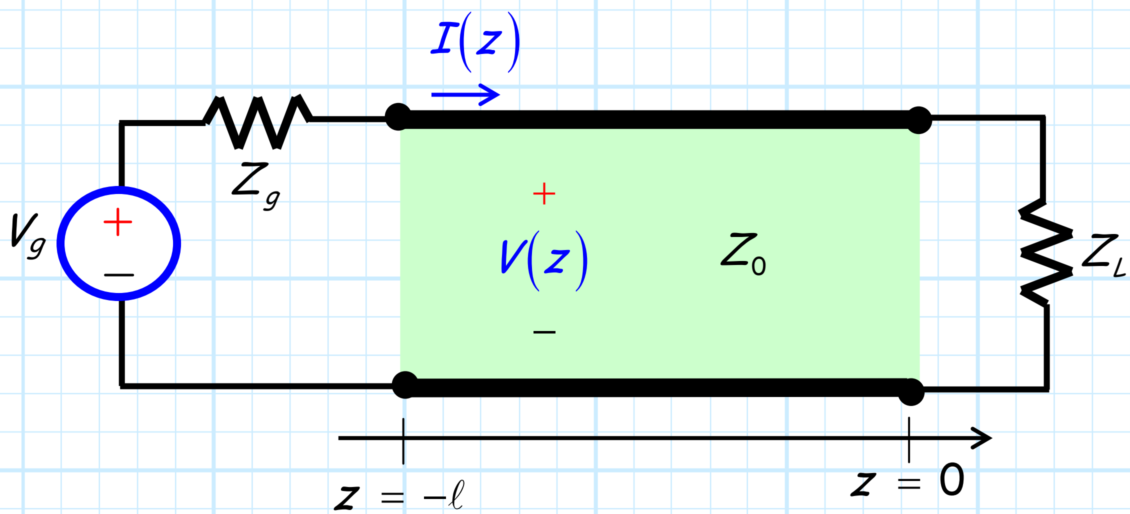
Delivered and Absorbed Power

Q: *If the purpose of a transmission line is to transfer **energy** from a source to a load, then exactly how at what rate is energy **absorbed** by load Z_L in the circuit shown below ??*



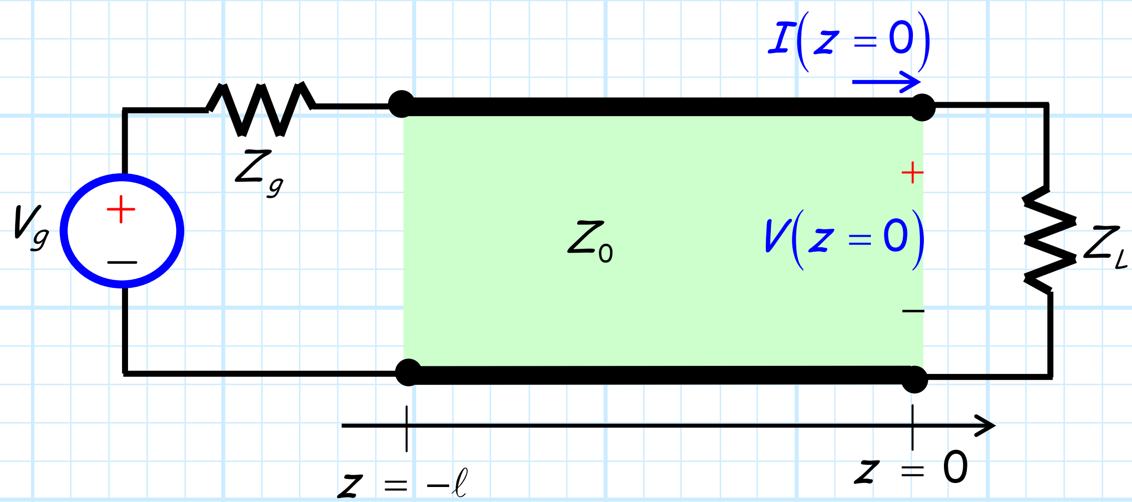
A: Now that we have a **complete** circuit (with **two** boundary conditions!), we of course can determine the numeric values of wave amplitudes V_0^+ and V_0^-.

$$V_0^+ = V_g\, e^{-j\beta\ell} \frac{Z_0}{Z_0\left(1+\Gamma_{in}\right)+Z_g\left(1-\Gamma_{in}\right)} \qquad \text{and} \qquad V_0^- = \Gamma_L\, V_0^+$$

Finally! We can calculate numbers

With knowledge of V_0^+ and V_0^-, we likewise can determine the **total** voltage $V(z)$ (**finally**!) and **total** current $I(z)$ along the transmission line.

Of course, we can **evaluate** this current and voltage **at the load** (i.e., at $z=0$), and then directly **compute** the rate at which energy is **absorbed by the load**.



$$P_{abs} = \frac{1}{2}\text{Re}\left\{V(z=0)\, I^*(z=0)\right\}$$

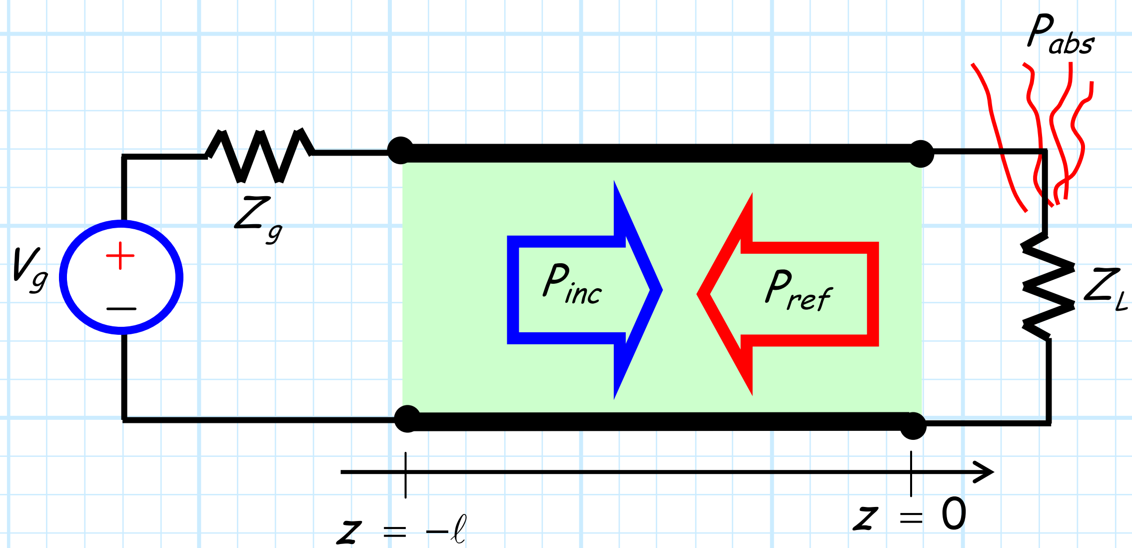
A truck with 30 cases of bananas…

Alternatively, we could likewise determine the absorbed power via the **"wave" viewpoint**, where we know that the absorbed power is simply the **difference** between the plus-wave (**incident**) and minus-wave (**reflected**).

$$P_{abs} = \frac{|V_0^+|^2}{2\,Z_0}\left(1 - |\Gamma_L|^2\right)$$

But remember, this simple equation is useful **only** if we first compute the **plus-wave amplitude**:

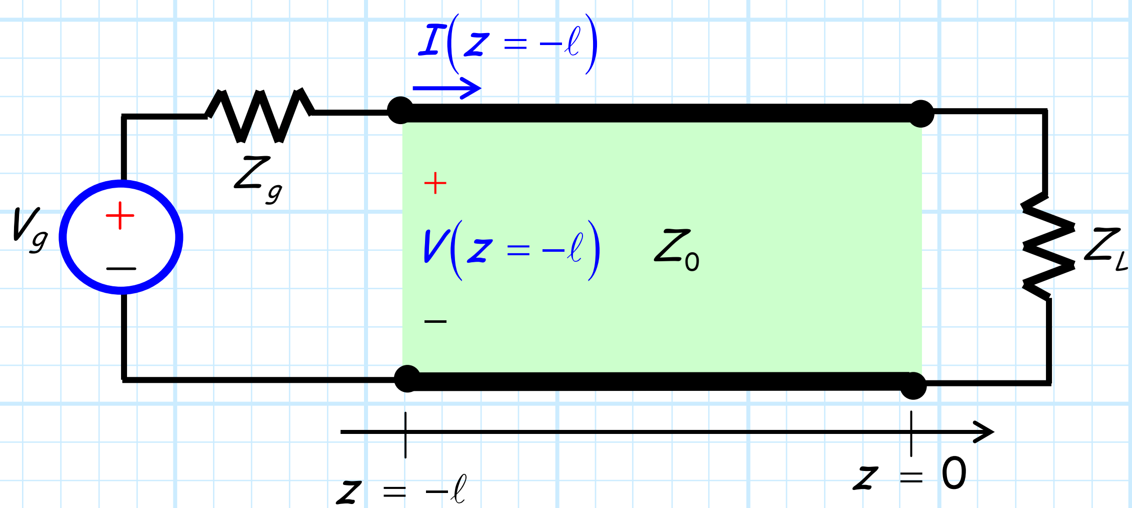
$$V_0^+ = V_g\, e^{-j\beta\ell}\, \frac{Z_0}{Z_0\left(1+\Gamma_{in}\right) + Z_g\left(1-\Gamma_{in}\right)}$$



Move to the beginning of the line

Q: *What about the* ***source****? At what rate does* ***it*** *deliver energy* ***to the circuit****?*

A: Well, we could of course (since we know V_0^+ and V_0^-) determine the total voltage and current **at the source** (i.e., at $z=-\ell$), and then directly compute the rate at which energy is delivered **by the source**.



$$P_{del} = \frac{1}{2}\text{Re}\left\{V\left(z=-\ell\right)I^*\left(z=-\ell\right)\right\}$$

Conservation of energy— it just makes thing so easy

However, since the transmission line is **lossless**, conservation of energy allows us to correctly conclude that this power **delivered by the source** is likewise the rate at which energy is **absorbed by the load**:

$$P_{abs} = P_{del}$$

Thus, all of the **three** calculations we just discussed will provide the **same correct value** for delivered/absorbed power!

$$P_{del} = \frac{1}{2}\text{Re}\left\{V\left(z=-\ell\right)\, I^*\left(z=-\ell\right)\right\} = \frac{\left|V_0^+\right|^2}{2\,Z_0}\left(1-\left|\Gamma_L\right|^2\right) = P_{abs} = \frac{1}{2}\text{Re}\left\{V\left(z=0\right)\, I^*\left(z=0\right)\right\} = P_{abs}$$

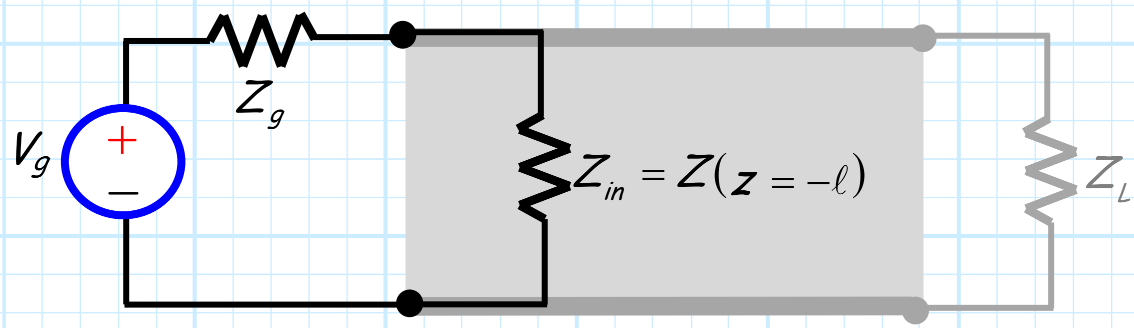
Again, for all three results, we **must** determine V_0^+ and V_0^-.

$$V_0^+ = V_g\, e^{-j\beta\ell}\, \frac{Z_0}{Z_0\left(1+\Gamma_{in}\right)+Z_g\left(1-\Gamma_{in}\right)} \qquad \text{and} \qquad V_0^- = \Gamma_L\, V_0^+$$

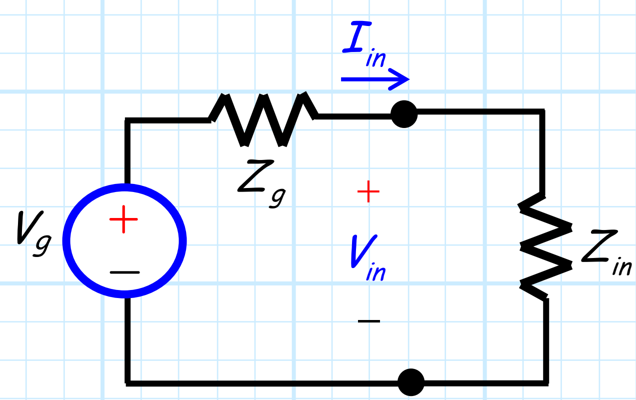
A 4th way—a way that is even easier

However, there is one method for determining the delivered/absorbed power that does **not** require the calculation of V_0^+!

Say we replace the transmission line and load with its equivalent circuit—its **input impedance**:



Recall that we do **not** need to know V_0^+ to determine input impedance—line impedance $Z(z)$ is independent of either V_0^+ or V_0^-. The **equivalent circuit** is thus:



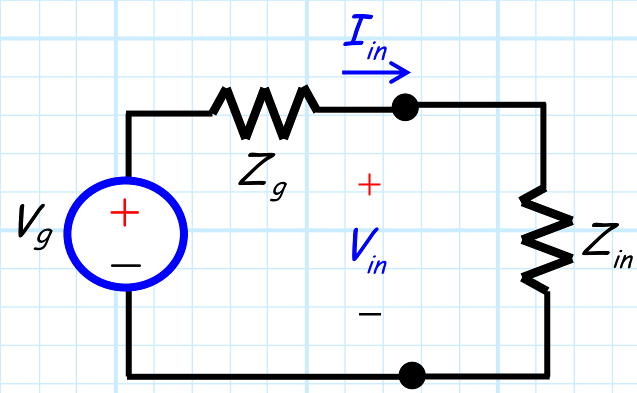
Dust off you introductory circuits text

Now we can easily find the power **delivered** by the source—it's simply **equal** to the rate at which energy is **absorbed** by the input impedance!

We simply have to apply beginning **circuit theory**.

Note by **voltage division** we can determine:

$$V(z=-\ell) = V_g \frac{Z_{in}}{Z_g + Z_{in}}$$



And from **Ohm's Law** we conclude:

$$I(z=-\ell) = \frac{V_g}{Z_g + Z_{in}}$$

A power equation without V_0^+

And thus, the **power** P_{in} absorbed by Z_{in} (and thus the **power** P_{del} delivered by the source) is:

$$
\begin{aligned}
P_{del} = P_{in} &= \frac{1}{2}\text{Re}\left\{V_{in}\, I_{in}^*\right\} \\
&= \frac{1}{2}\text{Re}\left\{V_g \frac{Z_{in}}{Z_g + Z_{in}} \frac{V_g^*}{\left(Z_g + Z_{in}\right)^*}\right\} \\
&= \frac{1}{2}\frac{\left|V_g\right|^2}{\left|Z_g + Z_{in}\right|^2}\text{Re}\left\{Z_{in}\right\}
\end{aligned}
$$

By conservation of energy, we know that this is likewise the rate at which the **load** Z_L **absorbs energy**:

$$
P_{del} = \frac{1}{2}\frac{\left|V_g\right|^2}{\left|Z_g + Z_{in}\right|^2}\text{Re}\left\{Z_{in}\right\} = P_{abs}
$$