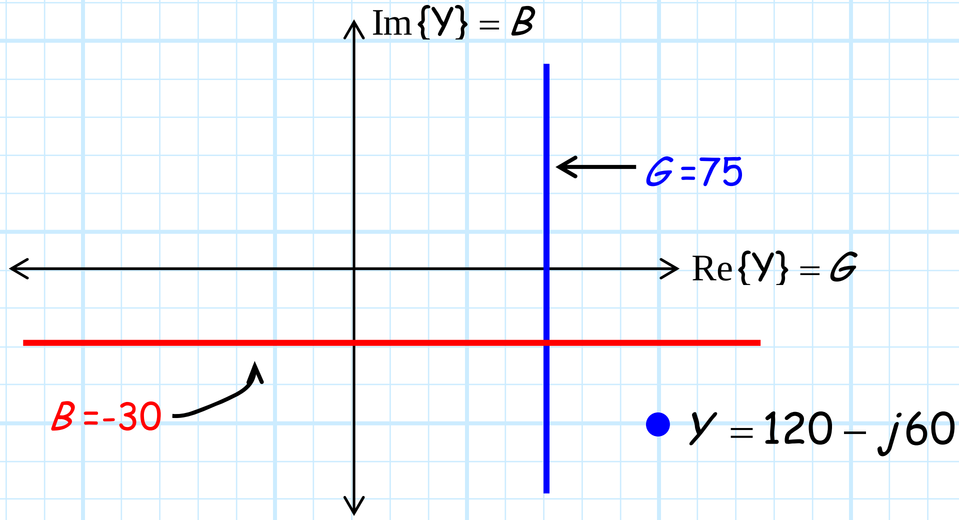
Admittance and the Smith Chart

Just like the complex impedance plane, we can plot points and contours on the complex **admittance** plane:



Q: *Can we also map **these** points and contours onto the complex Γ plane?*

A: You bet! Let's first rewrite the refection coefficient function in terms of **line admittance** $Y(z)$:

$$\begin{aligned}\Gamma(z) &= \frac{Z(z) - Z_0}{Z(z) + Z_0} \\ &= \frac{1/Y(z) - 1/Y_0}{1/Y(z) - 1/Y_0}\left(\frac{Y(z)Y_0}{Y(z)Y_0}\right) \\ &= \frac{Y_0 - Y(z)}{Y_0 + Y(z)}\end{aligned}$$

Thus,

$$\Gamma_L = \frac{Y_0 - Y_L}{Y_0 + Y_L} \qquad \text{and} \qquad \Gamma_{in} = \frac{Y_0 - Y_{in}}{Y_0 + Y_{in}}$$

We can therefore likewise express Γ in terms of **normalized** admittance:

$$\Gamma = \frac{Y_0 - Y}{Y_0 + Y} = \frac{1 - Y/Y_0}{1 + Y/Y_0} = \frac{1 - y'}{1 + y'}$$

Note this can likewise be expressed as:

$$\Gamma = \frac{1 - y'}{1 + y'} = -\frac{y' - 1}{y' + 1} = e^{j\pi}\,\frac{y' - 1}{y' + 1}$$

Contrast this to the mapping between normalized impedance and Γ:

$$\Gamma = \frac{z' - 1}{z' + 1}$$

The difference between the two is simply the factor $e^{j\pi}$—a rotation of 180° around the Smith Chart!.

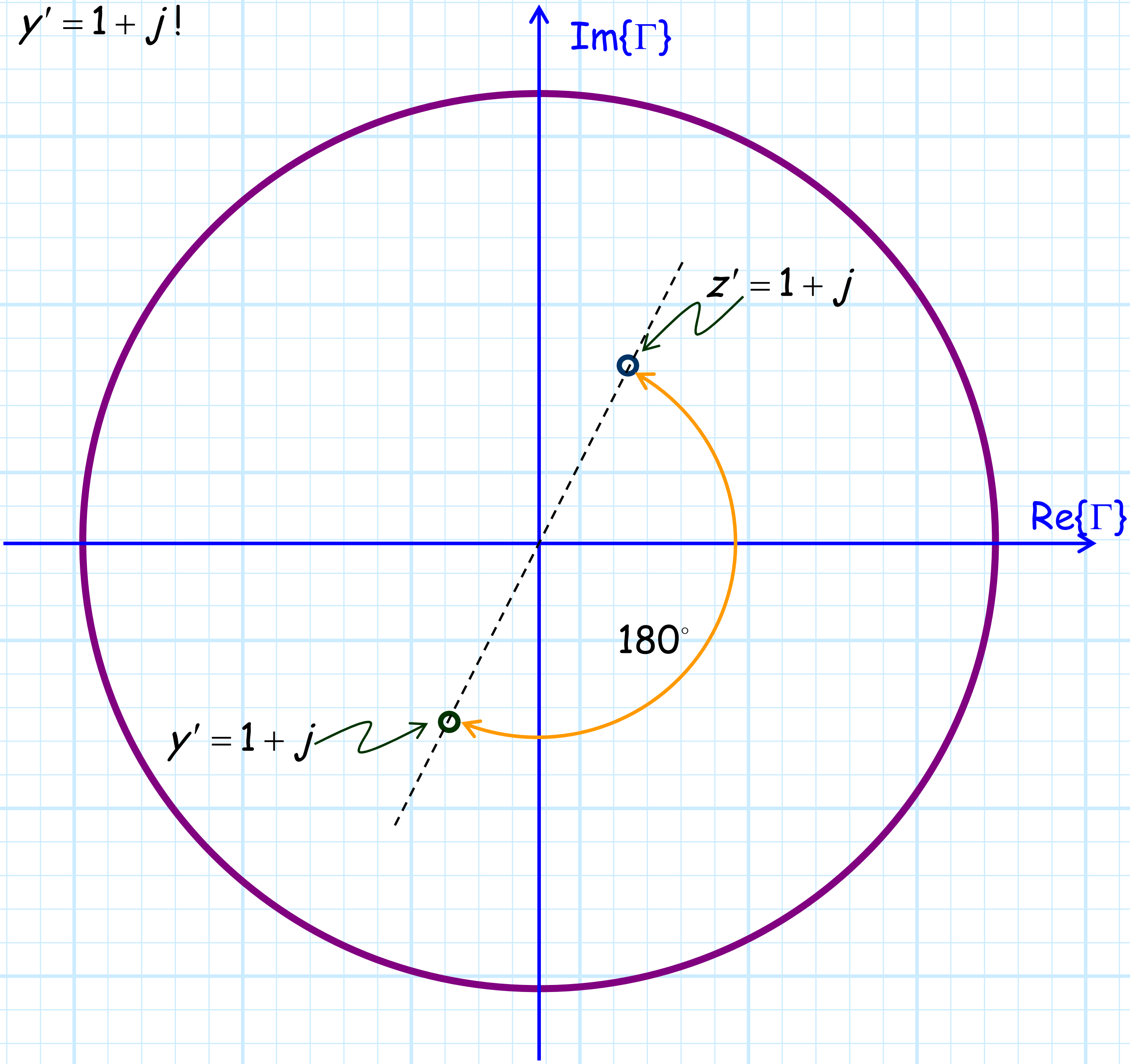
For example, let's pick some load at random; $z' = 1 + j$, for instance. We know where this point is mapped onto the complex Γ plane; we can locate it on our **Smith Chart**.

Now let's consider a different load, and express it in terms of its normalized admittance—an admittance that has the same

numerical value as the impedance of the first load (i.e., $y' = 1 + j$).

Q: ***Where*** *would this admittance value map onto the complex Γ plane?*

A: Start at the location $z' = 1 + j$ on the Smith Chart, and then rotate around the center 180°. You are now at the proper location on the complex Γ plane for the admittance $y' = 1 + j$!



We of course could just directly calculate Γ from the equation above, and then plot that point on the Γ plane.

Note the reflection coefficient for $z' = 1 + j$ is:

$$\Gamma = \frac{z'-1}{z'+1} = \frac{1+j-1}{1+j+1} = \frac{j}{2+j}$$

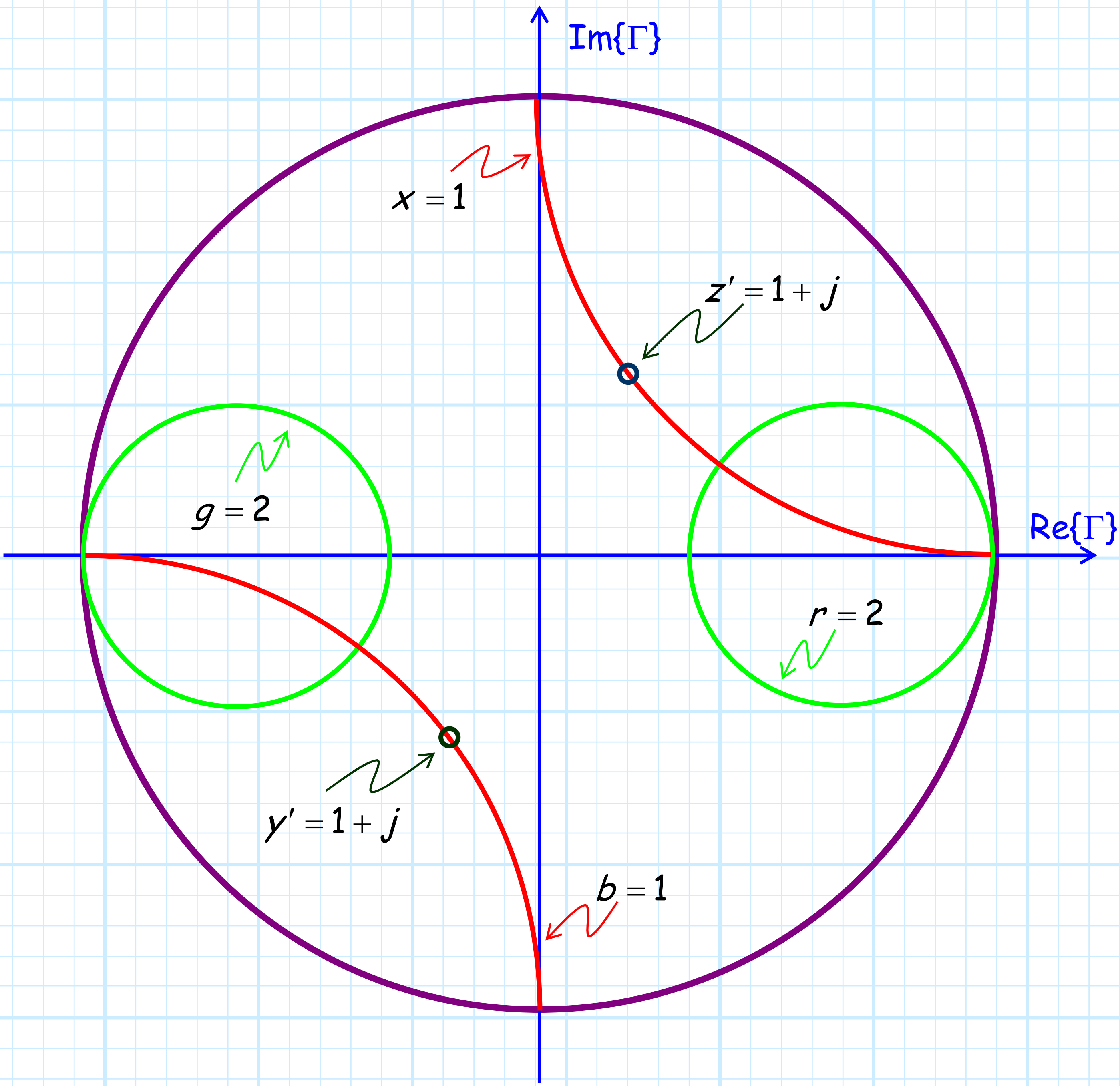
while the reflection coefficient for $y' = 1 + j$ is:

$$\Gamma = \frac{1-y'}{1+y'} = \frac{1-(1+j)}{1+(1+j)} = \frac{-j}{2+j}$$

Note the two results have **equal** magnitude, but are separated in **phase** by 180° $(-1 = e^{j\pi})$. This means that the two loads occupy points on the complex Γ plane that are a 180° **rotation** from each other!

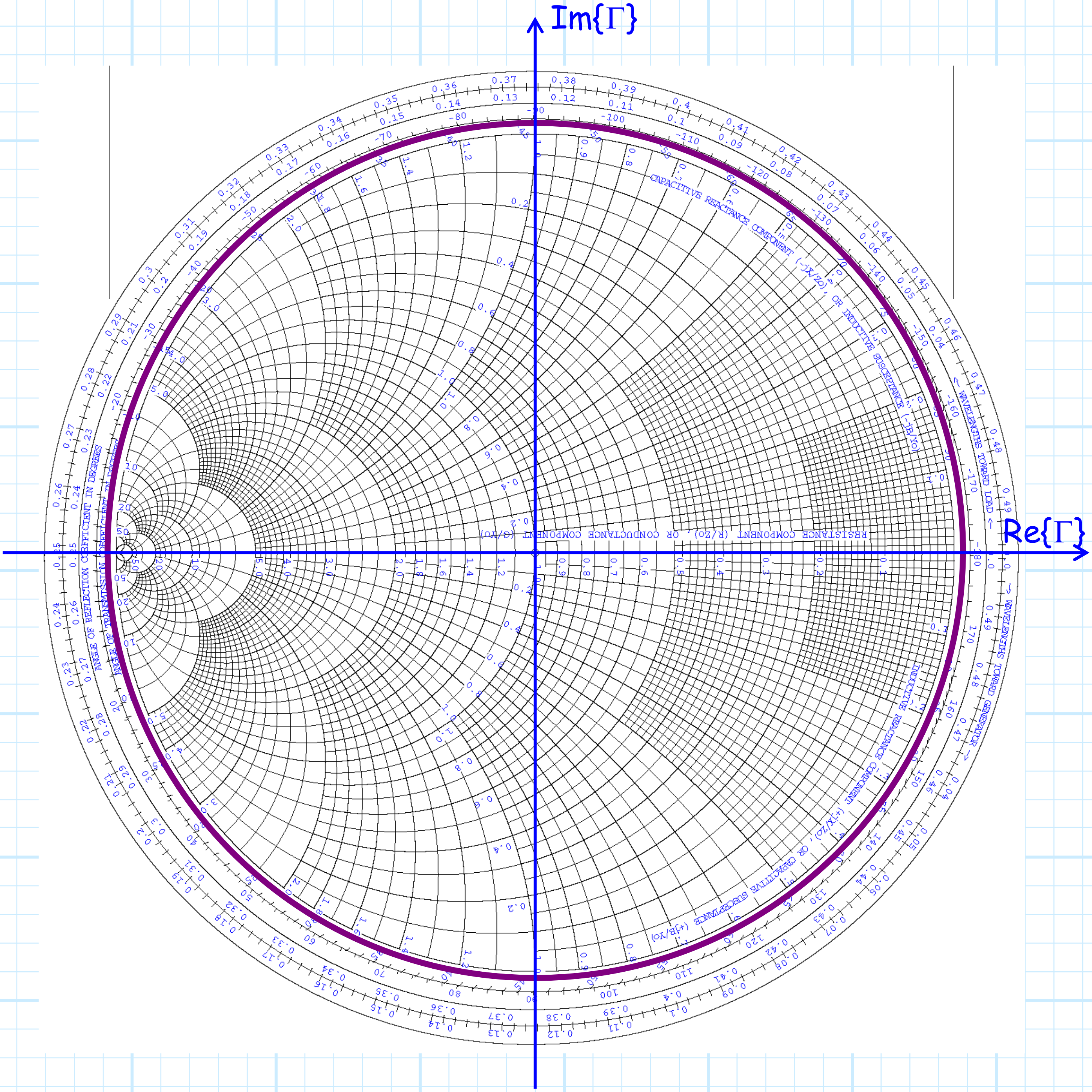
Moreover, this is a true statement not **just** for the point we randomly picked, but is true for **any** and **all** values of z' and y' mapped onto the complex Γ plane, provided that $z' = y'$.

For example, the g =2 circle mapped on the complex plane can be determined by **rotating** the r =2 circle 180° around the complex Γ plane, and the b =-1 contour can be found by rotating the x =-1 contour 180° around the complex Γ plane.



Thus, rotating **all** the resistance circles and reactance contours of the Smith Chart 180° around the complex Γ plane provides us a mapping of complex **admittance** onto the complex Γ plane:

The Admittance Smith Chart



Note that circles and contours have been rotated with **respect** to the complex Γ plane—the complex Γ plane remains **unchanged**!

This result should **not** surprise us. Recall the case where a transmission line of length $\ell = \lambda/4$ is terminated with a load of impedance z'_L (or equivalently, admittance y'_L). The input impedance (admittance) for this case is:

$$Z_{in} = \frac{Z_0^2}{Z_L} \quad \Rightarrow \quad \frac{Z_{in}}{Z_0} = \frac{Z_0}{Z_L} \quad \Rightarrow \quad z'_{in} = \frac{1}{z'_L} = y'_L$$

In other words, when $\ell = \lambda/4$, the input impedance is **numerically** equal to the load admittance—and **vice versa**!

But note that if $\ell = \lambda/4$, then $2\beta\ell = \pi$ --a rotation around the Smith Chart of 180°!