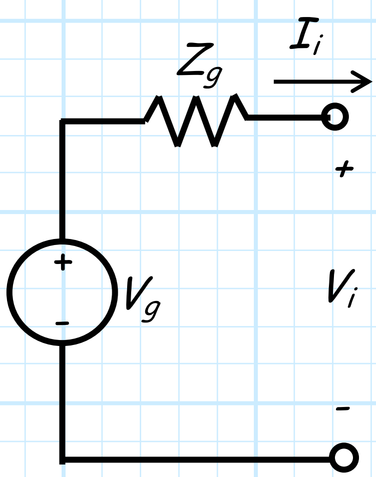
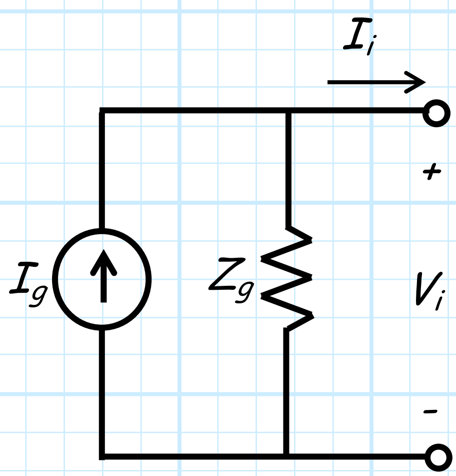
A Transmission Line Connecting Source & Load

We can think of a transmission line as a conduit that allows **power** to flow **from** an **output** of one device/network **to** an **input** of another.

To simplify our analysis, we can model the **input** of the device **receiving** the power with it input impedance (e.g., Z_L), while we can model the device **output delivering** the power with its Thevenin's or Norton's equivalent circuit.

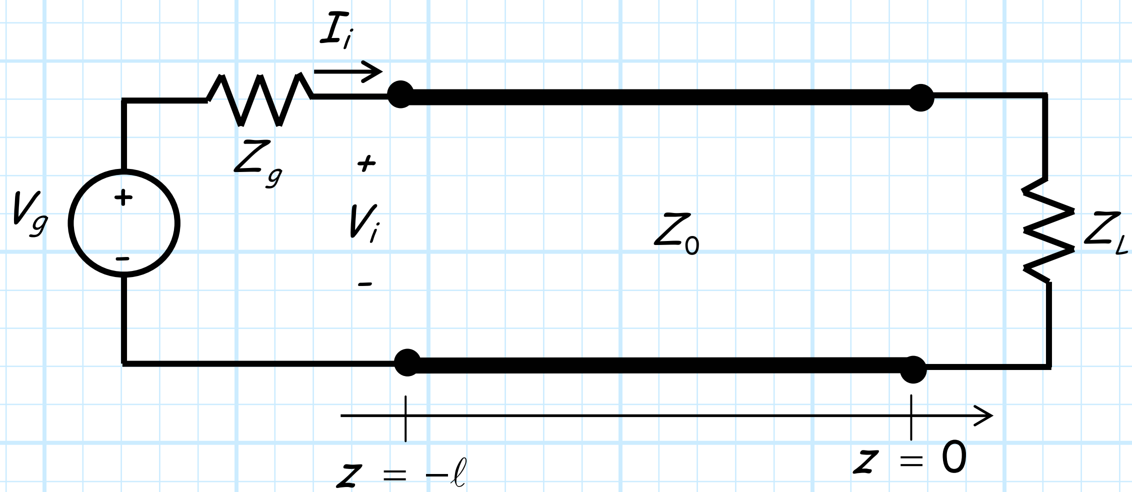




$$V_g = V_i + Z_g I_i$$

$$I_g = \frac{V_i}{Z_g} + I_i$$

Typically, the power source is modeled with its **Thevenin's** equivalent; however, we will find that the **Norton's** equivalent circuit is useful if we express the remainder of the circuit in terms of its **admittance** values (e.g., $Y_0, Y_L, Y(z)$).



Recall from the telegrapher's equations that the current and voltage along the transmission line are:

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{+j\beta z}$$

At $z = 0$, we enforced the **boundary condition** resulting from Ohm's Law:

$$Z_L = \frac{V_L}{I_L} = \frac{V(z=0)}{I(z=0)} = \frac{\left(V_0^+ + V_0^-\right)}{\left(\frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}\right)}$$

Which resulted in:

$$\frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} \doteq \Gamma_L$$

So therefore:

$$V(z) = V_0^+ \left[e^{-j\beta z} + \Gamma_L \, e^{+j\beta z} \right]$$

$$I(z) = \frac{V_0^+}{Z_0} \left[e^{-j\beta z} - \Gamma_L \, e^{+j\beta z} \right]$$

We are left with the **question**: just what is the **value** of complex constant V_0^+ ?!?

This constant depends on the signal **source**! To determine its exact **value**, we must now apply **boundary conditions** at $z = -\ell$.

We know that at the **beginning** of the transmission line:

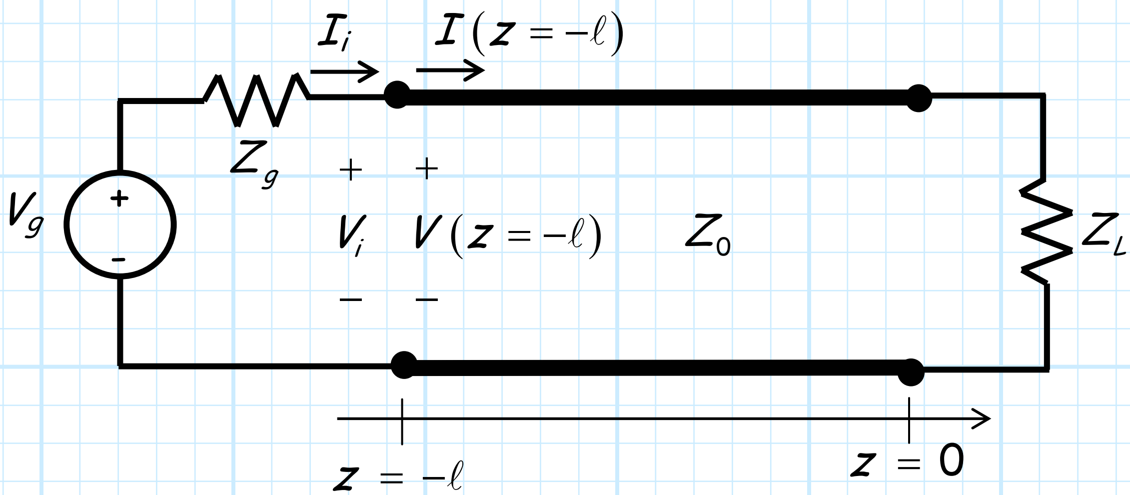
$$V(z = -\ell) = V_0^+ \left[e^{+j\beta \ell} + \Gamma_L \, e^{-j\beta \ell} \right]$$

$$I(z = -\ell) = \frac{V_0^+}{Z_0} \left[e^{+j\beta \ell} - \Gamma_L \, e^{-j\beta \ell} \right]$$

Likewise, we know that the **source** must satisfy:

$$V_g = V_i + Z_g \, I_i$$

To relate these **three** expressions, we need to apply **boundary conditions** at $z = -\ell$:



From **KVL** we find:

$$V_i = V(z = -\ell)$$

And from **KCL**:

$$I_i = I(z = -\ell)$$

Combining these equations, we find:

$$V_g = V_i + Z_g I_i$$

$$V_g = V_0^+ \left[e^{+j\beta\ell} + \Gamma_L e^{-j\beta\ell} \right] + Z_g \frac{V_0^+}{Z_0} \left[e^{+j\beta\ell} - \Gamma_L e^{-j\beta\ell} \right]$$

One equation → **one** unknown (V_0^+)!!

Solving, we find the value of V_0^+:

$$V_0^+ = V_g\, e^{-j\beta\ell} \frac{Z_0}{Z_0\left(1+\Gamma_{in}\right)+Z_g\left(1-\Gamma_{in}\right)}$$

where:

$$\Gamma_{in} = \Gamma\left(z=-\ell\right) = \Gamma_L\, e^{-j2\beta\ell}$$

Note this result looks different than the equation in your textbook (eq. 2.71):

$$V_0^+ = V_g \frac{Z_0}{Z_0+Z_g} \frac{e^{-j\beta\ell}}{\left(1-\Gamma_L\Gamma_g e^{-j2\beta\ell}\right)}$$

where:

$$\Gamma_g \doteq \frac{Z_g - Z_0}{Z_g + Z_0}$$

I like **my** expression better.

Although the two equations are equivalent, **my** expression is explicitly written in terms of $\Gamma_{in} = \Gamma\left(z=-\ell\right)$ (a very **useful**, **precise**, and **unambiguous** value), while the book's expression is written in terms of this **so-called** "source reflection coefficient" Γ_g (a **misleading**, **confusing**, **ambiguous**, and mostly **useless** value).

Specifically, we might be **tempted** to equate Γ_g with the value $\Gamma\left(z=-\ell\right)=\Gamma_{in}$, but it is **not** $\left(\Gamma_g \neq \Gamma\left(z=-\ell\right)\right)$!

There is one **very important** point that must be made about the result:

$$V_0^+ = V_g\, e^{-j\beta\ell} \frac{Z_0}{Z_0\left(1+\Gamma_{in}\right)+Z_g\left(1-\Gamma_{in}\right)}$$

And that is—the wave $V_0^+(z)$ **incident** on the load Z_L is actually dependent **on** the value of load Z_L !!!!!

Remember:

$$\Gamma_{in} = \Gamma(z=-\ell) = \Gamma_L\, e^{-j2\beta\ell}$$

We tend to think of the incident wave $V_0^+(z)$ being "**caused**" by the source, and it is certainly true that $V_0^+(z)$ **depends** on the source—after all, $V_0^+(z)=0$ if $V_g=0$. However, we find from the equation above that it **likewise** depends on the value of the load!

Thus we **cannot**—in general—consider the incident wave to be the "**cause**" and the reflected wave the "**effect**". Instead, each wave must obtain the proper **amplitude** (e.g., V_0^+, V_0^-) so that the boundary conditions are satisfied at **both** the beginning and end of the transmission line.