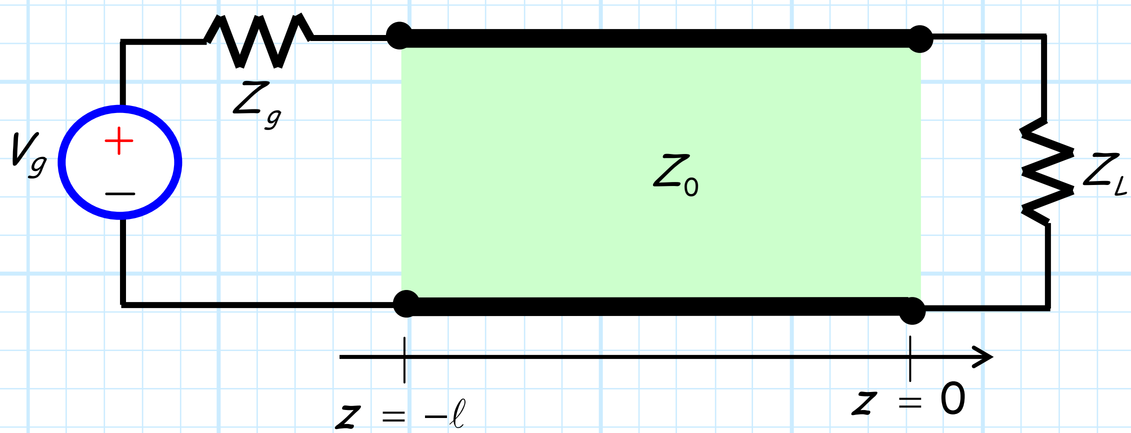
A Transmission Line Connecting Source and Load

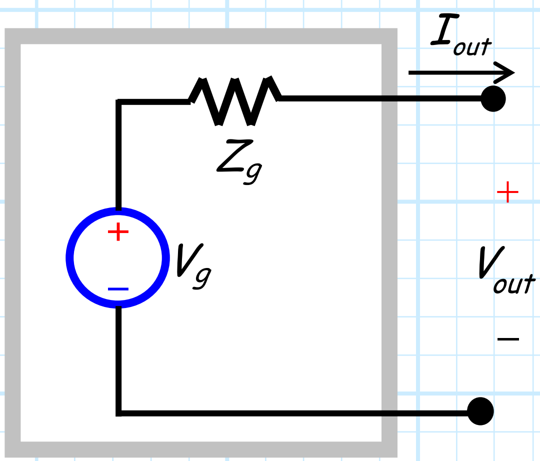
We can think of a transmission line as a conduit that allows **power** to flow **from** an **output** of one device/network **to** an **input** of another.

To simplify our analysis, we modeled the **input** of the device **receiving** the power with it input impedance (e.g., Z_L).



The sources are equivalent circuits

We can similarly model the device **delivering** the power with its Thevenin's or Norton's equivalent circuit.

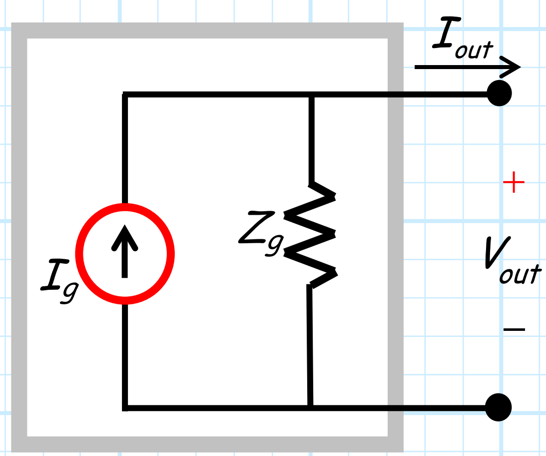


$$V_{out} = V_g - Z_g I_{out}$$

$$I_{out} = \frac{V_g - V_{out}}{Z_g}$$

$$I_{out} = I_g - V_{out}/Z_g$$

$$V_{out} = \left(I_g - I_{out}\right) Z_g$$



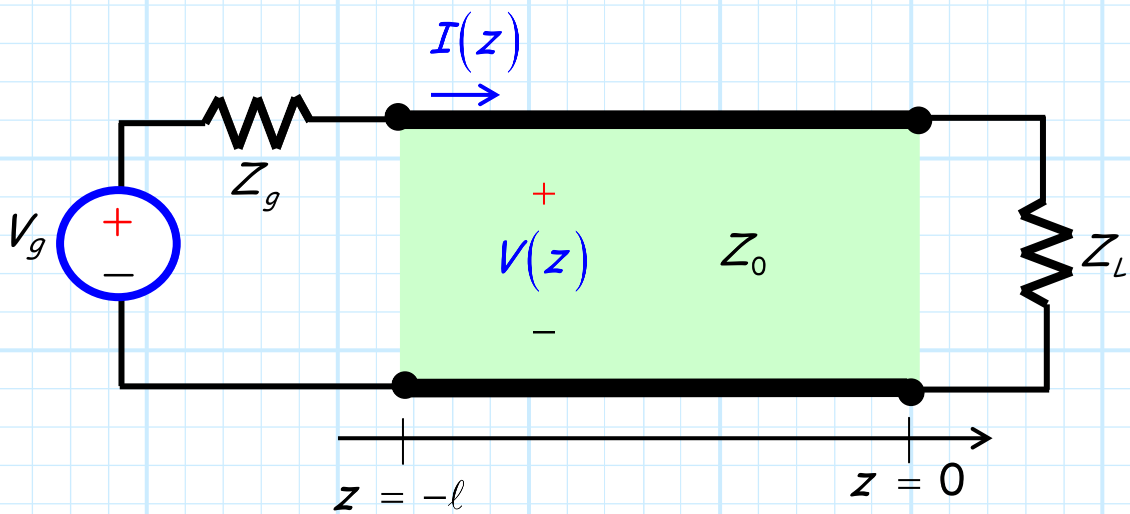
Typically, the power source is modeled with its **Thevenin's** equivalent.

However, we will find that the **Norton's** equivalent circuit is useful if we express the remainder of the circuit in terms of its **admittance** values (e.g., $Y_0, Y_L, Y(z)$).

We've already satisfied one boundary condition

Recall that we applied **boundary conditions** at location $z = 0$ to find that:

$$V(z) = V_0^+ \left[e^{-j\beta z} + \Gamma_L \, e^{+j\beta z} \right] \qquad\qquad I(z) = \frac{V_0^+}{Z_0} \left[e^{-j\beta z} - \Gamma_L \, e^{+j\beta z} \right]$$

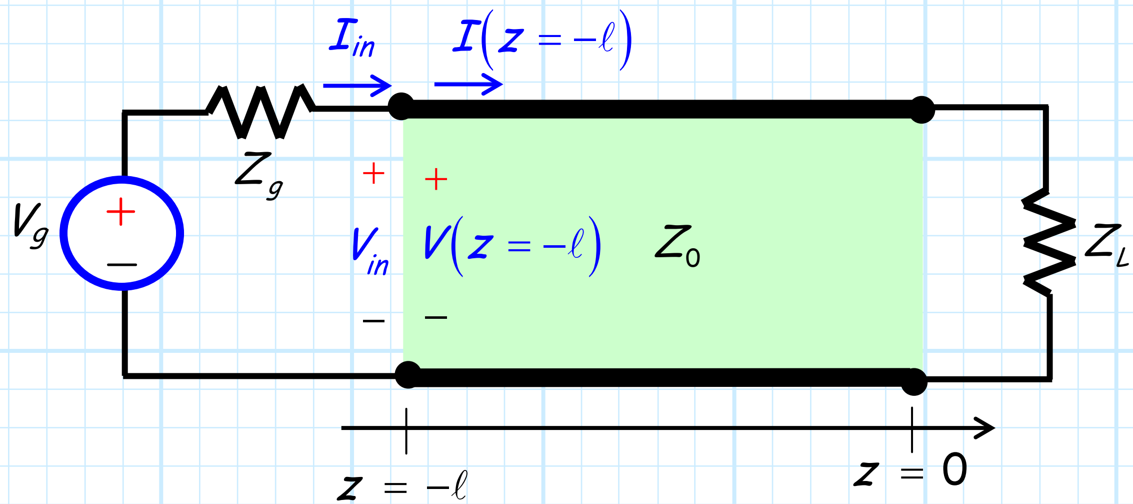


We are left with the **question**: just what is the **value** of complex constant V_0^+?!?

→**This** constant depends on the signal **source**!

We must now satisfy a second boundary condition

To determine its exact **value**, we must now apply **boundary conditions** at $z=-\ell$.



We know that at the **beginning** of the transmission line:

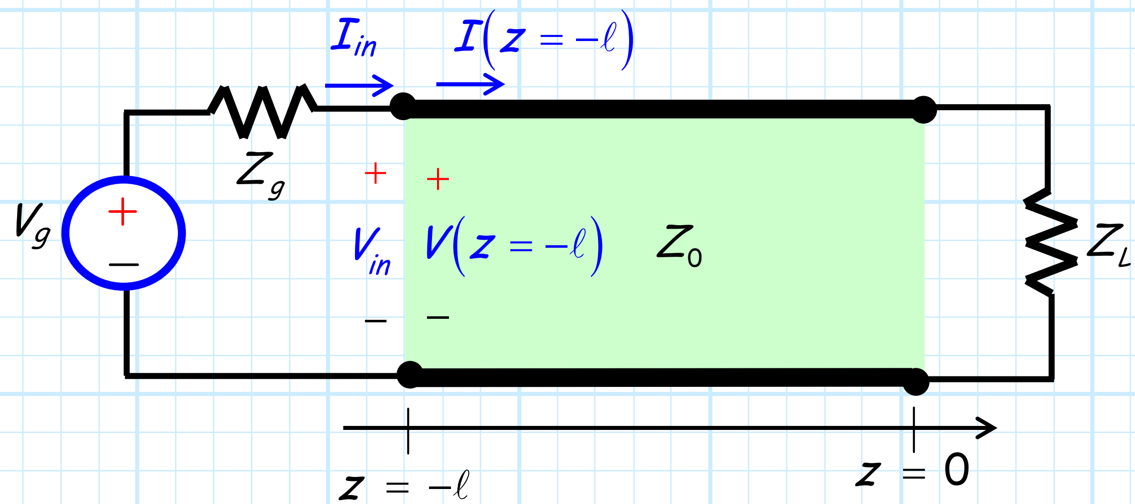
$$V(z=-\ell)=V_0^+\left[e^{+j\beta\ell}+\Gamma_L\, e^{-j\beta\ell}\right] \qquad I(z=-\ell)=\frac{V_0^+}{Z_0}\left[e^{+j\beta\ell}-\Gamma_L\, e^{-j\beta\ell}\right]$$

Likewise, we know that the **source** must satisfy:

$$V_g=V_{in}+Z_g\, I_{in}$$

Good ol' Kirchoff!

To relate these **three** expressions, we need to apply **boundary conditions** at $z=-\ell$:

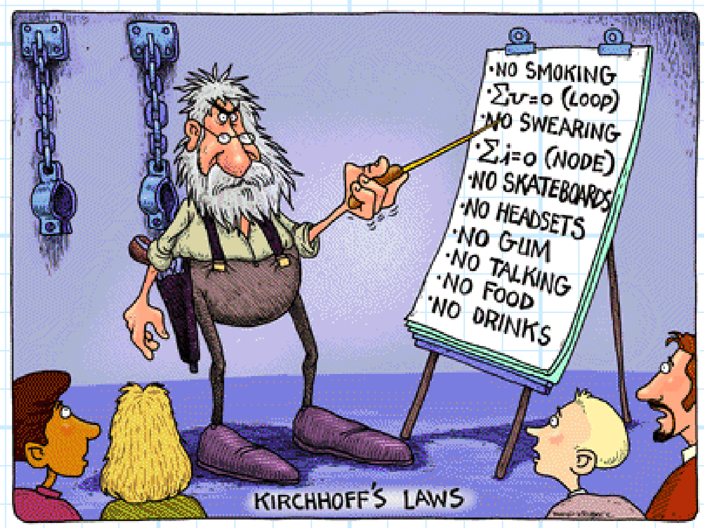


From **KVL** we find:

$$V_i = V(z=-\ell)$$

And from **KCL**:

$$I_i = I(z=-\ell)$$



The solution (yuck)!

Combining these equations, we get:

$$V_g = V_{in} + Z_g I_{in}$$

$$V_g = V_0^+ \left[e^{+j\beta\ell} + \Gamma_L e^{-j\beta\ell} \right] + Z_g \frac{V_0^+}{Z_0} \left[e^{+j\beta\ell} - \Gamma_L e^{-j\beta\ell} \right]$$

One equation → **one** unknown (V_0^+)!!

Solving, we find the value of V_0^+:

$$V_0^+ = V_g e^{-j\beta\ell} \frac{Z_0}{Z_0 \left(1 + \Gamma_{in}\right) + Z_g \left(1 - \Gamma_{in}\right)}$$

where:

$$\Gamma_{in} = \Gamma\left(z = -\ell\right) = \Gamma_L e^{-j2\beta\ell}$$

The plus wave depends on the source *and* load

There is one **very important** point that must be made about the result:

$$V_0^+ = V_g \, e^{-j\beta\ell} \frac{Z_0}{Z_0\left(1+\Gamma_{in}\right)+Z_g\left(1-\Gamma_{in}\right)}$$

And that is—the wave $V_0^+(z)$ **incident** on the load Z_L is actually dependent **on** the value of load Z_L !!!!!

$$\text{Remember:} \quad \Gamma_{in} = \Gamma\left(z=-\ell\right) = \Gamma_L \, e^{-j2\beta\ell}$$

We tend to think of the incident wave $V_0^+(z)$ being "**caused**" by the source, and it is certainly true that $V_0^+(z)$ **depends** on the source—after all, $V_0^+(z)=0$ if $V_g=0$.

However, we find from the equation above that it **likewise** depends on the value of the **load**!

Thus we **cannot**—in general—consider the incident wave to be the "**cause**" and the reflected wave the "**effect**".

Instead, each wave must obtain the proper **amplitude** (e.g., V_0^+, V_0^-) so that the boundary conditions are satisfied at **both** the beginning and end of the transmission line.