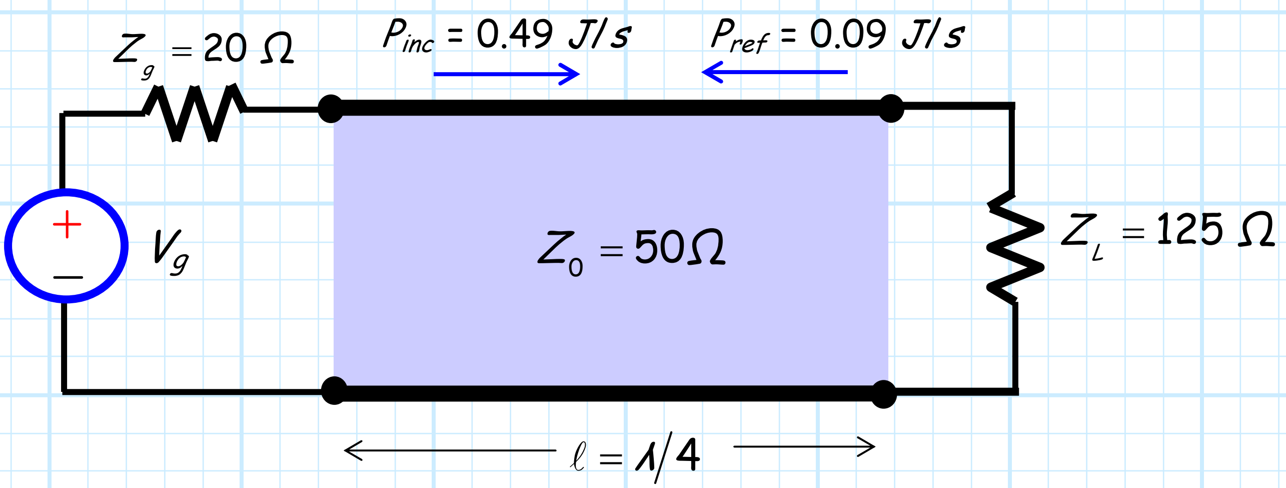
Example: Conservation of Energy and You

Consider this circuit, where the transmission line is **lossless** and has length $\ell = \lambda/4$:



The wave **incident** on the load Z_L has power of $P_{inc} = 0.49\ J/s$.

The wave **reflected** from the load Z_L has power of $P_{ref} = 0.09\ J/s$.

Determine the **magnitude** of source voltage V_g (i.e., determine $|V_g|$).

Hint: This is **not** a boundary condition problem. Do **not** attempt to find $V(z)$ and/or $I(z)$!

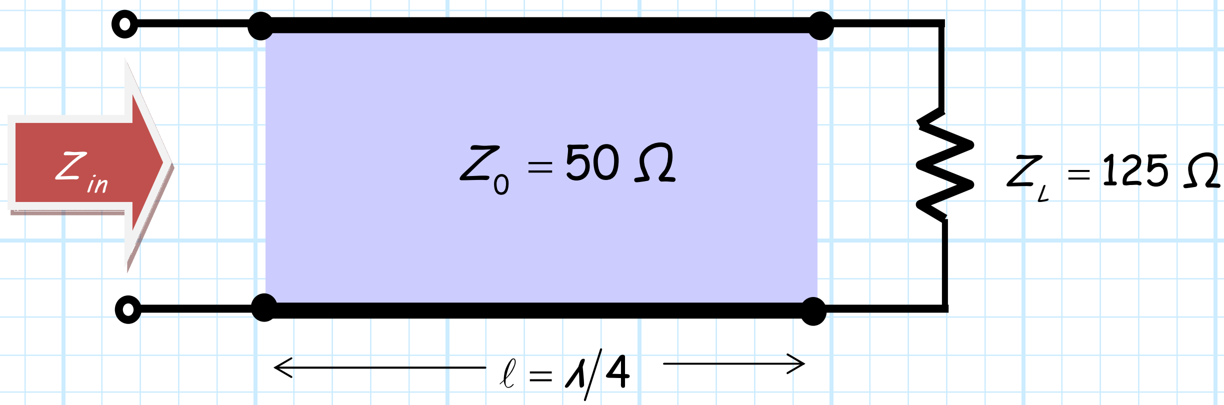
Solution

From **conservation of energy**, we find the load absorbes energy at a rate of:

$$\begin{aligned} P_{abs} &= P_{inc} - P_{ref} \\ &= 0.49 - 0.09 \\ &= 0.4\ W \end{aligned}$$

Since the transmission line is **lossless**, this absorbed power **must** likewise be the power **delivered** by the source to the input of the transmission line (i.e., the power absorbed by input impedance Z_{in}).

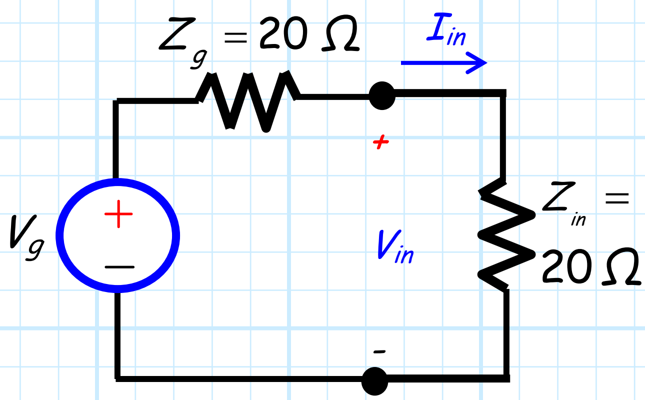
$$P_{in} = P_{abs} = 0.40 \quad J/s$$



Note the transmission line length has the **special case** $\ell = \lambda/4$, therefore the input impedance is easily computed:

$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{50^2}{125} = 20\Omega = Z_g$$

A **conjugate match** ($Z_{in} = Z_g^*$)!



Thus, the power **delivered** by the source is:

$$\begin{aligned}P_{del} &= \frac{1}{2} Re\left\{V_{in} I_{in}^*\right\} \\ &= \frac{1}{2} Re\left\{\left(\frac{V_g}{2}\right)\left(\frac{V_g^*}{20+20}\right)\right\} \\ &= \frac{1}{8} Re\left\{\left|V_g\right|^2\right\}\frac{1}{20} \\ &= \frac{\left|V_g\right|^2}{160}\end{aligned}$$

And since we know that $P_{abs} = P_{del} = 0.4\ W$, we can conclude:

$$P_{del} = 0.4 = \frac{\left|V_g\right|^2}{160} \qquad \Rightarrow \qquad \left|V_g\right| = \sqrt{160(0.4)} = 8.0 \quad V$$