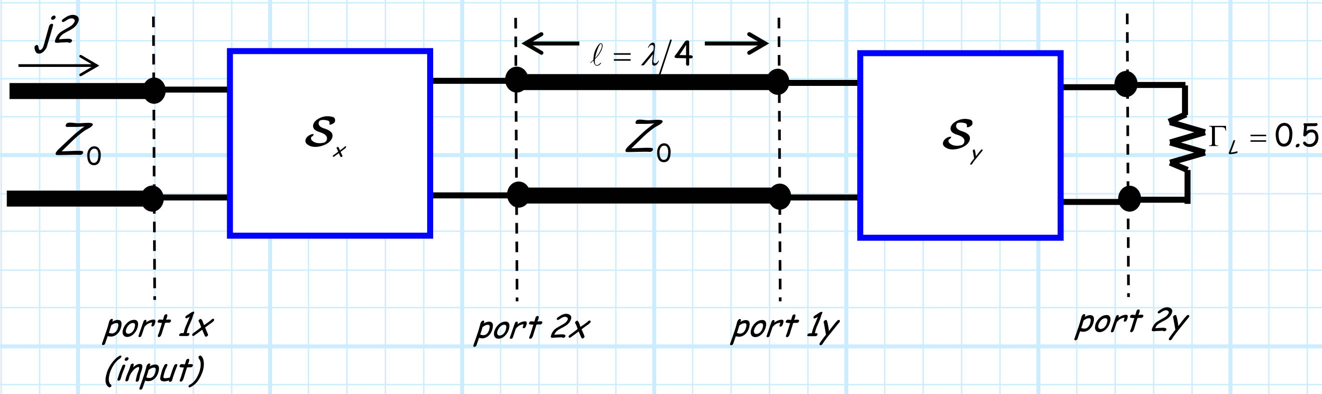
Example: Analysis Using Signal Flow Graphs

Below is a **single**-port device (with **input** at port 1a) constructed with two two-port devices ($\mathcal{S}_x$ and $\mathcal{S}_y$), a quarter wavelength transmission line, and a load impedance.



Where $Z_0 = 50\Omega$.

The scattering matrices of the two-port devices are:

$$\mathcal{S}_x = \begin{bmatrix} 0.35 & 0.5 \\ 0.5 & 0 \end{bmatrix} \qquad \mathcal{S}_y = \begin{bmatrix} 0 & 0.8 \\ 0.8 & 0.4 \end{bmatrix}$$

Likewise, we know that the value of the voltage wave **incident** on port 1 of device $\mathcal{S}_x$ is:

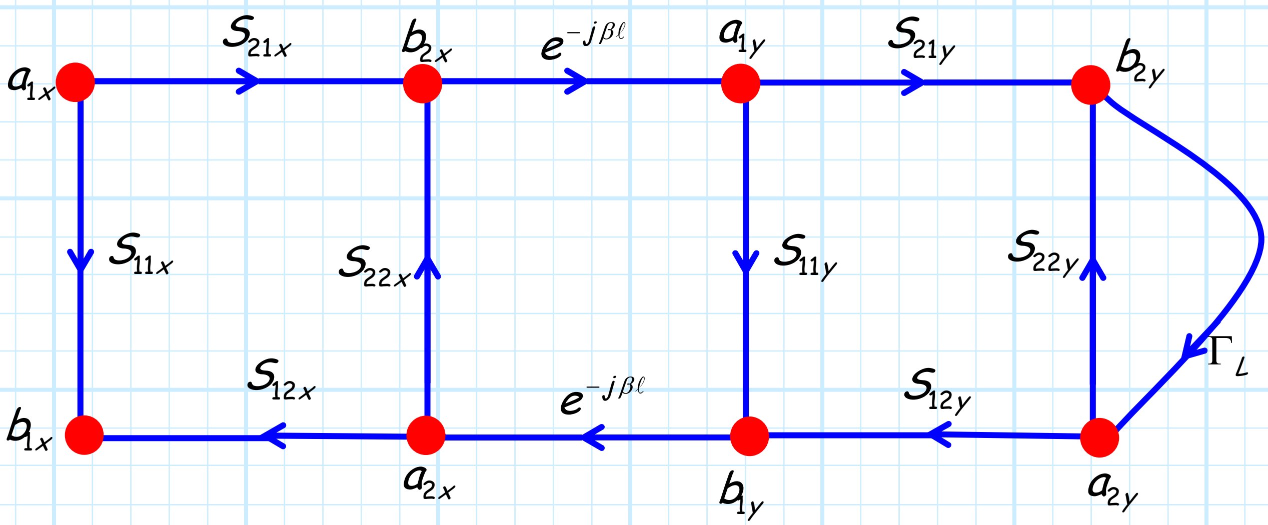
$$a_{1x} \doteq \frac{V_{01x}^{+}\left(z_{1x} = z_{1xP}\right)}{\sqrt{Z_0}} = \frac{j2}{\sqrt{50}} = \frac{j\sqrt{2}}{5} \quad V$$

Now, let's draw the complete **signal flow graph** of this circuit, and then reduce the graph to determine:

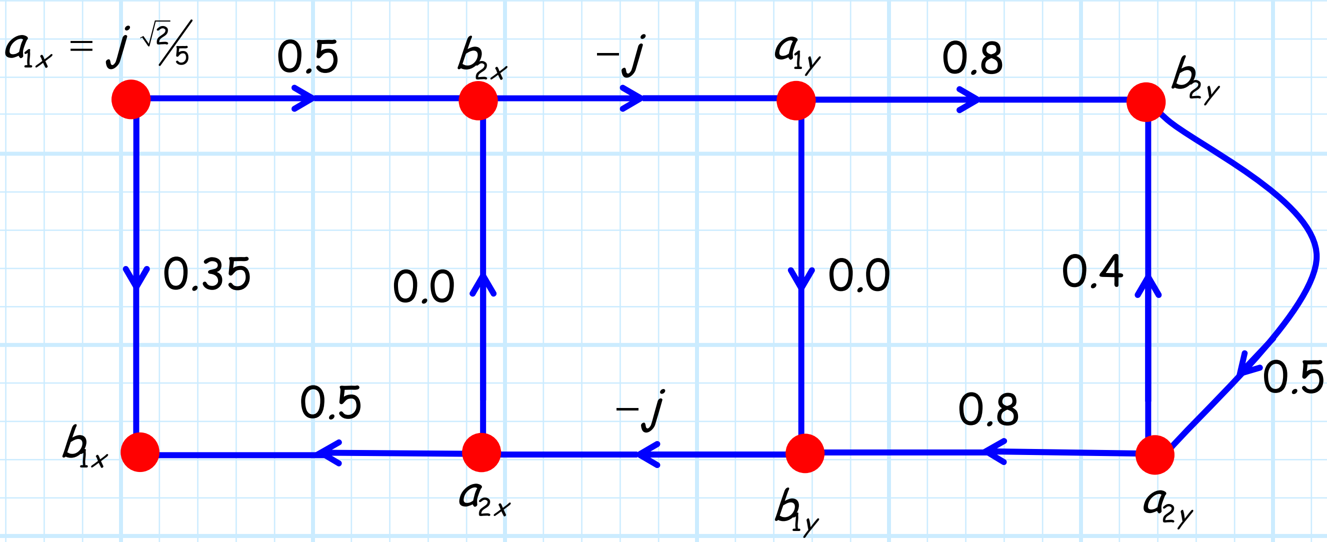
a) The total current through load Γ_L .

b) The power delivered to (i.e., absorbed by) port *1x*.

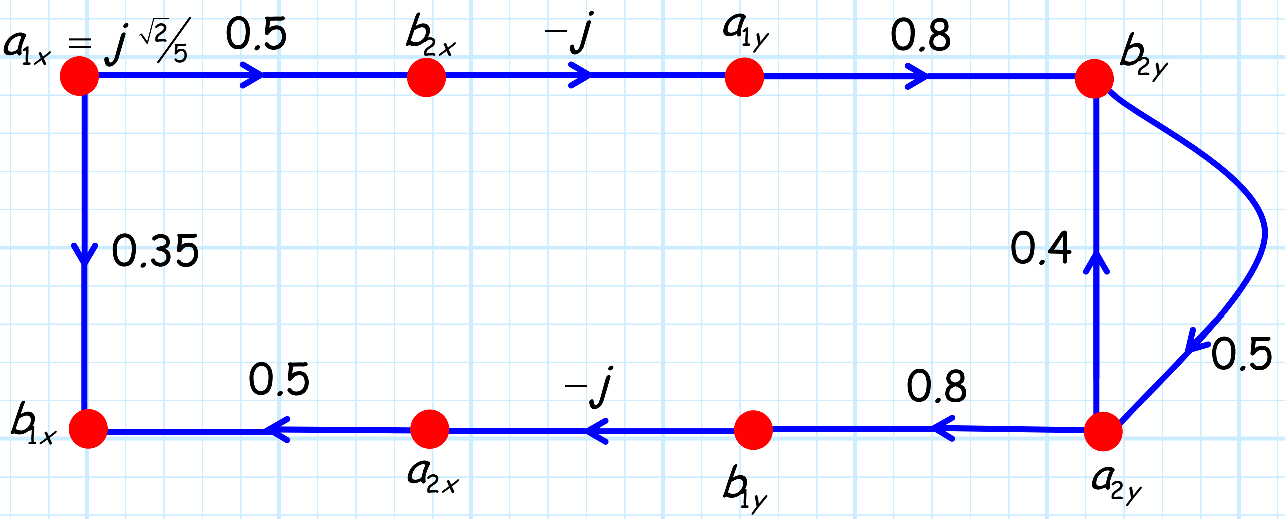
The signal flow graph describing this network is:



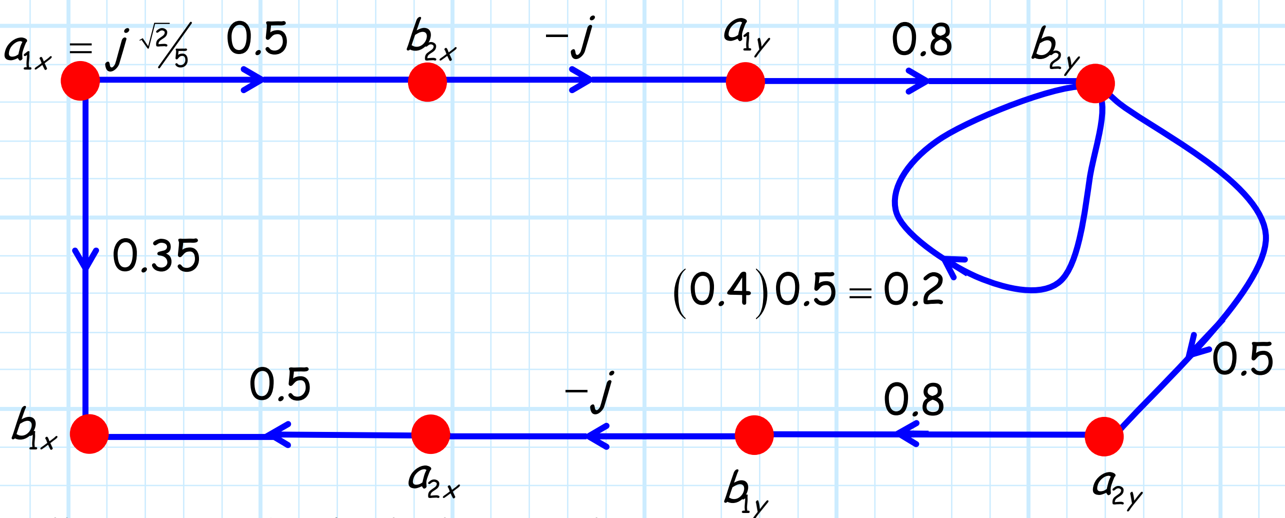
Inserting the numeric values of branches:



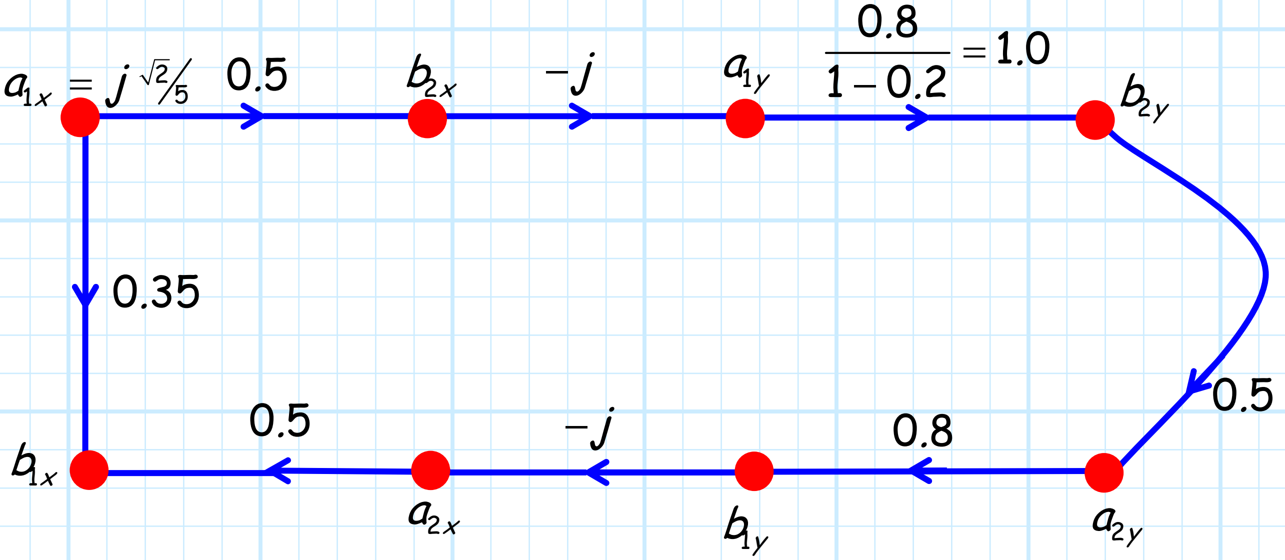
Removing the zero valued branches:



And now applying "splitting" rule 4:



Followed by the "self-loop" rule 3:



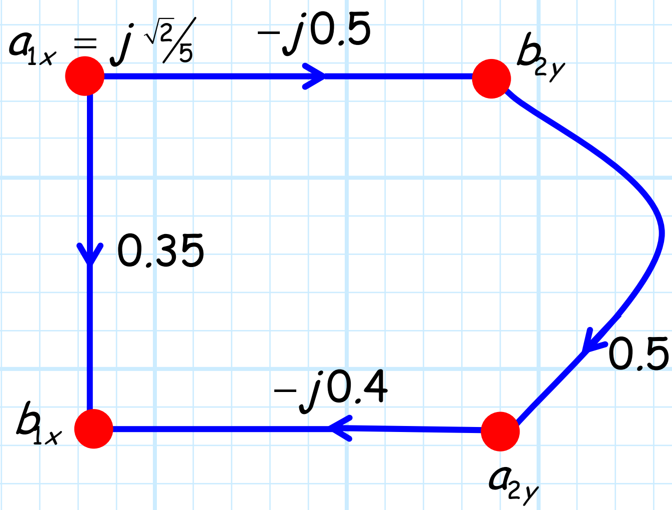
Now, let's used this simplified signal flow graph to find the solutions to our questions!

a) The total current through load Γ_L .

The total current through the load is:

$$\begin{aligned} I_L &= -I\left(z_{2y} = z_{2yP}\right) \\ &= -\frac{V_{02y}^{+}\left(z_{2y} = z_{2yP}\right) - V_{02y}^{-}\left(z_{2y} = z_{2yP}\right)}{Z_0} \\ &= -\frac{a_{2y} - b_{2y}}{\sqrt{Z_0}} \\ &= \frac{b_{2y} - a_{2y}}{\sqrt{50}} \end{aligned}$$

Thus, we need to determine the value of nodes a_{2y} and b_{2y}. Using the "series" rule 1 on our signal flow graph:



Note we've simply **ignored** (i.e., neglected to plot) the node for which we have **no interest**!

From this graph we can conclude:

$$b_{2y} = -j0.5\, a_{1x} = -j0.5\left(\frac{j\sqrt{2}}{5}\right) = 0.1\sqrt{2}$$

and:

$$a_{2y} = 0.5\, b_{2y} = 0.5\left(0.1\sqrt{2}\right) = 0.05\sqrt{2}$$

Therefore:

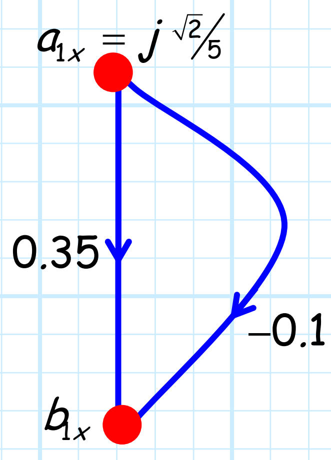
$$I_L = \frac{b_{2y} - a_{2y}}{\sqrt{50}} = \frac{(0.1 - 0.05)\sqrt{2}}{\sqrt{50}} = \frac{0.05}{5} = 10.0\ mA$$

b) The **power** delivered to (i.e., absorbed by) port *1x*.

The power delivered to port 1x is:

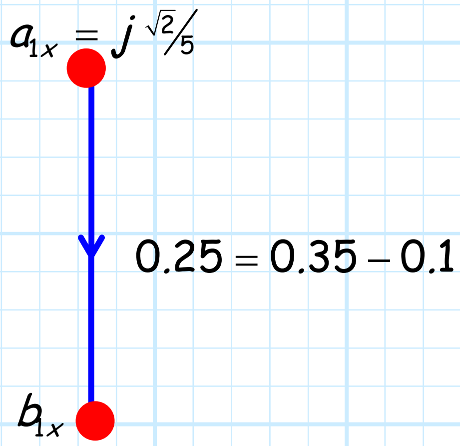
$$\begin{aligned} P_{abs} &= P^+ - P^- \\ &= \frac{\left|V_{1x}^+\left(z_{1x} = z_{1xP}\right)\right|^2}{2Z_0} - \frac{\left|V_{1x}^-\left(z_{1x} = z_{1xP}\right)\right|^2}{2Z_0} \\ &= \frac{\left|a_{1x}\right|^2 - \left|b_{1x}\right|^2}{2} \end{aligned}$$

Thus, we need determine the values of nodes a_{1x} and b_{1x}. Again using the series rule 1 on our signal flow graph:



Again we've simply **ignored** (i.e., neglected to plot) the node for which we have **no interest**!

And then using the "parallel" rule 2:



Therefore:

$$b_{1x} = 0.25\, a_{1x} = 0.25\left(j\,\sqrt{2}/5\right) = j0.05\sqrt{2}$$

and:

$$P_{abs} = \frac{\left|j\,\sqrt{2}/5\right|^2 - \left|j0.05\sqrt{2}\right|^2}{2} = \frac{0.08 - 0.005}{2} = 37.5\ mW$$