\,mW$ can be expressed as $P(dBw) = -30\,dBw$.

Q: *OK, so what does **dBm** mean?*

A: This notation simply means that we have normalized some power P to one **Milliwatt** (i.e., $P/1\,mW$)—as opposed to one Watt. Therefore:

$$P(dBm) \doteq 10 \log_{10}\left(\frac{P}{1\,mW}\right)$$

For example, $P = 100$ Watts can alternatively be expressed as $P(dBm) = +50\,dBm$. Likewise, $P = 1\,mW$ can be expressed as $P(dBm) = 0\,dBm$.

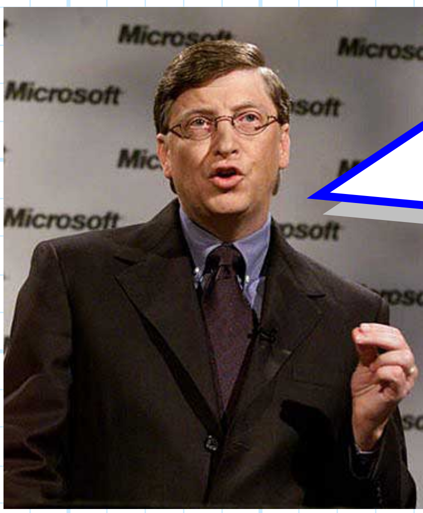
Make sure you are very **careful** when doing math with decibels!

Standard dB Values

Note that $10\,log_{10}(10) = 10dB$

Therefore an amplifier with a gain G = **10** is likewise said to have a gain of **10** dB.

Now consider an amplifier with a gain of **20** dB……



Q: *Yes, yes, I know. A **20 dB** amplifier has gain **G=20**, a **30 dB** amp has **G=30**, and so forth.*

*Please speed this lecture up and quit wasting my valuable time making such **obvious** statements!*

A: NO! Do **not** make this **mistake**!

Recall from **your** knowledge of logarithms that:

$$10\log_{10}\left[10^{n}\right] = n\,10\log_{10}\left[10\right] = 10n$$

Therefore, if we express gain as $G = 10^n$, we conclude:

$$G = 10^n \quad \leftrightarrow \quad G(dB) = 10n$$

In other words, G=**100** = 10^2 (n=2) is expressed as **20 dB**, while **30 dB** (n=3) indicates G= **1000** = 10^3.

Likewise **100 mW** is denoted as **20 dBm**, and **1000 Watts** is denoted as **30 dBW**.

Note also that **0.001 mW** = 10^{-3} mW is denoted as **–30 dBm**.

Another important relationship to keep in mind when using decibels is $10\log_{10}[2] \approx 3.0$. This means that:

$$10\log_{10}[2^n] = n\, 10\log_{10}[2] \simeq 3n$$

Therefore, if we express gain as $G = 2^n$, we conclude:

$$G = 2^n \quad \leftrightarrow \quad G(dB) \simeq 3n$$

As a result, a **15 dB** (n=5) gain amplifier has G= 2^5 = **32**. Similarly, **1/8** = 2^{-3} mW (n=-3) is denoted as **–9 dBm**.

Multiplicative Products and Decibels

Other logarithmic relationships that we will find **useful** are:

$$10\log_{10}[x\,y] = 10\log_{10}[x] + 10\log_{10}[y]$$

and its close cousin:

$$10\log_{10}\left[\frac{x}{y}\right] = 10\log_{10}[x] - 10\log_{10}[y]$$

Thus, the relationship $P_{out} = G\,P_{in}$ is written in **decibels** as:

$$\begin{aligned} P_{out} &= G\,P_{in} \\ \frac{P_{out}}{1mW} &= \frac{G\,P_{in}}{1mW} \\ 10\log_{10}\left[\frac{P_{out}}{1mW}\right] &= 10\log_{10}\left[\frac{G\,P_{in}}{1mW}\right] \\ 10\log_{10}\left[\frac{P_{out}}{1mW}\right] &= 10\log_{10}[G] + 10\log_{10}\left[\frac{P_{in}}{1mW}\right] \end{aligned}$$

$$P_{out}(dBm) = G(dB) + P_{out}(dBm)$$

It is evident that "deebees" are **not** a unit! The units of the result can be found by **multiplying** the units of each term in a **summation** of decibel values.

For example, say some power $P_1 = 6\,dBm$ is **combined** with power $P_2 = 10\,dBm$. What is the resulting **total** power $P_T = P_1 + P_2$?

Q: *This result really **is** obvious—of **course** the total power is:*

$$P_T(dBm) = P_1(dBm) + P_2(dBm)$$
$$= 6\;dBm + 10\;dBm$$
$$= 16\;dBm$$

A: **NO!** Never do **this** either!

Logarithms are very helpful in expressing **products** or **ratios** of parameters, but they are **not** much help when our math involves sums and differences!

$$10\log_{10}[x+y] = ????$$

So, if you wish to add P_1=6 dBm of power to P_2=10 dBm of power, you must first **explicitly** express power in Watts:

$$P_1 = 10\;dBm = 10\;mW \quad \text{and} \quad P_2 = 6\;dBm = 4\;mW$$

Thus, the total power P_T is:

$$\begin{aligned} P_T &= P_1 + P_2 \\ &= 4.0\ mW + 10.0\ mW \\ &= 14.0\ mW \end{aligned}$$

Now, we can express this total power in *dBm*, where we find:

$$P_T\left(dBm\right) = 10\ log_{10}\left(\frac{14.0\ mW}{1.0\ mW}\right) = 11.46\ dBm$$

The result is **not** 16.0 *dBm* !.

We **can** mathematically add 6 *dBm* and 10 *dBm*, but we must understand what result means (**nothing useful**!).

$$\begin{aligned} 6\,dBm\ + 10\ dBm &= 10\log_{10}\left[\frac{4mW}{1mW}\right] + 10\log_{10}\left[\frac{10mW}{1mW}\right] \\ &= 10\log_{10}\left[\frac{40\ mW^2}{1mW^2}\right] \\ &= 16\ dB \text{ relative to } 1\ \text{mW}^2 \end{aligned}$$

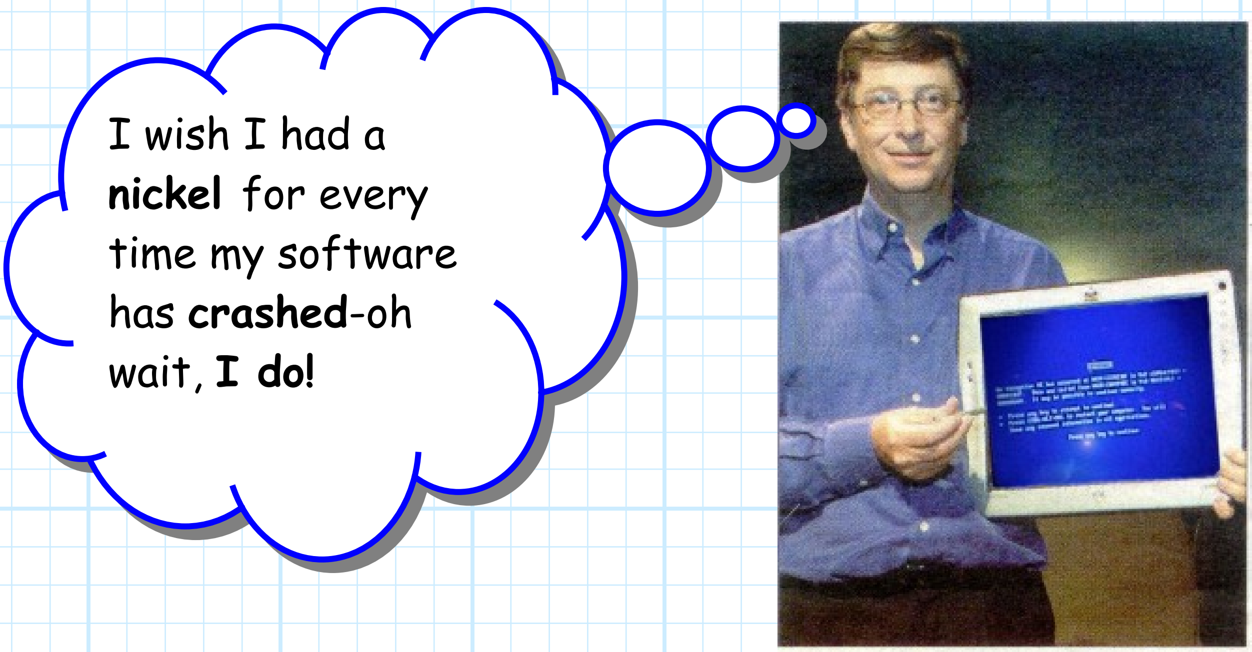
Thus, mathematically speaking, 6 *dBm* + 10 *dBm* implies a multiplication of power, resulting in a value with units of **Watts squared** !

A few more **tidbits** about decibels:

1. $1.0 \quad \leftrightarrow \quad 0 \; dB$

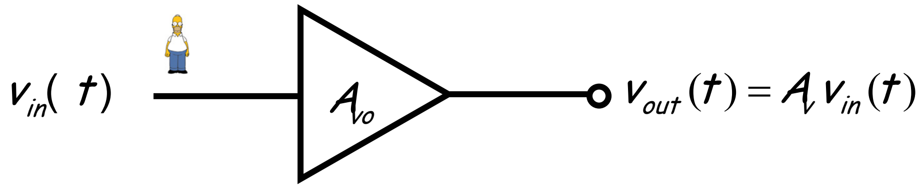
2. $0.0 \quad \leftrightarrow \quad -\infty \; dB$

3. $5^n \quad \leftrightarrow \quad \simeq 7n \; dB$ (can **you** show why?)



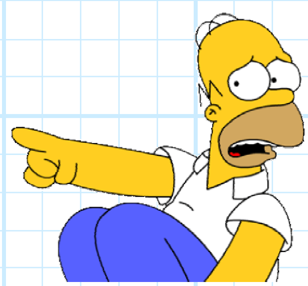
Amplifiers

An **ideal** amplifier takes an input signal and reproduces it **exactly** at its output, only with a **larger** magnitude!



where A_v is the **voltage gain** of the amplifier.

Now, let's express this result using our knowledge of **linear circuit theory** !



Recall, the output $v_{out}(t)$ of a linear device can be determined by **convolving** its input $v_{in}(t)$ with the device **impulse response** $h(t)$:

$$v_{out}(t) = \int_{-\infty}^{t} h(t - t')v_{in}(t')dt'$$

The impulse response for the **ideal** amplifier would therefore be:

$$h(t) = A_v \, \delta(t)$$

so that:

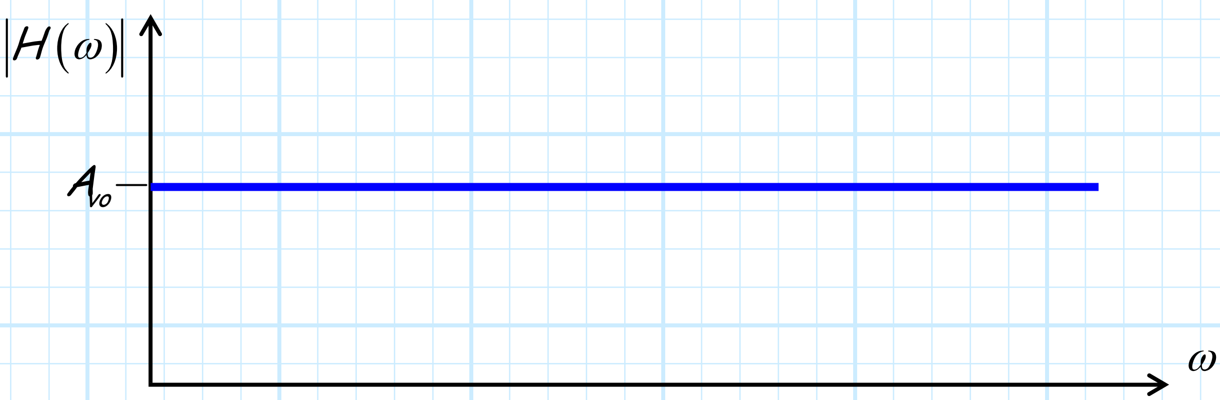
$$
\begin{aligned}
v_{out}(t) &= \int_{-\infty}^{t} h(t-t')\,v_{in}(t')\,dt' \\
&= \int_{-\infty}^{t} A_v\,\delta(t-t')\,v_{in}(t')\,dt' \\
&= A_v\,v_{in}(t)
\end{aligned}
$$

→ **Any** and **every** function $v_{in}(t)$ is an **eigen function** of an **ideal** amplifier!!

We can alternatively represent the ideal amplifier response in the **frequency domain**, by taking the **Fourier Transform** of the impulse response:

$$
\begin{aligned}
H(\omega) &= \int_{-\infty}^{\infty} h(t)\,e^{-j\omega t}\,dt \\
&= \int_{-\infty}^{\infty} A_v\,\delta(t)\,e^{-j\omega t}\,dt \\
&= A_v + j0
\end{aligned}
$$

This result, although simple, has an interesting interpretation. It means that the amplifier exhibits gain of A_v for sinusoidal signals of **any** and **all** frequencies!



BUT, there is one **big** problem with an ideal amplifier:

→ They are **impossible** to build !!

The **ideal** amplifier has a frequency response of $|H(\omega)| = A_v$. Note this means that the amplifier gain is A_v for **all** frequencies $0 < \omega < \infty$ (D.C. to daylight !).

The **bandwidth** of the ideal amplifier is therefore **infinite** !

* Since every electronic device will exhibit **some** amount of inductance, capacitance, and resistance, every device will have a **finite** bandwidth.

* Moreover, we discussed that a matching network likewise exhibits **finite** bandwidth.

* In other words, there will be frequencies ω where the device does **not work** !

* From the standpoint of an amplifier, "not working" means $|H(\omega)| \ll A_v$ (i.e., **low gain**).

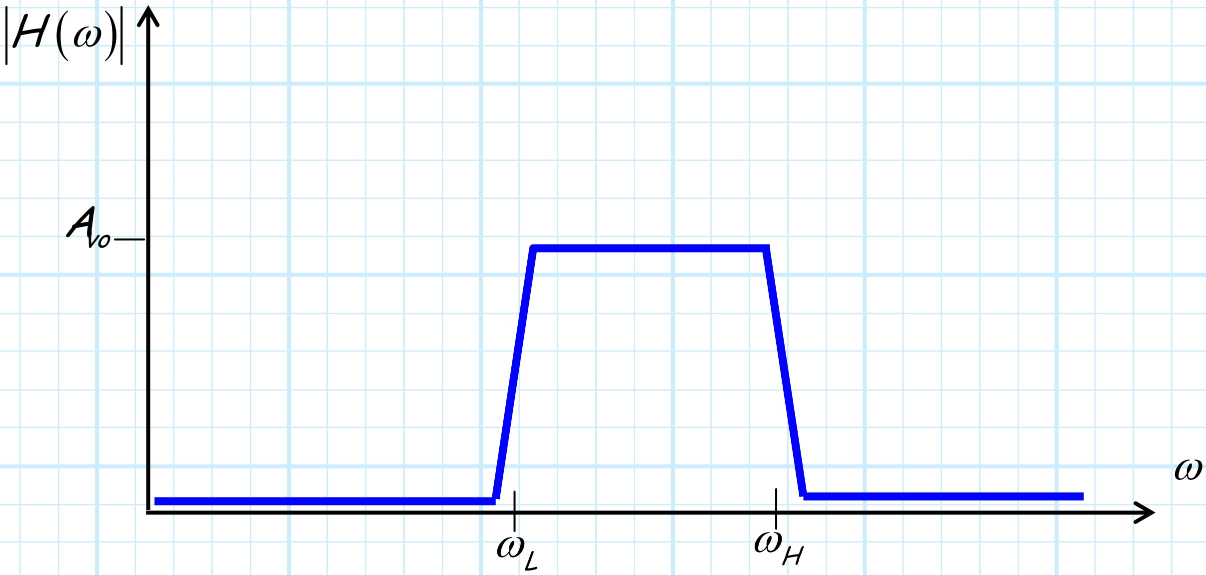
→ Amplifiers therefore have **finite** bandwidths.

There is a range of frequencies ω between ω_L and ω_H where the gain will (approximately) be A_v. For frequencies outside this range, the gain will typically be small (i.e. $|H(\omega)| \ll A_v$):

$$|H(\omega)| = \begin{cases} \approx A_v & \omega_L < \omega < \omega_H \\ \ll A_v & \omega < \omega_L, \omega > \omega_H \end{cases}$$

The **width** of this frequency range is called the amplifier **bandwidth**:

$$Bandwidth \doteq \omega_H - \omega_L \quad \text{(radians/sec)}$$
$$\doteq f_L - f_H \quad \text{(cycles/sec)}$$



One result of having a **finite bandwidth** is that the amplifier impulse response is **not** an impulse function!

$$h(t) = \int_{-\infty}^{\infty} H(\omega)\, e^{+j\omega t}\, dt \neq A_{vo}\ \delta(t)$$

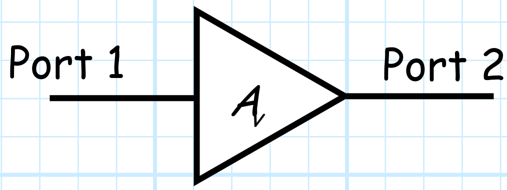
therefore **generally** speaking:

$$v_{out}(t) \neq A_v\, v_{in}(t)$$

The **ideal** amplifier is **not** possible!

Amplifier Gain

Note that an amplifier is a **two-port** device.



As a result, we can describe an amplifier with a 2 x 2 **scattering matrix**:

$$\mathcal{S} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

Q: *What is the scattering matrix of an **ideal** amplifier??*

A: Let's start with S_{11} and S_{22}.

To insure maximum power transfer, the input and output ports would ideally be matched:

$$S_{11} = S_{22} = 0$$

Now, let's look at scattering parameter S_{21}. We know that **if** the amplifier is connected to matched devices:

$$P_2^- = \left|S_{21}\right|^2 P_1^+$$

or, stated **another** way:

$$P_{out} = \left|S_{21}\right|^2 P_{in}$$

Therefore, we can **define** the amplifier **power gain** as:

$$G \doteq \frac{P_{out}}{P_{in}} = \left|S_{21}\right|^2$$

As the purpose of an amplifier is to boost the signal power, we can conclude that **ideally**:

$$\left|S_{21}\right| \gg 1$$

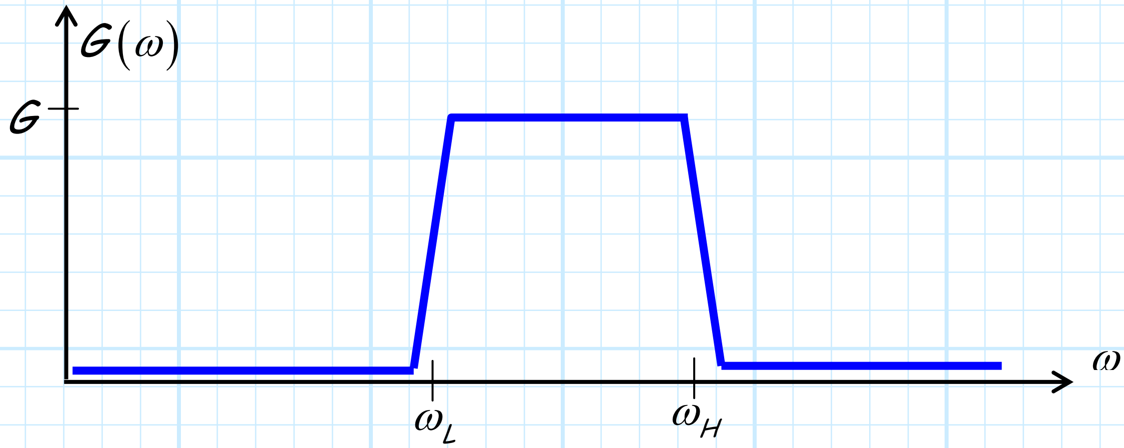
Clearly, an amplifier must be an **active** device!

As discussed earlier, the gain of an amplifier will change with signal frequency:

$$G(\omega) = \left|S_{21}(\omega)\right|^2$$

When radio engineers speak of amplifier **gain**, they almost always are speaking of this **power gain G**. However, they do not generally state it as a specific function of frequency!

Rather, amplifier gain is typically specified as a **numeric** value such as G =20 or G =13 dB. This value is a statement of the approximate amplifier gain **within** the amplifier **bandwidth**.

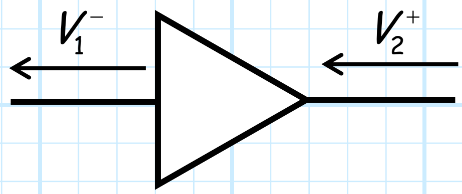


Thus, amplifier **gain** and **bandwidth** are the two most fundamental performance specifications of any microwave amplifier—together they (approximately) describe the amplifier transfer function!

Additionally, radio engineers almost always speak of amplifier gain in **decibels** (dB):

$$G(dB) = 10 \log_{10} G$$

Finally, let's consider S_{12}. This scattering parameter relates the wave into port 2 (the output) to the wave out of port 1 (the input).



Q: *Are amplifiers **reciprocal** devices? In other words, is* $S_{12} = S_{21}$ *??*

A: No! An amplifier is strictly a **directional** device; there is a specific input, and a specific output—it does **not** work in reverse!

Ideally, $S_{12} = 0$. Any other value can just cause problems!

Typically though, S_{12} is small, but **not** zero. Generally speaking, radio engineers express S_{12} as a value called **reverse isolation**:

$$\text{reverse isolation} \doteq -10\,log_{10}\left|S_{12}\right|^2$$

Note when $S_{12} = 0$, reverse isolation will be **infinite**. Thus, the **larger** the reverse isolation, the **better**!

Summarizing, we find that the scattering matrix of the **ideal amplifier** is:

$$\mathcal{S}_{ideal} = \begin{bmatrix} 0 & 0 \\ S_{21} & 0 \end{bmatrix} \qquad \text{where } \left|S_{21}\right| \gg 1$$

The **non-ideal** reality is that the zero valued terms will be **small**, but not **precisely** zero. Moreover, each scattering parameter will change with signal **frequency**—although they remain **approximately** constant within the amplifier **bandwidth**.

Amplifier Output Power

Say we have an amplifier with gain G = 30 dB (i.e., G = 1000).

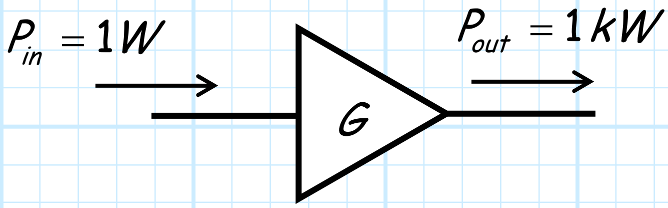
If the input power to this amplifier is 0 dBw (i.e., P_{in} = 1W), then the output power is:

$$P_{in}\, G = P_{out}$$

$$(1\text{ W})\, 1000 = 1000\text{ W}$$

Or, in dB:

$$0\text{ dBw} + 30\text{ dB} = 30\text{ dBw}$$



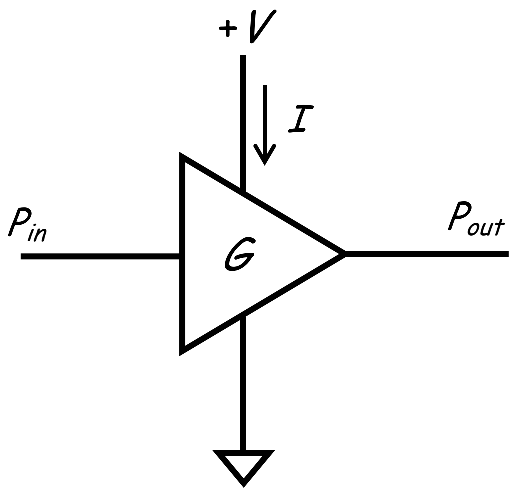
WOW! We created 999 Watts !

The energy crisis is solved !

Of course, the amplifier cannot **create** energy.

Q: *Then, where does the power come from ???*

A: The **D.C.** power supply ! (Every amplifier has one).



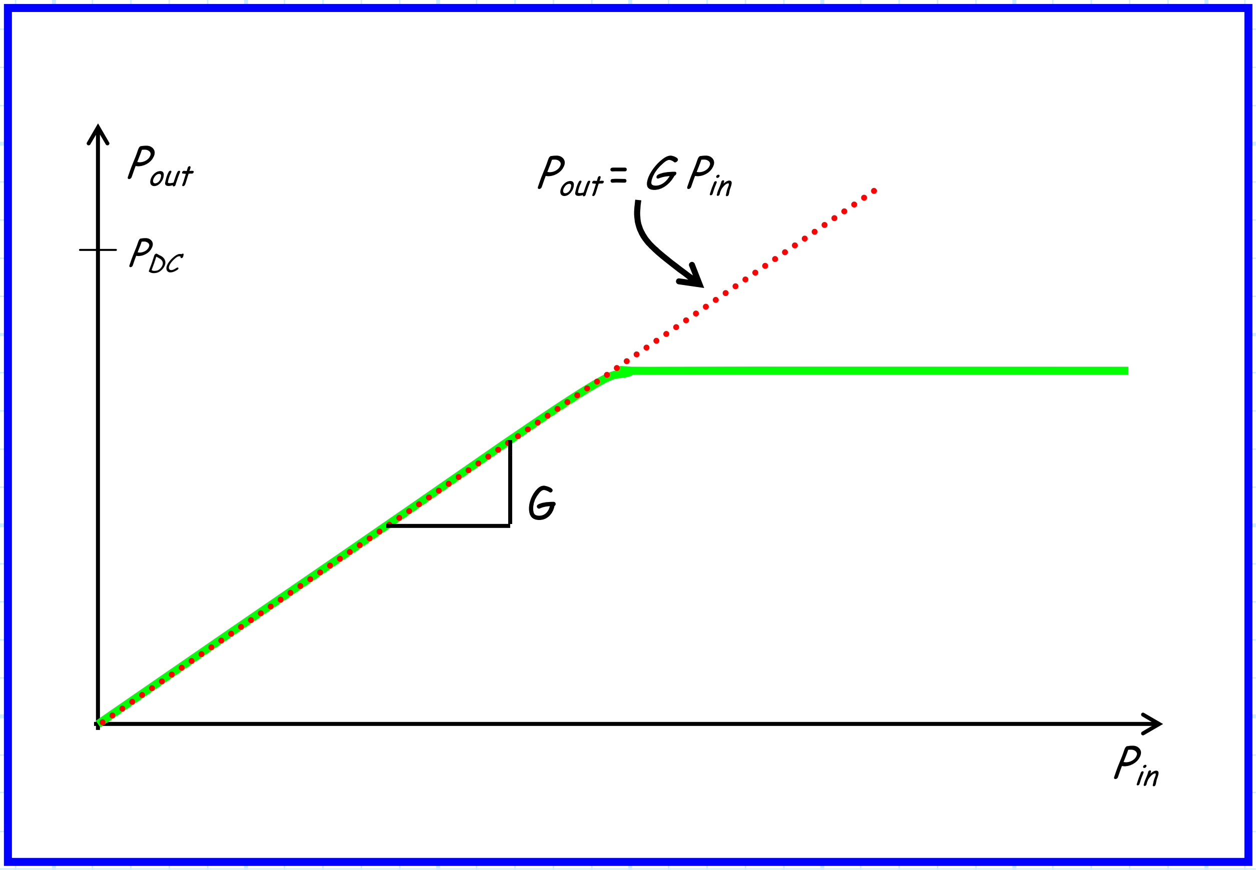
D. C. Power $= P_{DC} = V\,I > P_{out}$

The output power P_{out} cannot exceed the power delivered by the D.C. supply.

Q: *What happens to the D.C. power not converted to signal power P_{out} ??*

A:

So, if we were to plot P_{out} vs. P_{in} for a microwave amplifier, we would get something like this:



We notice that the output power **compresses**, or saturates.

Note there is **one** point on this curve where the amplifier output power P_{out} is 1 dB less than its ideal value of $G\,P_{in}$. In other words, there is one (and only one!) value of P_{in} and P_{out} that will satisfy the equation:

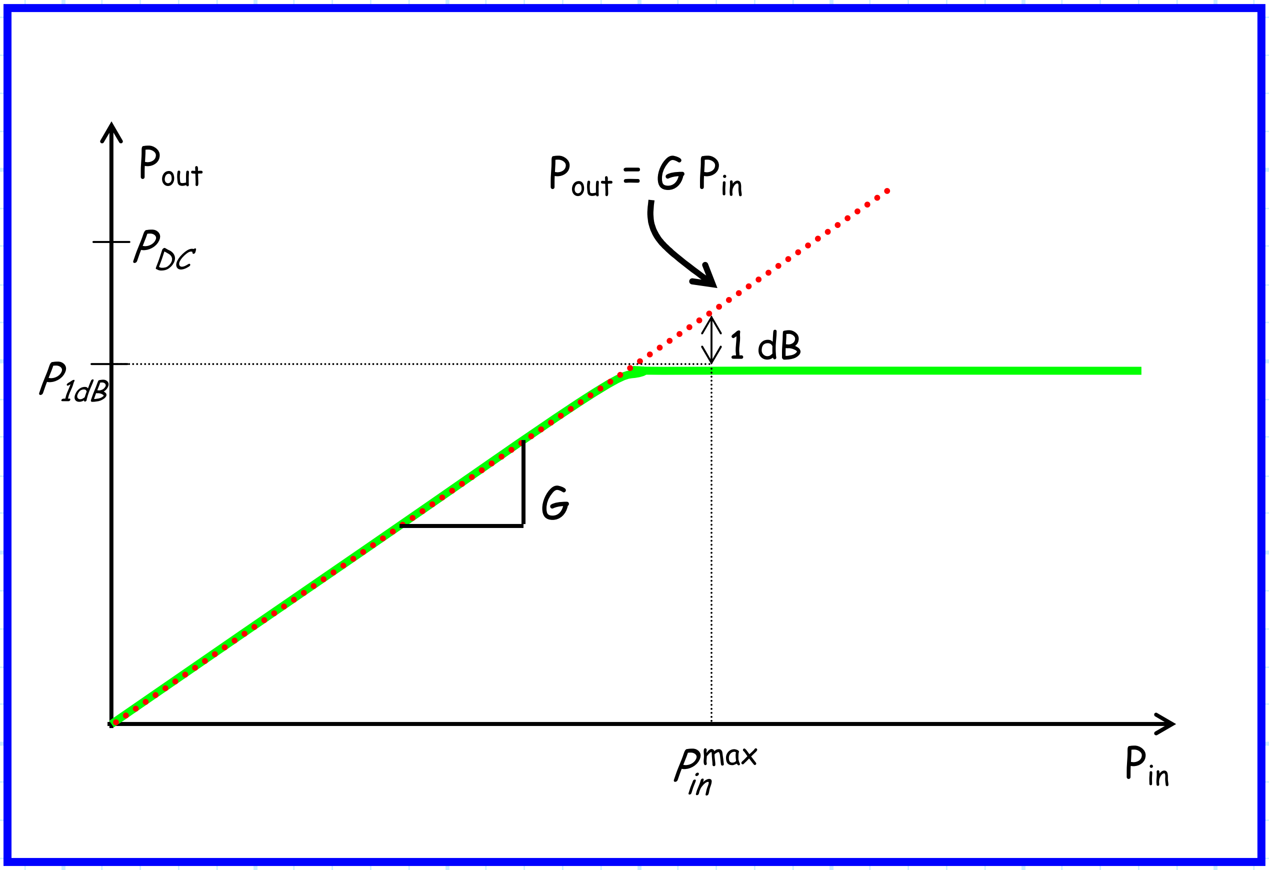
$$P_{out}(dB) = \left[P_{in}(dB) + G(dB)\right] - 1\ dB$$

At this point, the amplifier is said to be compressed 1 dB. Therefore, a 10 dB amplifier would appear to be a 9 dB amplifier!

The output power when the amplifier has compressed 1dB is called the **1 dB compression point** $\doteq$ $\mathbf{P_{1dB}}$ of the amplifier.

The 1 dB compression point is generally considered to be the **maximum power output** of the amplifier.

The input power at the 1 dB compression point is said to be the **maximum input power** (P_{in}^{max}) of the amplifier. We of course **can** put more than P_{in}^{max} into the amplifier—but we **won't** get much more power out!



Note the equation $P_{out}(dB) = \left[P_{in}(dB) + G(dB)\right] - 1\ dB$ alone is **not sufficient** to determine the 1 dB compression point, as we have two uknowns (P_{in} and P_{out}). We need **another** equation!

This second "equation" is the actual **curve** or **table** of data relating P_{in} to P_{out} for a **specific** amplifier.

Amplifier Efficiency

We can define **amplifier efficiency** e as the ratio of the maximum output power (P_{1dB}) to the D.C. power:

$$e = \frac{P_{1dB}}{P_{DC}}$$ (**don't** use decibels here!)

For example, if e=0.4, then up to 40% of the D.C. power **can** be converted to **output power**, while the remaining 60% is converted to **heat**.

→ We require **high power** amps to be **very** efficient!

Intermodulation Distortion

The 1 dB compression curve shows that amplifiers are only **approximately** linear.

Actually, this should be **obvious**, as amplifiers are constructed with transistors—**non-linear** devices!

So, instead of the **ideal** case:

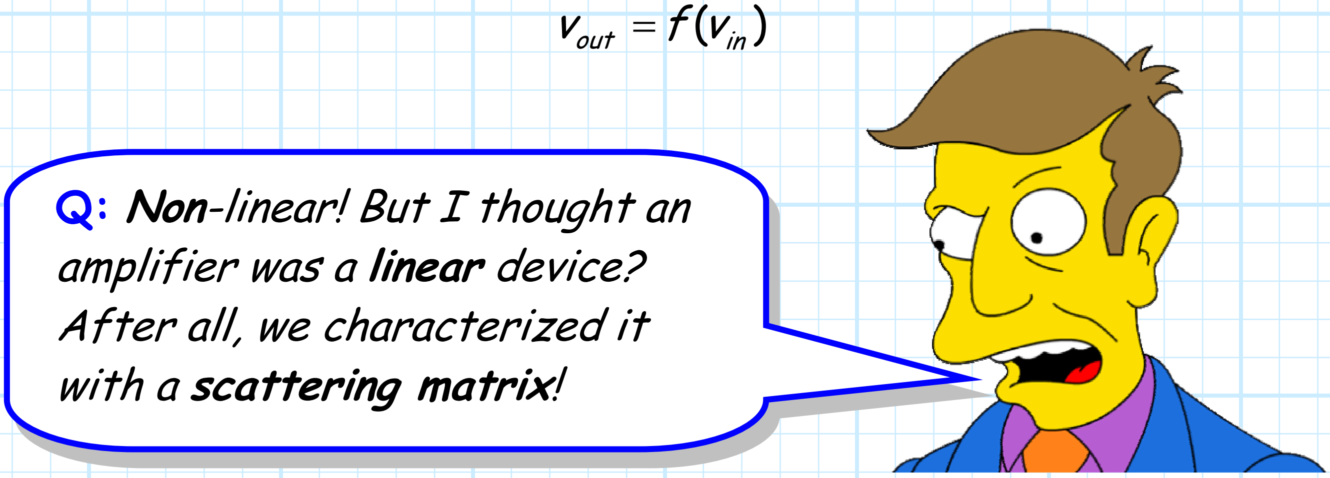
$$v_{out} = A_v \, v_{in}$$

Actual amplifier behavior requires more terms to describe!

$$v_{out} = A_v \, v_{in} + B \, v_{in}^2 + C \, v_{in}^3 + \cdots$$

This representation is simply a **Taylor Series** representation of the **non-linear** function:

$$v_{out} = f(v_{in})$$



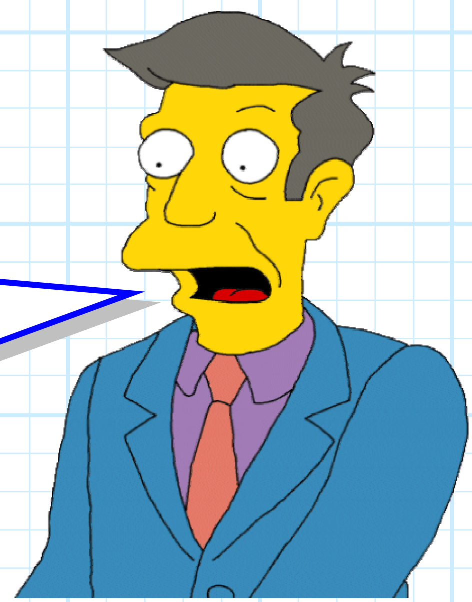
A: Generally speaking, the constants B, C, D, etc. are **very** small compared to the voltage gain A_v. Therefore, **if** v_{in} is likewise small, we can **truncate** the Taylor Series and **approximate** amplifier behavior as the linear function:

$$v_{out} \approx A_v v_{in}$$

BUT, as v_{in} gets large, the values v_{in}^2 and v_{in}^3 will get **really** large! In that case, the terms $B v_{in}^2$ and $C v_{in}^3$ will become **significant**.

As a result, the output will not simply be a larger version of the input. The output will instead be **distorted**—a phenomenon known as **Intermodulation Distortion.**

Q: *Good heavens! This sounds terrible. What exactly is* ***Intermodulation Distortion,*** *and what will it do to our signal output?!?*



A: Say the input to the amplifier is sinusoidal, with magnitude a:

$$v_{in} = a\cos\omega t$$

Using our knowledge of trigonometry, we can determine the result of the second term of the output Taylor series:

$$\begin{aligned} B\,v_{in}^2 &= B\,a^2 \cos^2 \omega t \\ &= \frac{B\,a^2}{2} + \frac{B\,a^2}{2} \cos 2\omega t \end{aligned}$$

We have created a **harmonic** of the input signal!

In other words, the input signal is at a frequency ω, while the output includes a signal at **twice** that frequency (2ω).

We call this signal a **second** order product, as it is a result of **squaring** the input signal.

Note we also have a **cubed** term in the output signal equation:

$$v_{out} = A_v\, v_{in} + B\, v_{in}^2 + C\, v_{in}^3 + \cdots$$

Using a trig identity, we find that:

$$\begin{aligned} C\,v_{in}^3 &= C\,a^3 \cos^3 \omega t \\ &= \frac{C\,a^3}{2} \cos \omega t + \frac{C\,a^3}{4} \cos 3\omega t \end{aligned}$$

Now we have produced a **second harmonic** (i.e., 3ω)!

As you might expect, we call this harmonic signal a **third-order** product (since it's produced from v_{in}^3).

Q: *I confess that I am still a bit* ***befuddled****. You said that values B and C are typically* ***much*** *smaller that that of voltage gain* A_v *. Therefore it would seem that these harmonic signals would be* ***tiny*** *compared to the fundamental output signal* $A_v \, a \cos \omega t$ *. Thus, I* ***don't*** *why there's a problem!*

To understand why intermodulation distortion can be a problem in amplifiers, we need to consider the **power** of the output signals.

We know that the power of a sinusoidal signal is proportional to its **magnitude squared**. Thus, we find that the power of each output signal is related to the input signal power as:

$$\text{1rst-order output power} \doteq P_1^{out} = A_v^2 \, P_{in} = G \; P_{in}$$

$$\text{2nd-order output power} \doteq P_2^{out} = \frac{B^2}{4} P_{in}^2 = G_2 \, P_{in}^2$$

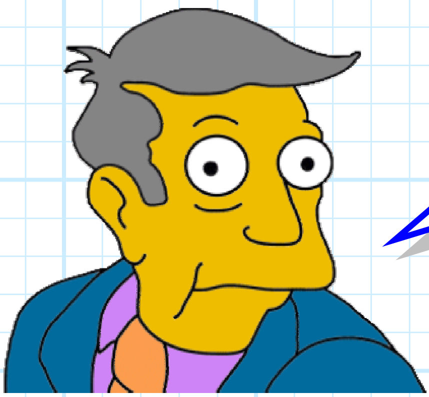
$$\text{3rd-order output power} \doteq P_3^{out} = \frac{C^2}{16} P_{in}^3 = G_3 \, P_{in}^3$$

where we have obviously **defined** $G_2 \doteq B^2/4$ and $G_3 \doteq C^2/16$.

Note that unlike G, the values G_2 and G_3 are **not coefficients** (i.e., not unitless!). The value G_2 obviously has units of inverse power (e.g., mW^{-1} or W^{-1}), while G_3 has units of inverse power squared (e.g., mW^{-2} or W^{-2}).

We know that typically, G_2 and G_3 are much **smaller** than G. Thus, we are **tempted** to say that P_1^{out} is much **larger** than P_2^{out} or P_3^{out}.

But, we might be **wrong** !



Q: ***Might*** *be wrong! Now I'm more confused than ever. Why can't we say* ***definitively*** *that the second and third order products are* ***insignificant****??*

Look **closely** at the expressions for the output power of the first, second, and third order products:

$$P_1^{out} = G \; P_{in}$$
$$P_2^{out} = G_2 \; P_{in}^2$$
$$P_3^{out} = G_3 \; P_{in}^3$$

This **first** order output power is of course **directly** proportional to the input power. However, the **second** order output power is proportional to the input power **squared**, while the **third** order output is proportional to the input power **cubed**!

Thus we find that if the input power is **small**, the second and third order products **are** insignificant. But, as the input power **increases**, the second and third order products get **big** in a hurry!

For example, if we **double** the input power, the **first** order signal will of course likewise **double**. However, the **second** order power will **quadruple**, while the **third** order power will increase **8 times**.

For **large** input powers, the second and third order output products can in fact be **almost as large** as the first order signal!

Perhaps this can be most easily seen by expressing the above equations in **decibels**, e.g.,:

$$P_1^{out}(dBm) = G(dB) + P_{in}(dBm)$$
$$P_2^{out}(dBm) = G_2(dBm^{-1}) + 2\left[P_{in}(dBm)\right]$$
$$P_3^{out}(dBm) = G_3(dBm^{-2}) + 3\left[P_{in}(dBm)\right]$$

where we have used the fact that $\log x^n = n \log x$. Likewise, we have defined:

$$\begin{aligned} G_2(dBm^{-1}) &= 10\log_{10}\left[\frac{G_2}{\left(\frac{1}{1.0mW}\right)}\right] \\ &= 10\log_{10}\left[G_2\,(1.0mW)\right] \end{aligned}$$

and:

$$\begin{aligned} G_3(dBm^{-2}) &= 10\log_{10}\left[\frac{G_3}{\left(\frac{1}{1.0mW^2}\right)}\right] \\ &= 10\log_{10}\left[G_3\,(1.0mW^2)\right] \end{aligned}$$

Hint: Just express everything in milliwatts!

Note the value $2\left[P_{in}(dBm)\right]$ does **not** mean the value $2P_{in}$ expressed in decibels. The value $2\left[P_{in}(dBm)\right]$ is fact the value of P_{in} expressed in decibels—**times two**!

For **example**, if $P_{in}(dBm) = -30\,dBm$, then $2\left[P_{in}(dBm)\right] = -60\,dBm$. Likewise, if $P_{in}(dBm) = 20\,dBm$, then $2\left[P_{in}(dBm)\right] = 40\,dBm$.

What this means is that for every **1dB** increase in **input** power P_{in} the fundamental (**first-order**) signal will increase **1dB**; the **second-order** power will increase **2dB**; and the **third-order** power will increase **3dB**.

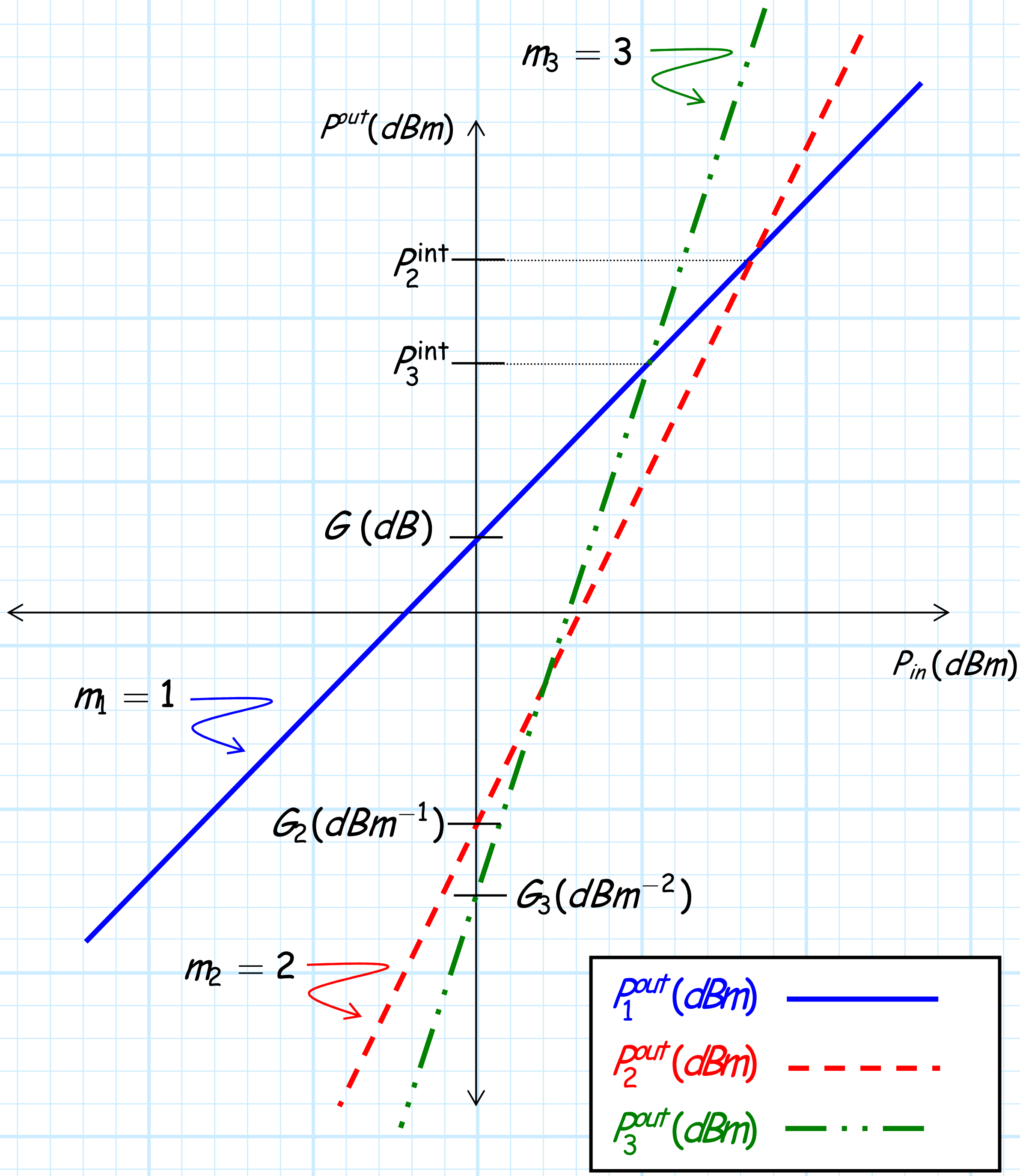
This is evident when we look at the three power equations (in decibels), as each is an equation of a **line** (i.e., $y = m\,x + b$).

For example, the equation:

$$P_3^{out}(dBm) = 3\left[P_{in}(dBm)\right] + G_3(dBm^{-2})$$
$$y = m\,x + b$$

describes a line with **slope** $m = 3$ and "y **intercept**" $b = G_3(dBm^{-2})$ (where $x = P_{in}(dBm)$ and $y = P^{out}(dBm)$).

Plotting each of the three equations for a typical amplifier, we would get something that looks like this:



Note that for $P_{in}(dBm) < 0\,dBm$ (the left side of the plot), the second and third-order products are small compared to the fundamental (first-order) signal.

However, when the input power increases **beyond** 0 dBm (the right side of the plot), the second and third order products rapidly **catch up**! In fact, they will (theoretically) become **equal** to the first order product at some large input power.

The point at which each higher order product **equals** the first-order signal is defined as the **intercept point**. Thus, we define the **second order intercept** point as the output power **when**:

$$P_2^{out} = P_1^{out} \doteq P_2^{\text{int}} \quad \textit{Second - order intercept power}$$

Likewise, the **third order intercept** point is defined as the third-order output power **when**:

$$P_3^{out} = P_1^{out} \doteq P_3^{\text{int}} \quad \textit{Third - order intercept power}$$

Using a little algebra **you** can show that:

$$P_2^{\text{int}} = \frac{G^2}{G_2} \quad \text{and} \quad P_3^{\text{int}} = \sqrt{\frac{G^3}{G_3}}$$

Or, expressed in **decibels**:

$$P_2^{int}(dBm) = 2\,G(dB) - G_2(dBm^{-1})$$

$$P_3^{int}(dBm) = \frac{3\,G(dB) - G_3(dBm^{-2})}{2}$$

* Radio engineers specify the intermodulation distortion performance of a specific amplifier in terms of the **intercept points**, rather than values G_2 and G_3.

* Generally, only the **third-order** intercept point is provided by amplifier manufactures (we'll see why later).

* **Typical** values of P_3^{int} for a **small-signal** amplifier range from +20 dBm to +50 dBm

* Note that as G_2 and G_3 **decrease**, the intercept points **increase**.

Therefore, the **higher** the intercept point of an amplifier, the better the amplifier !

One other **important point**: the **intercept points** for most amplifiers are much **larger** than the **compression point**! I.E.,:

$$P^{int} > P_{1dB}$$

In other words the intercept points are "theoretical", in that we can **never**, in fact, increase the input power to the point that the higher order signals are **equal** to the fundamental signal power.

All signals, including the higher order signals, have a **maximum** limit that is determined by the amplifier **power supply**.

Two-Tone Intermodulation

Q: *It doesn't seem to me that this* ***dad-gum*** *intermodulation distortion is really that much of a problem.*

I mean, the first and second ***harmonics*** *will likely be well* ***outside*** *the amplifier bandwidth, right?*

A: True, the **harmonics** produced by intermodulation distortion typically are **not** a problem in radio system design. There is a problem, however, that is **much worse** than **harmonic** distortion!

> **This** problem is called **two-tone** intermodulation distortion.

Say the input to an amplifier consists of **two** signals at **dissimilar** frequencies:

$$v_{in} = a\cos\omega_1 t + a\cos\omega_2 t$$

Here we will assume that both frequencies ω_1 and ω_2 are within the **bandwidth** of the amplifier, but are **not** equal to each other $(\omega_1 = \omega_2)$.

This of course is a much more **realistic** case, as typically there will be **multiple** signals at the input to an amplifier!

For example, the two signals considered here could represent two **FM radio stations**, operating at frequencies within the FM band (i.e., $88.1 \text{ MHz} \leq f_1 \leq 108.1 \text{ MHz}$ and $88.1 \text{ MHz} \leq f_2 \leq 108.1 \text{ MHz}$).

Q: *My point exactly! Intermodulation distortion will produce those **dog-gone** second-order products:*

$$\frac{a^2}{2}\cos 2\omega_1 t \quad \text{and} \quad \frac{a^2}{2}\cos 2\omega_2 t$$

*and **gul-durn** third order products:*

$$\frac{a^3}{4}\cos 3\omega_1 t \quad \text{and} \quad \frac{a^3}{4}\cos 3\omega_2 t$$

*but these harmonic signals will lie well **outside** the FM band!*

A: True! Again, the **harmonic** signals are **not** the problem. The problem occurs when the **two input** signals combine together to form **additional** second and third order products.

Recall an amplifier output is accurately described as:

$$v_{out} = A_v v_{in} + B v_{in}^2 + C v_{in}^3 + \cdots$$

Consider first the **second-order** term if **two** signals are at the input to the amplifier:

$$\begin{aligned} v_2^{out} &= B v_{in}^2 \\ &= B\left(a\cos\omega_1 t + a\cos\omega_2 t\right)^2 \\ &= B\left(a^2\cos^2\omega_1 t + 2a^2\cos\omega_1 t\ \cos\omega_2 t + a^2\cos^2\omega_2 t\right) \end{aligned}$$

Note the first and third terms of the above expression are **precisely** the same as the terms we examined on the previous handout. They result in **harmonic** signals at frequencies $2\omega_1$ and $2\omega_2$, respectively.

The **middle** term, however, is something **new**. Note **it** involves the product of $\cos\omega_1 t$ and $\cos\omega_2 t$. Again using our knowledge of **trigonometry**, we find:

$$2a^2\cos\omega_1 t\ \cos\omega_2 t = a^2\cos\left(\omega_2 - \omega_1\right)t + a^2\cos\left(\omega_2 + \omega_1\right)t$$

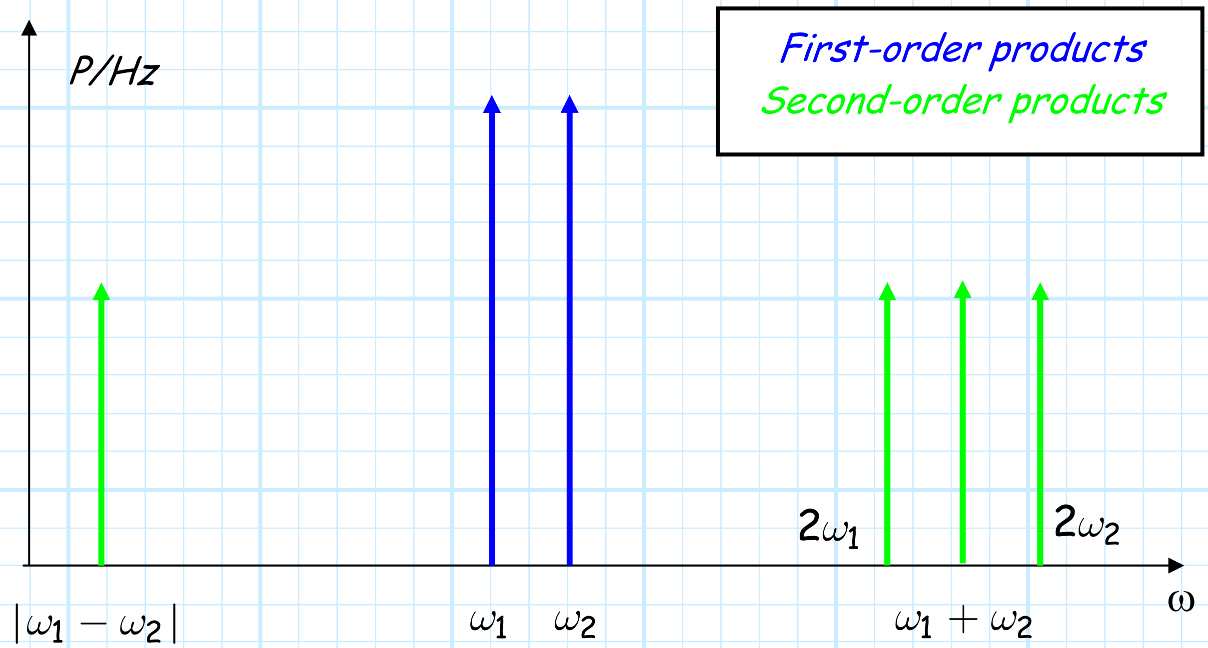
Note that since $\cos(-x) = \cos x$, we can **equivalently** write this as:

$$2a^2\cos\omega_1 t\ \cos\omega_2 t = a^2\cos\left(\omega_1 - \omega_2\right)t + a^2\cos\left(\omega_1 + \omega_2\right)t$$

Either way, the result is obvious—we produce **two new signals**!

These new **second-order** signals oscillate at frequencies $(\omega_1 + \omega_2)$ and $|\omega_1 - \omega_2|$.

Thus, if we looked at the **frequency spectrum** (i.e., signal power as a function of frequency) of an amplifier **output** when two sinusoids are at the input, we would see something like this:



Note that the new terms have a frequency that is either much **higher** than both ω_1 and ω_2 (i.e., $(\omega_1 + \omega_2)$), or much **lower** than both ω_1 and ω_2 (i.e., $|\omega_1 - \omega_2|$).

Either way, these new signals will typically be **outside** the amplifier bandwidth!

Q: *I thought you said these "two-tone" intermodulation products were some **"big problem"**. These sons of a gun appear to be **no more** a problem than the harmonic signals!*

A: This observation is indeed correct for **second**-order, two-tone intermodulation products. But, we have **yet** to examine the **third**-order terms! I.E.,

$$\begin{aligned} v_3^{out} &= C\, v_{in}^3 \\ &= C\left(a\cos\omega_1 t + a\cos\omega_2 t\right)^3 \end{aligned}$$

If we multiply this all out, and again apply our trig knowledge, we find that a **bunch** of new **third-order** signals are created.

Among these signals, of course, are the second **harmonics** $\cos 3\omega_1 t$ and $\cos 3\omega_2 t$. Additionally, however, we get these **new** signals:

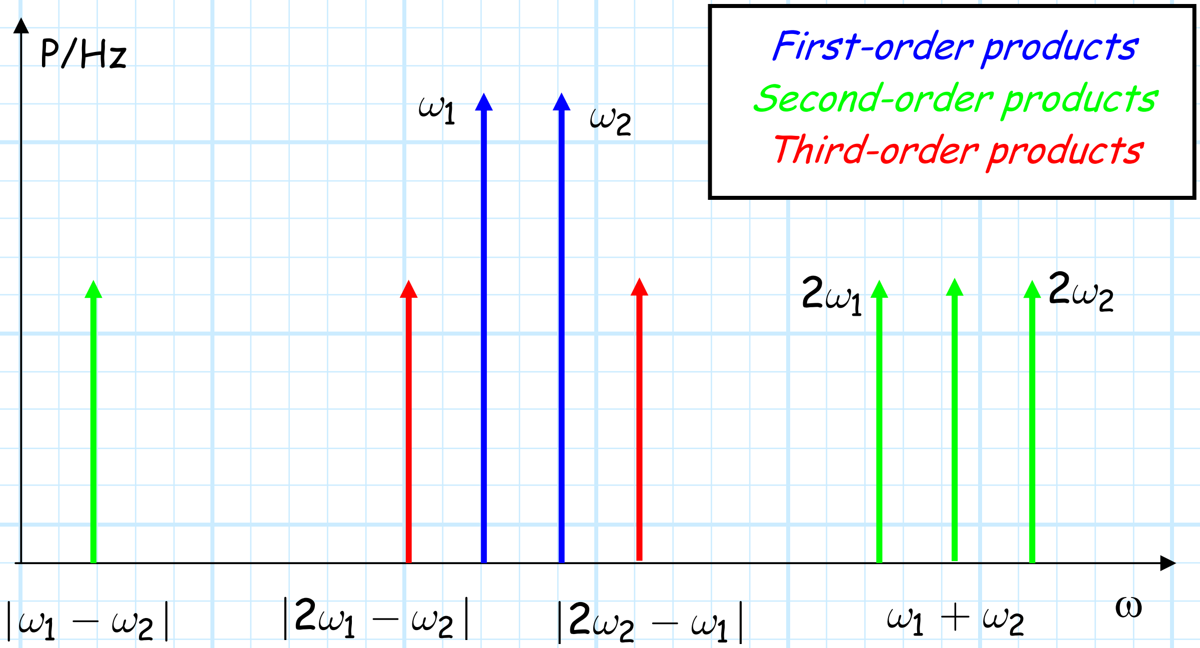
$$\cos(2\omega_2 - \omega_1)t \quad \text{and} \quad \cos(2\omega_1 - \omega_2)t$$

Note since $\cos(-x) = \cos x$, we can **equivalently** write these terms as:

$$\cos(\omega_1 - 2\omega_2)t \quad \text{and} \quad \cos(\omega_2 - 2\omega_1)t$$

Either way, it is apparent that the **third-order** products include signals at frequencies $|\omega_1 - 2\omega_2|$ and $|\omega_2 - 2\omega_1|$.

Now lets look at the output spectrum with **these new** third-order products included:

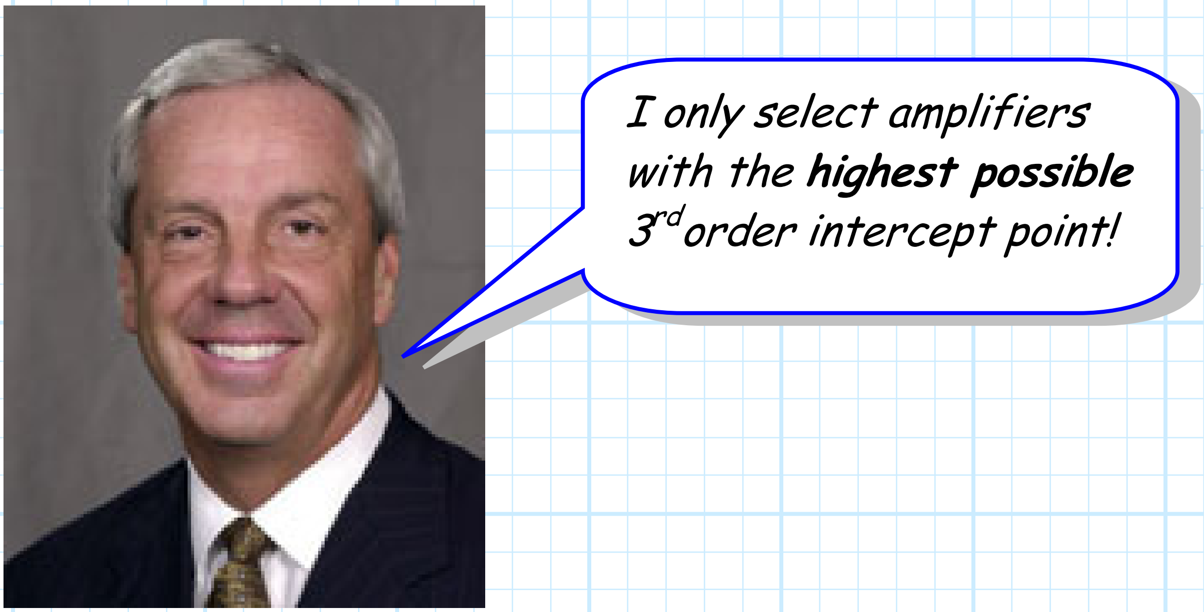


Now **you** should see the problem! **These** third-order products are very **close** in frequency to ω_1 and ω_2. They will likely lie **within** the bandwidth of the amplifier!

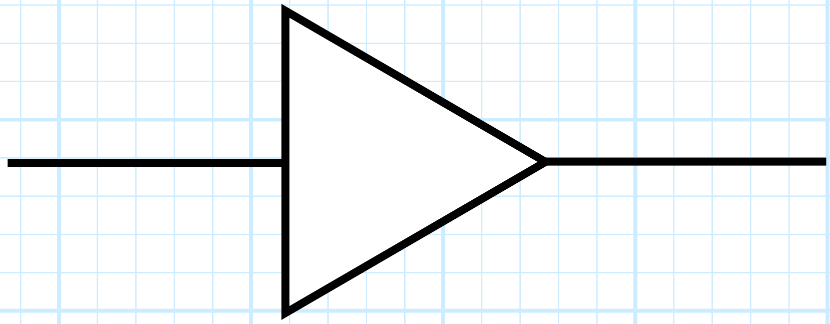
For example, if f_1=100 MHz and f_2=101 MHz, then $2f_2 - f_1$=102 MHz and $2f_1 - f_2$= 99 MHz. All frequencies are **well** within the FM radio bandwidth!

Thus, these **third-order**, **two-tone** intermodulation products are the **most significant** distortion terms.

This is why we are most concerned with the **third-order intercept point** of an amplifier!



The Amplifier Spec Sheet



Here's a list of some of the most **important** amplifier specifications:

Gain (dB)

Provides the "nominal" (← a vendor weasel word) gain within the bandwidth of the amplifier.

Gain Flatness(dB)

Usually expressed as +/- (e.g. +/- 0.5 dB), the specification indicates how much the gain varies across the "middle" portion of the amplifier bandwidth.

Bandwidth (Hz)

Expressed in terms of a low-end frequency (i.e., f_L) and a high-end frequency (i.e., f_H), this value is generally specified as the frequencies where the gain has reduced to 3dB less than its nominal value.

Input Impedance

Can be expressed in a number of ways: S_{11}, Z_{in}, Γ, return loss, or VSWR.

Ouput Impedance

Can be expressed in a number of ways: S_{22}, Z_{in}, Γ, return loss, or VSWR.

Reverse Isolation (dB)

The larger the number, the better.

D.C. Power

Simply the product of the DC current and DC voltage.

1 dB compression point (Watts, dBm, dBw)

Regarded as the maximum **output power** the amplifier can produce.

3^{rd} order intercept point (Watts, dBm, dBw)

The larger the number, the better.

Noise Figure (dB)

We will learn about this later!