Impedance & Admittance

As an alternative to impedance Z, we can define a complex parameter called **admittance** Y:

$$Y = \frac{I}{V}$$

where V and I are complex voltage and current, respectively.

Clearly, admittance and impedance are not independent parameters, and are in fact simply geometric **inverses** of each other:

$$Y = \frac{1}{Z} \qquad Z = \frac{1}{Y}$$

Thus, all the impedance parameters that we have studied can be **likewise** expressed in terms of admittance, e.g.:

$$Y(z) = \frac{1}{Z(z)} \qquad Y_L = \frac{1}{Z_L} \qquad Y_{in} = \frac{1}{Z_{in}}$$

Moreover, we can define the **characteristic admittance** Y_0 of a transmission line as:

$$Y_0 = \frac{I^+(z)}{V^+(z)}$$

And thus it is similarly evident that characteristic impedance and characteristic admittance are geometric **inverses**:

$$Y_0 = \frac{1}{Z_0} \qquad\qquad Z_0 = \frac{1}{Y_0}$$

As a result, we can define a **normalized admittance** value y':

$$y' = \frac{Y}{Y_0}$$

An therefore (not surprisingly) we find:

$$y' = \frac{Y}{Y_0} = \frac{Z_0}{Z} = \frac{1}{z'}$$

Note that we can express normalized impedance and admittance more compactly as:

$$y' = Y\,Z_0 \qquad \text{and} \qquad z' = Z\,Y_0$$

Now since admittance is a **complex** value, it has both a real and imaginary component:

$$Y = G + j\,B$$

where:

$$\text{Re}\{Y\} \doteq G = \text{ Conductance}$$

$$\text{Im}\{Z\} \doteq B = \text{ Susceptance}$$

Now, since $Z = R + jX$, we can state that:

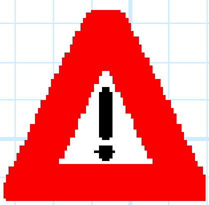
$$G + jB = \frac{1}{R + jX}$$

Steve Marcus / Reuters

Q: *Yes yes, I see, and from this we can conclude:*

$$G = \frac{1}{R} \quad \text{and} \quad B = \frac{-1}{X}$$

*and so forth. Please speed this up and quit wasting my valuable time making such **obvious** statements!*



A: **NOOOO!** We find that $G \neq 1/R$ and $B \neq 1/X$ (generally). Do **not** make this mistake!

In fact, we find that

$$\begin{aligned} G + jB &= \frac{1}{R + jX}\frac{R - jX}{R - jX} \\ &= \frac{R - jX}{R^2 + X^2} \\ &= \frac{R}{R^2 + X^2} - j\frac{X}{R^2 + X^2} \end{aligned}$$

Thus, equating the real and imaginary parts we find:

$$G = \frac{R}{R^2 + X^2} \qquad \text{and} \qquad B = \frac{-X}{R^2 + X^2}$$

Note then that **IF** $X = 0$ (i.e., $Z = R$), we get, as expected:

$$G = \frac{1}{R} \qquad \text{and} \qquad B = 0$$

And that **IF** $R = 0$ (i.e., $Z = R$), we get, as expected:

$$G = 0 \qquad \text{and} \qquad B = \frac{-1}{X}$$