**Delivered Power**

**Q:** If the purpose of a transmission line is to transfer power from a source to a load, then exactly how much power is delivered to \( Z_L \) for the circuit shown below??

\[ I(z) \]

\[ V(z) \]

\[ Z_{in} \]

\[ Z_g \]

\[ + \]

\[ - \]

\[ V_g \]

\[ Z_L \]

\[ z = -\ell \]

\[ z = 0 \]

**A:** We of course could determine \( V^+_0 \) and \( V^-_0 \), and then determine the power absorbed by the load (\( P_{abs} \)) as:

\[
P_{abs} = \frac{1}{2} \text{Re} \{ V(z = 0) I^*(z = 0) \}
\]

However, if the transmission line is lossless, then we know that the power delivered to the load must be equal to the power "delivered" to the input (\( P_{in} \)) of the transmission line:

\[
P_{abs} = P_{in} = \frac{1}{2} \text{Re} \{ V(z = -\ell) I^*(z = -\ell) \}
\]
However, we can determine this power \textbf{without} having to solve for \( V_0^+ \) and \( V_0^- \) (i.e., \( V(z) \) and \( I(z) \)). We can simply use our knowledge of \textbf{circuit theory}!

We can \textbf{transform} load \( Z_L \) to the beginning of the transmission line, so that we can replace the transmission line with its \textbf{input impedance} \( Z_{in} \):

\[ \begin{align*}
I(z = -\ell) &= V_g \frac{Z_{in}}{Z_g + Z_{in}} \\
V(z = -\ell) &= \frac{V_g}{Z_g + Z_{in}} \\
Z_{in} &= Z(z = -\ell)
\end{align*} \]

\textbf{Note by voltage division} we can determine:\

\[ V(z = -\ell) = V_g \frac{Z_{in}}{Z_g + Z_{in}} \]

And from \textbf{Ohm's Law} we conclude:

\[ I(z = -\ell) = \frac{V_g}{Z_g + Z_{in}} \]

And thus, the \textbf{power} \( P_{in} \) delivered to \( Z_{in} \) (and thus the \textbf{power} \( P_{abs} \) delivered to the load \( Z_L \)) is:
\[ P_{abs} = P_{in} = \frac{1}{2} \text{Re}\{V(z = -\ell) I^*(z = -\ell)\} \]

\[ = \frac{1}{2} \text{Re} \left\{ V_g \frac{Z_{in}}{Z_g + Z_{in}} \frac{V_g^*}{(Z_g + Z_{in})^*} \right\} \]

\[ = \frac{1}{2} \left| V_g \right|^2 \frac{\left| Z_{in} \right|^2}{\left| Z_g + Z_{in} \right|^2} \text{Re}\{V_{in}\} \]

Note that we could also determine \( P_{abs} \) from our earlier expression:

\[ P_{abs} = \frac{\left| V_0^+ \right|^2}{2 Z_0} \left( 1 - \left| \Gamma_L \right|^2 \right) \]

But we would of course have to first determine \( V_0^+(\ell) \):

\[ V_0^+ = V_g e^{-j\beta\ell} \frac{Z_0}{Z_0 (1 + \Gamma_{in}) + Z_g (1 - \Gamma_{in})} \]