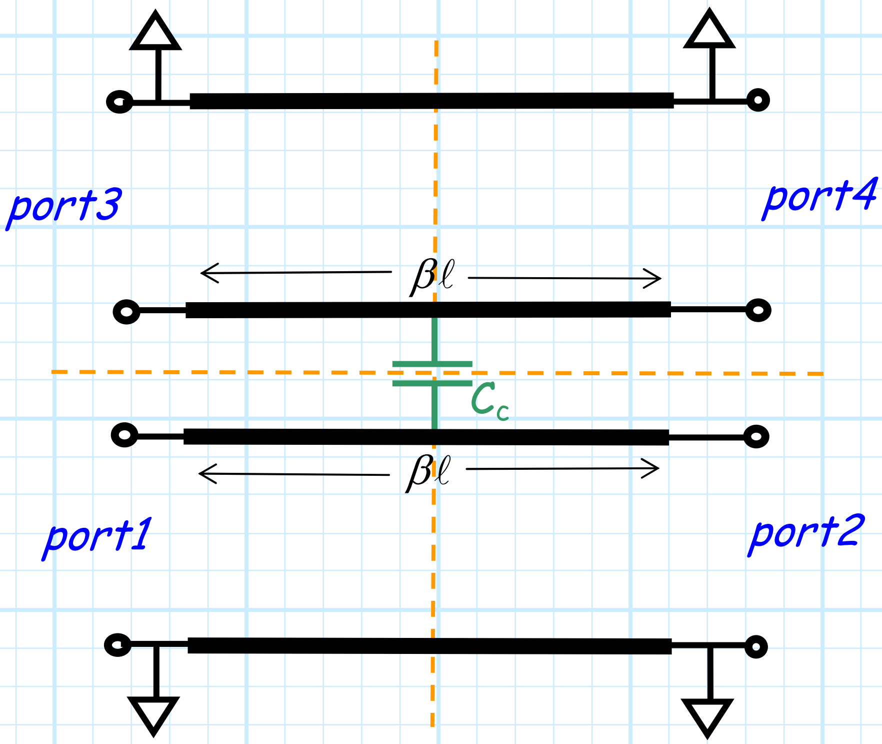
# Analysis and Design of Coupled-Line Couplers

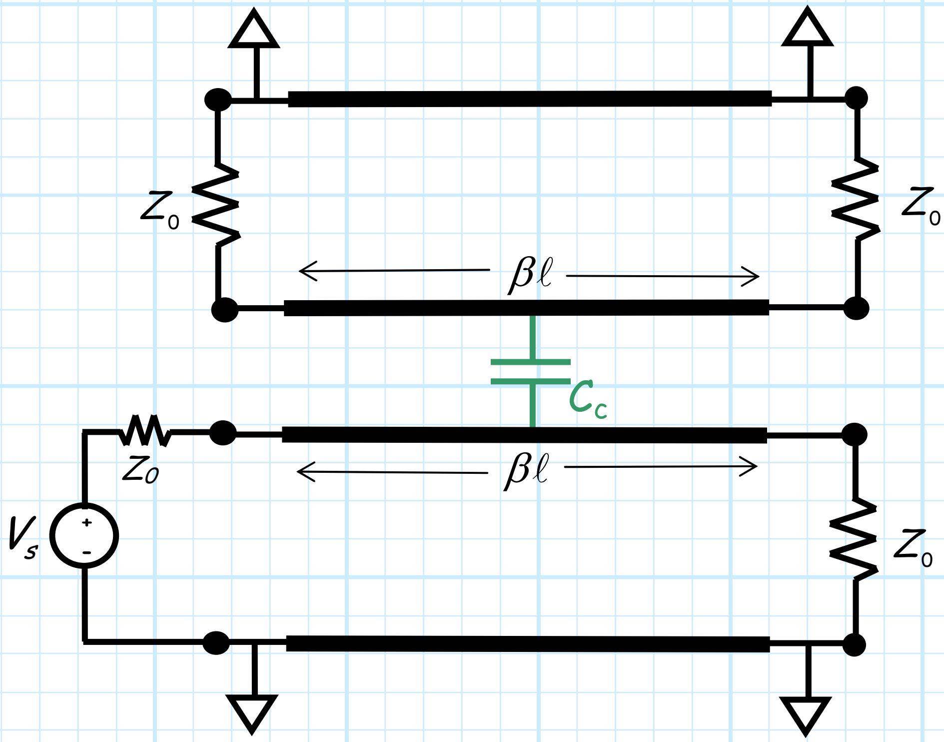
A pair of coupled lines forms a **4-port** device with **two** planes of reflection symmetry—it exhibits $D_4$ symmetry.



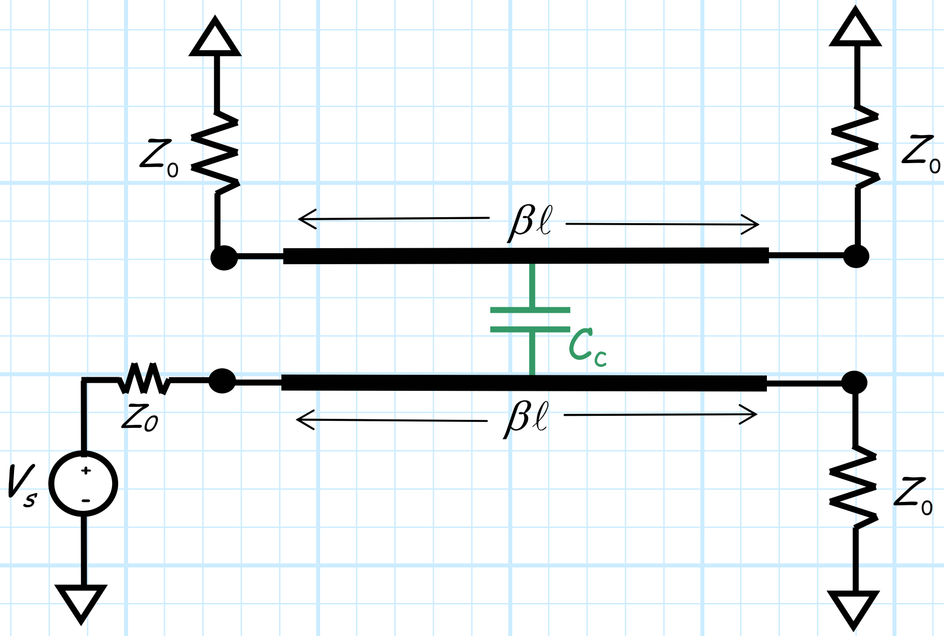

As a result, we know that the **scattering matrix** of this four-port device has just **4 independent** elements:

$$\mathcal{S} = \begin{bmatrix} S_{11} & S_{21} & S_{31} & S_{41} \\ S_{21} & S_{11} & S_{41} & S_{31} \\ S_{31} & S_{41} & S_{11} & S_{21} \\ S_{41} & S_{31} & S_{21} & S_{11} \end{bmatrix}$$

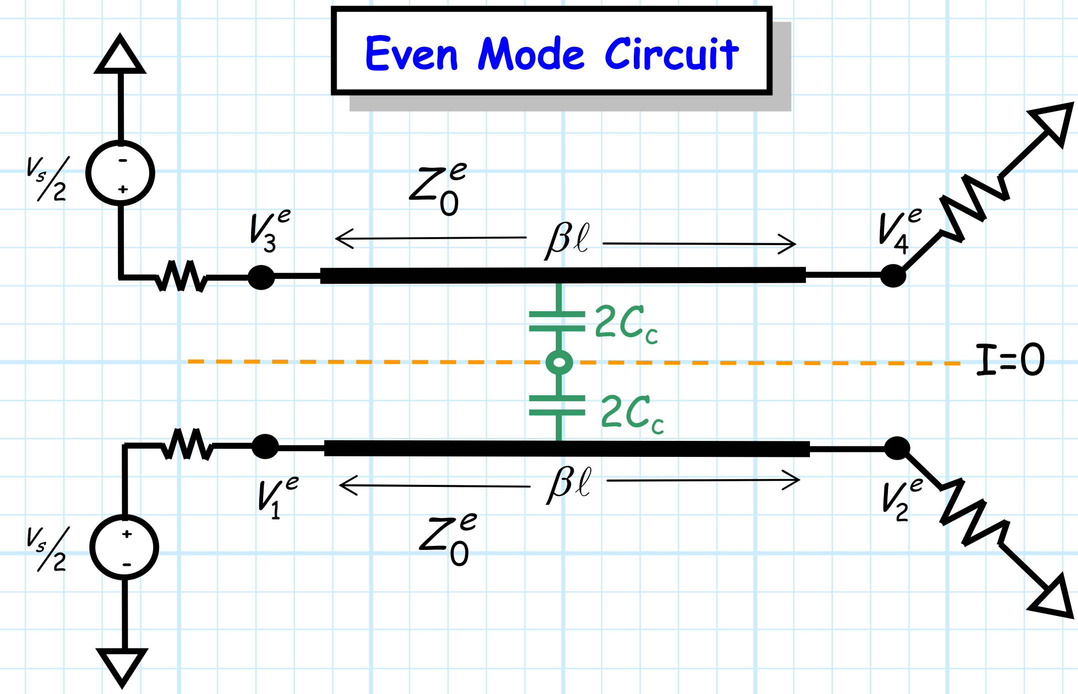
To determine these four elements, we can apply a **source to port 1** and then **terminate** all other ports:

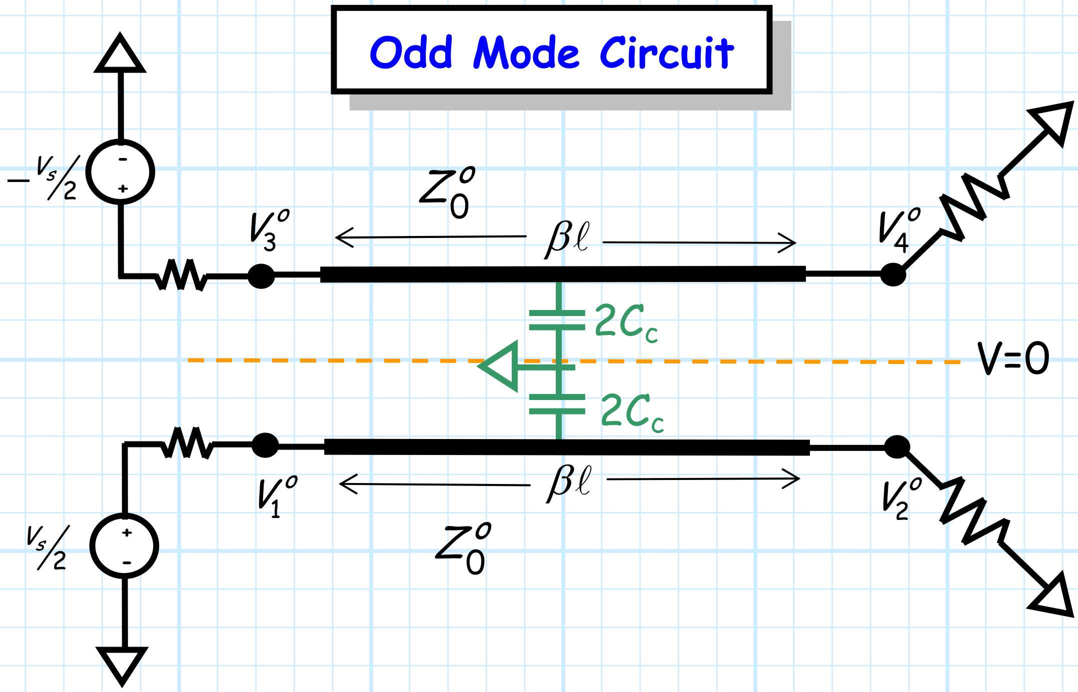



Typically, a coupled-line coupler schematic is drawn **without** explicitly showing the **ground conductors** (i.e., without the ground plane):



To analyze this circuit, we must apply **odd/even mode analysis**. The two circuit analysis modes are:



Note that the **capacitive coupling** associated with these modes are different, resulting in a **different** characteristic impedance of the lines for the two cases (i.e., $Z_0^e, Z_0^o$).

**Q:** *So what?*

**A:** Consider what would happen if the characteristic impedance of each line where **identical** for **each mode**:

$$Z_0^e = Z_0^o = Z_0$$

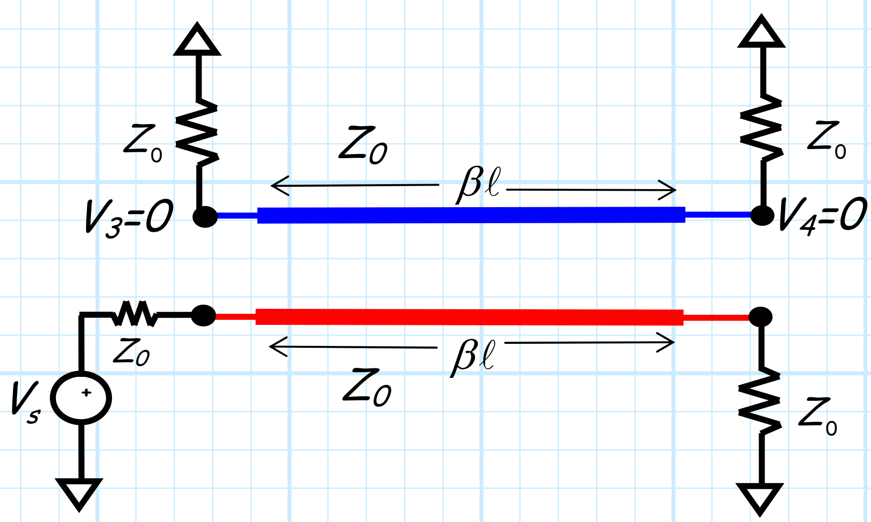
For this situation we would find that:

$$V_3^e = -V_3^o \qquad \text{and} \qquad V_4^e = -V_4^o$$

and thus when applying **superposition**:

$$V_3 = V_3^e + V_3^o = 0 \qquad \text{and} \qquad V_4 = V_4^e + V_4^o = 0$$

indicating that **no power is coupled** from the **"energized"** transmission line onto the **"passive"** transmission line.



This makes sense! After all, **if** no coupling occurs, then the characteristic impedance of each line is **unaltered** by the presence of the other—their characteristic impedance is $Z_0$ **regardless** of "mode".

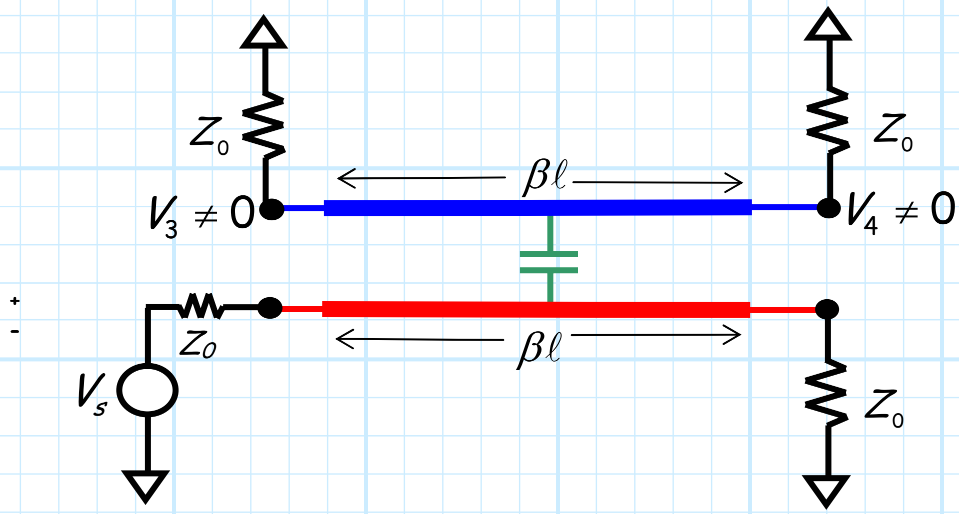
However, if coupling **does** occur, then $Z_0^e \neq Z_0^o$, meaning in general:

$$V_3^e \neq -V_3^o \qquad \text{and} \qquad V_4^e \neq -V_4^o$$

and thus in general:

$$V_3 = V_3^e + V_3^o \neq 0 \qquad \text{and} \qquad V_4 = V_4^e + V_4^o \neq 0$$

The odd/even mode analysis thus reveals the amount of **coupling from** the energized section **onto** the passive section!




Now, our **first step** in performing the odd/even mode analysis will be to determine scattering parameter $S_{11}$. To accomplish this, we will need to determine voltage $V_1$:

$$V_1 = V_1^e + V_1^o$$

The result is a bit complicated, so it won't be presented here. However, a question we might ask is, what value **should** $S_{11}$ be?

**Q:** *What value should $S_{11}$ be?*

**A:** For the device to be a **matched** device, it must be **zero**!

From the value of $S_{11}$ derived from our odd/even analysis, ICBST (it can be shown that) $S_{11}$ will be equal to zero **if** the odd and even mode characteristic impedances are related as:

$$Z_0^e Z_o^o = Z_0^2$$

In other words, we should design our coupled line coupler such that the **geometric mean** of the even and odd mode impedances is **equal to** $Z_0$.

Now, assuming this design rule has been implemented, we also find (from odd/even mode analysis) that the scattering parameter $S_{31}$ is:

$$S_{31}(\beta) = \frac{j\left(Z_0^e - Z_0^o\right)}{2Z_0 \cot \beta\ell + j\left(Z_0^e + Z_0^o\right)}$$

Thus, we find that **unless** $Z_0^e = Z_0^o$, power must be coupled from port 1 to port 3!

**Q:** *But what is the value of line **electrical length** $\beta\ell$ ?*

**A:** The **electrical length** of the coupled transmission lines is also a **design parameter**. Assuming that we want to **maximize** the coupling onto port 3 (at design frequency $\omega_0$), we find from the expression above that this is accomplished if we set $\beta_0 \ell$ such that:

$$cot\, \beta_0 \ell = 0$$

Which occurs when the **line length** is set to:

$$\beta_0 \ell = \pi/2 \qquad \Rightarrow \qquad \ell = \lambda_0/4$$

**Once again**, our design rule is to set the transmission line length to a value equal to **one-quarter wavelength** (at the design frequency).

$$\boxed{\ell = \lambda_0/4}$$

Implementing these **two** design rules, we find that at the design frequency:

$$S_{31} = \frac{Z_0^e - Z_0^o}{Z_0^e + Z_0^o}$$

This value is a **very** important one with respect to coupler performance. Specifically, it is the **coupling coefficient** $c$!

$$\boxed{c = \frac{Z_0^e - Z_0^o}{Z_0^e + Z_0^o}}$$

Given this definition, we can **rewrite** the scattering parameter $S_{31}$ as:

$$S_{31}(\beta) = \frac{jc \;\; tan\, \beta\ell}{\sqrt{1-c^2} + j\, tan\, \beta\ell}$$

**Continuing** with our odd/even mode analysis, we find (given that $Z_0^e Z_o^o = Z_0^2$:

$$S_{21} = \frac{\sqrt{1-c^2}}{\sqrt{1-c^2}\; cos\, \beta\ell + j\, sin\, \beta\ell}$$

and so at our **design frequency**, where $\beta_0 \ell = \pi/2$, we find:

$$S_{21}(\beta)\Big|_{\beta\ell = \pi/2} = \frac{\sqrt{1-c^2}}{\sqrt{1-c^2}\,(0) + j\,(1)} = -j\sqrt{1-c^2}$$

**Finally**, our odd/even analysis reveals that at our design frequency:

$$S_{41} = 0$$

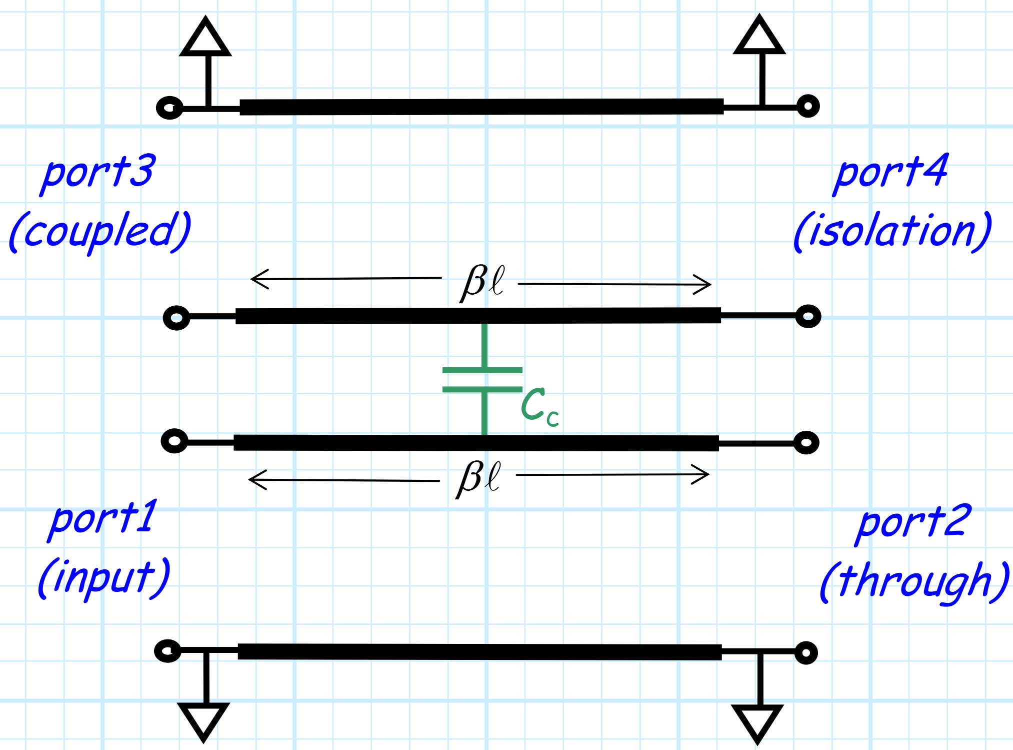
**Combining** these results, we find that at our **design frequency**, the **scattering matrix** of our coupled-line coupler is:

$$\mathcal{S} = \begin{bmatrix} 0 & -j\sqrt{1-c^2} & c & 0 \\ -j\sqrt{1-c^2} & 0 & 0 & c \\ c & 0 & 0 & -j\sqrt{1-c^2} \\ 0 & c & -j\sqrt{1-c^2} & 0 \end{bmatrix}$$

**Q:** *Hey! Isn't this the same scattering matrix as the* ***ideal symmetric directional coupler*** *we studied in the first section of this chapter?*

**A:** The very same! The coupled-line coupler—**if** our design rules are followed—results in an "ideal" directional coupler.

If the **input** is port 1, then the **through** port is port 2, the **coupled** port is port 3, and the **isolation** port is port 4!




**Q:** *But, how do we* ***design*** *a coupled-line coupler with a* ***specific*** *coupling coefficient c?*

**A:** Given our **two design constraints**, we know that:

$$Z_0^e Z_o^o = Z_0^2 \qquad \text{and} \qquad c = \frac{Z_0^e - Z_0^o}{Z_0^e + Z_0^o}$$

We can **rearrange** these two expressions to find **solutions** for our odd and even mode impedances:

$$Z_0^e = Z_0\sqrt{\frac{1+c}{1-c}} \qquad Z_0^o = Z_0\sqrt{\frac{1-c}{1+c}}$$

Thus, **given** the desired values $Z_0$ and $c$, we can determine the proper values of $Z_0^e$ and $Z_o^o$ for an ideal directional coupler.

**Q:** *Yes, but the odd and even mode impedance depends on the* ***physical structure*** *of the coupled lines, such as substrate dielectric $\varepsilon_r$, substrate thickness (d or b), conductor width W, and separation distance S.*

*How do we determine* ***these*** *physical design parameters for desired values of $Z_0^e$ and $Z_o^o$ ??*

**A:** That's a much more difficult question to answer! Recall that there is **no** direct formulation relating microstrip and stripline parameters to **characteristic impedance** (we only have numerically derived **approximations**).

* So it's no surprise that there is likewise **no direct formulation** relating odd and even mode characteristic impedances to the specific physical parameters of microstrip and stripline coupled lines.

* Instead, we again have numerically derived **approximations** that allow us to determine (approximately) the required microstrip and stripline parameters, or we can use a **microwave CAD packages** (like ADS!).

* For example, **figures 7.29 and 7.30** provide **charts** for selecting the required values of W and S, given some $\varepsilon_r$ and $b$ (or $d$).

* Likewise, example 7.7 on page 345 provides a good **demonstration** of the single-section coupled-line coupler **design synthesis**.