Example: Determining Transmission Line Length

A load **terminating** at transmission line has a normalized impedance $z_L' = 2.0 + j2.0$. What should the **length** ℓ of transmission line be in order for its input impedance to be:

a) purely **real** (i.e., $x_{in} = 0$)?

b) have a real (resistive) part equal to **one** (i.e., $r_{in} = 1.0$)?

Solution:

a) Find $z_L' = 2.0 + j2.0$ on your Smith Chart, and then rotate **clockwise** until you "bump into" the contour $x = 0$ (recall this is contour lies on the Γ_r **axis!**).

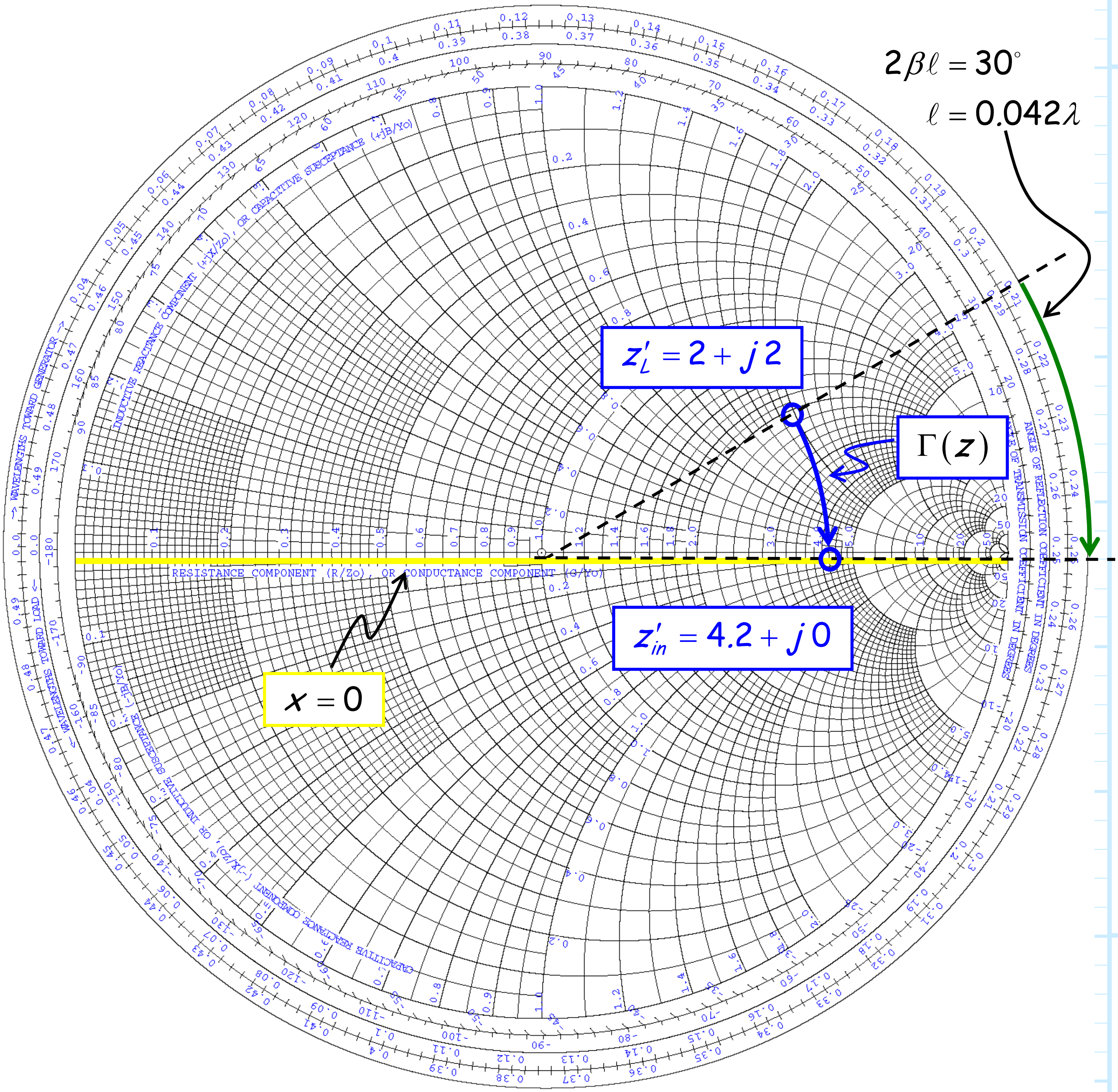
When you reach the $x = 0$ contour—**stop!** Lift your pencil and note that the impedance value of this location is **purely real** (after all, $x = 0$!).

Now, measure the **rotation angle** that was required to move clockwise from $z_L' = 2.0 + j2.0$ to an impedance on the $x = 0$ contour—this **angle** is equal to $2\beta\ell$!

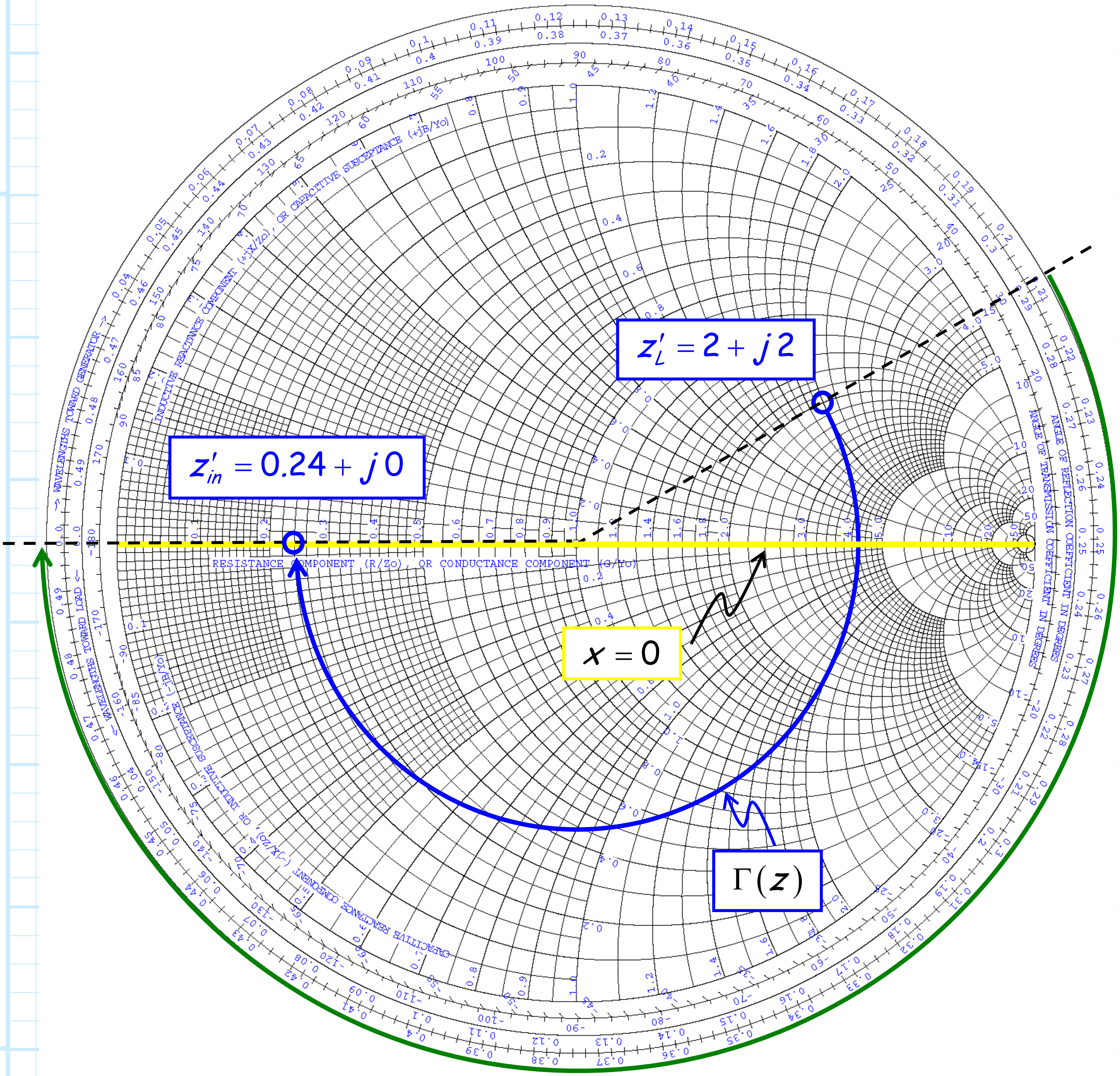
You can now **solve** for ℓ, or alternatively use the **electrical length scale** surrounding the Smith Chart.

One more important point—there are **two** possible solutions!

Solution 1:



Solution 2:



$$2\beta\ell = 210^\circ$$

$$\ell = 0.292\lambda$$

b) Find $z'_L = 2.0 + j2.0$ on your Smith Chart, and then rotate **clockwise** until you "bump into" the **circle** $r = 1$ (recall this circle intersects the **center** point or the Smith Chart!).

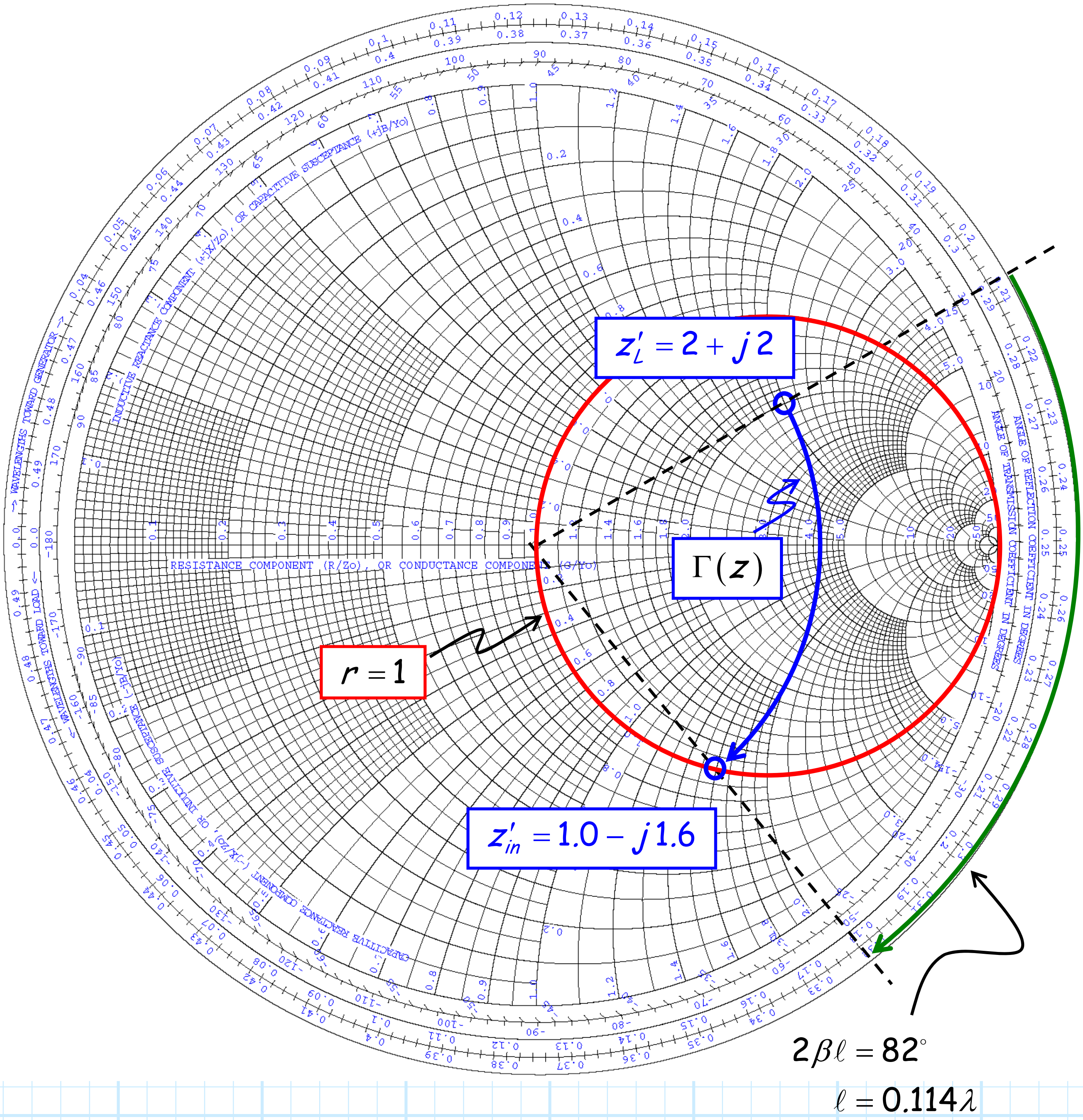
When you reach the $r = 1$ circle—**stop**! Lift your pencil and note that the impedance value of this location has a real value equal to **one** (after all, $r = 1$!).

Now, measure the **rotation angle** that was required to move clockwise from $z'_L = 2.0 + j2.0$ to an impedance on the $r = 1$ circle—this **angle** is equal to $2\beta\ell$!

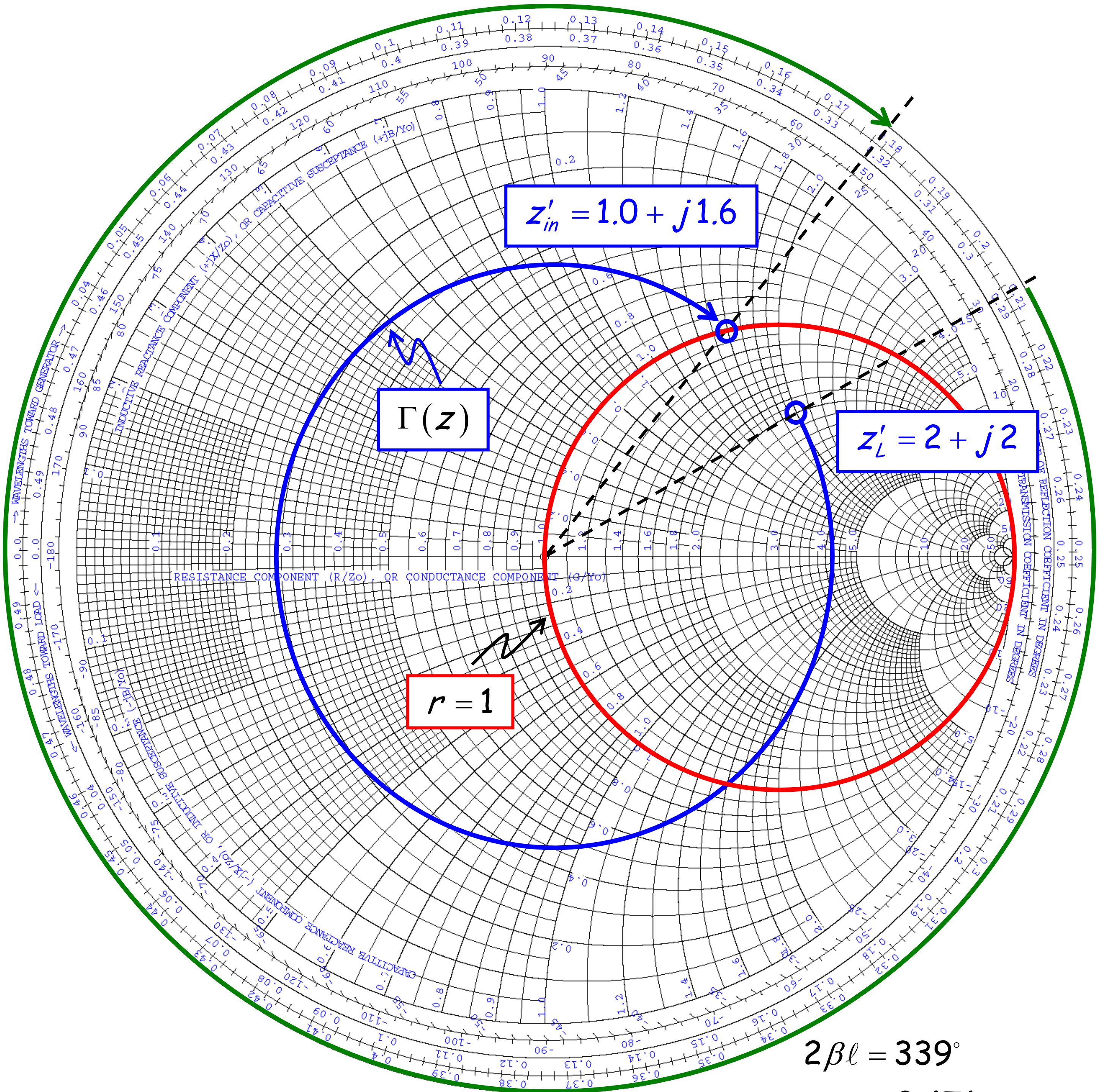
You can now **solve** for ℓ, or alternatively use the **electrical length scale** surrounding the Smith Chart.

Again, we find that there are **two** solutions!

Solution 1:



Solution 2:



$$2\beta\ell = 339^\circ$$

$$\ell = 0.471\lambda$$

Q: *Hey! For part b), the solutions resulted in $z'_{in} = 1 - j\,1.6$ and $z'_{in} = 1 + j\,1.6$ --the **imaginary** parts are equal but **opposite!** Is this just a coincidence?*

A: Hardly! Remember, the two impedance solutions must result in the **same magnitude** for Γ--for this example we find $|\Gamma(z)| = 0.625$.

Thus, for impedances where r=1 (i.e., $z' = 1 + j\,x$):

$$\Gamma = \frac{z'-1}{z'+1} = \frac{(1+jx)-1}{(1+jx)+1} = \frac{j\,x}{2+j\,x}$$

and therefore:

$$|\Gamma|^2 = \frac{|j\,x|^2}{|2+j\,x|^2} = \frac{x^2}{4+x^2}$$

Meaning:

$$x^2 = \frac{4\,|\Gamma|^2}{1-|\Gamma|^2}$$

of which there are **two** equal by opposite solutions!

$$x = \pm\frac{2\,|\Gamma|}{\sqrt{1-|\Gamma|^2}}$$

Which for **this** example gives us our solutions $x = \pm 1.6$.