Special Problem III.A-1

The vector wave equation:

$$\nabla^2 \bar{E}(\bar{r},t) - \frac{1}{c^2} \frac{\partial^2 \bar{E}(\bar{r},t)}{\partial t^2} = 0$$

can be written in Cartesian coordinates as:

$$\hat{x}\, \nabla^2 E_x(\bar{r},t) + \hat{y}\, \nabla^2 E_y(\bar{r},t) + \hat{z}\, \nabla^2 E_z(\bar{r},t) - \frac{1}{c^2} \frac{\partial^2 \bar{E}(\bar{r},t)}{\partial t^2} = 0$$

where:

$$\bar{E}(\bar{r},t) = E_x(\bar{r},t)\,\hat{x} + E_y(\bar{r},t)\,\hat{y} + E_z(\bar{r},t)\,\hat{z}$$

and $\nabla^2 f(x,y,z) = \nabla \cdot \nabla f(x,y,z)$.

a) Using direct substitution, show that the plane wave:

$$\bar{E}(\bar{r},t) = \hat{x}\, e^{j(kz - \omega t)}$$

is a solution to the wave equation.