

Example: Scattering Parameters

Consider a **two-port device** with a scattering matrix (at some specific frequency ω_0):

$$\mathcal{S}(\omega = \omega_0) = \begin{bmatrix} 0.1 & j0.7 \\ j0.7 & -0.2 \end{bmatrix}$$

and $Z_0 = 50\Omega$.

Say that the transmission line connected to **port 2** of this device is terminated in a **matched load**, and that the wave **incident on port 1** is:

$$V_1^+(z_1) = -j2 e^{-j\beta z_1}$$

where $z_{1p} = z_{2p} = 0$.

Determine:

1. the port voltages $V_1(z_1 = z_{1p})$ and $V_2(z_2 = z_{2p})$.
2. the port currents $I_1(z_1 = z_{1p})$ and $I_2(z_2 = z_{2p})$.
3. the net power flowing into port 1

1. Since the **incident** wave on port 1 is:

$$V_1^+(z_1) = -j2 e^{-j\beta z_1}$$

we can conclude (since $z_{1\rho} = 0$):

$$\begin{aligned} V_1^+(z_1 = z_{1\rho}) &= -j2 e^{-j\beta z_{1\rho}} \\ &= -j2 e^{-j\beta(0)} \\ &= -j2 \end{aligned}$$

and since port 2 is **matched** (and **only** because its matched!), we find:

$$\begin{aligned} V_1^-(z_1 = z_{1\rho}) &= S_{11} V_1^+(z_1 = z_{1\rho}) \\ &= 0.1(-j2) \\ &= -j0.2 \end{aligned}$$

The voltage at port 1 is thus:

$$\begin{aligned} V_1(z_1 = z_{1\rho}) &= V_1^+(z_1 = z_{1\rho}) + V_1^-(z_1 = z_{1\rho}) \\ &= -j2.0 - j0.2 \\ &= -j2.2 \\ &= 2.2 e^{-j\pi/2} \end{aligned}$$

Likewise, since port 2 is **matched**:

$$V_2^+(z_2 = z_{2\rho}) = 0$$

And also:

$$\begin{aligned} V_2^-(z_2 = z_{2P}) &= S_{21} V_1^+(z_1 = z_{1P}) \\ &= j0.7 (-j2) \\ &= 1.4 \end{aligned}$$

Therefore:

$$\begin{aligned} V_2(z_2 = z_{2P}) &= V_2^+(z_2 = z_{2P}) + V_2^-(z_2 = z_{2P}) \\ &= 0 + 1.4 \\ &= 1.4 \\ &= 1.4 e^{-j0} \end{aligned}$$

2. The port **currents** can be easily determined from the results of the previous section.

$$\begin{aligned} I_1(z_1 = z_{1P}) &= I_1^+(z_1 = z_{1P}) - I_1^-(z_1 = z_{1P}) \\ &= \frac{V_1^+(z_1 = z_{1P})}{Z_0} - \frac{V_1^-(z_1 = z_{1P})}{Z_0} \\ &= -j \frac{2.0}{50} + j \frac{0.2}{50} \\ &= -j \frac{1.8}{50} \\ &= -j0.036 \\ &= 0.036 e^{-j\pi/2} \end{aligned}$$

and:

$$\begin{aligned}
 I_2(z_2 = z_{2P}) &= I_2^+(z_2 = z_{2P}) - I_2^-(z_2 = z_{2P}) \\
 &= \frac{V_2^+(z_2 = z_{2P})}{Z_0} - \frac{V_2^-(z_2 = z_{2P})}{Z_0} \\
 &= \frac{0}{50} - \frac{1.4}{50} \\
 &= -0.028 \\
 &= 0.028 e^{+j\pi}
 \end{aligned}$$

3. The net power flowing into port 1 is:

$$\begin{aligned}
 \Delta P_1 &= P_1^+ - P_1^- \\
 &= \frac{|V_{01}^+|^2}{2Z_0} - \frac{|V_{01}^-|^2}{2Z_0} \\
 &= \frac{(2)^2 - (0.2)^2}{2(50)} \\
 &= 0.0396 \text{ Watts}
 \end{aligned}$$