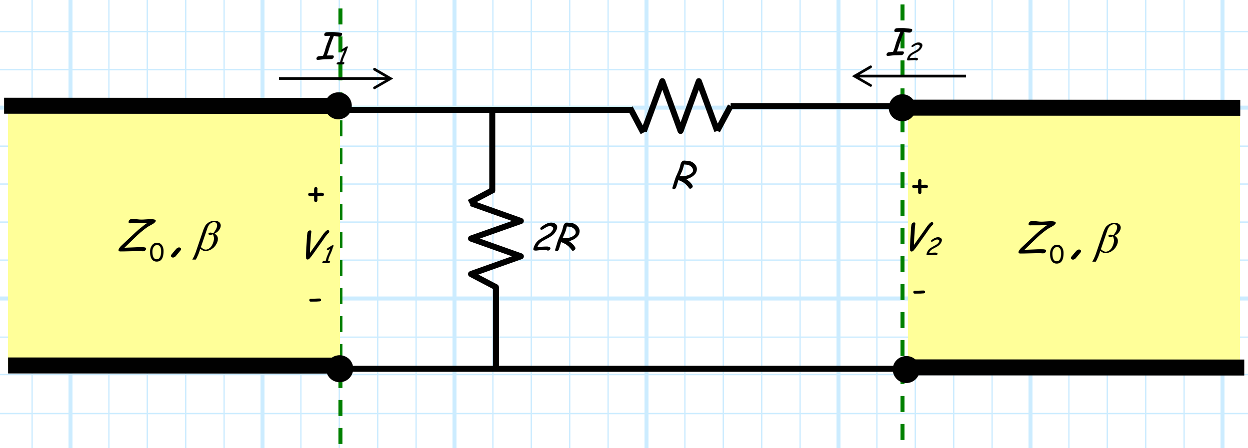
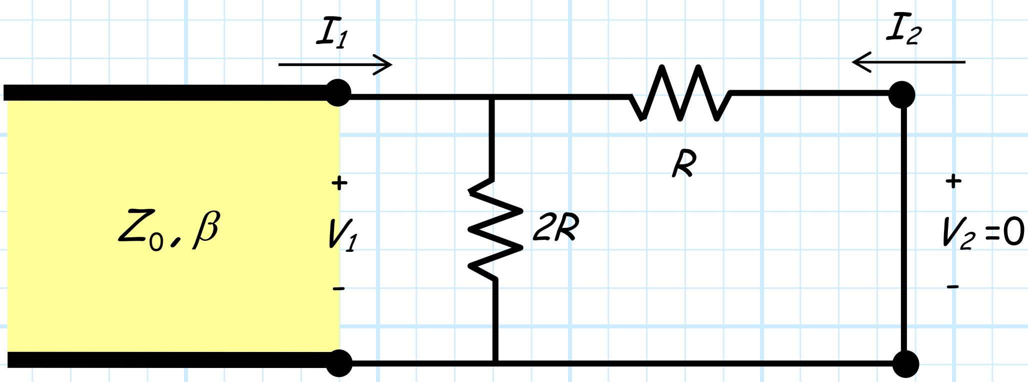
Example: Evaluating the Admittance Matrix

Consider the following **two-port** device:



Let's determine the **admittance matrix** of this device!

Step 1: Place a **short** at port 2.



Step 2: Determine **currents** I_1 and I_2 .

Note that **after** the short was placed at port 2, both resistors are in **parallel**, with a potential V_2 across each.

The current I_1 is thus simply the **sum** of the two currents through **each** resistor:

$$I_1 = \frac{V_1}{2R} + \frac{V_1}{R} = \frac{3V_1}{2R}$$

The current I_2 is simply the **opposite** of the current through R:

$$I_2 = -\frac{V_1}{R}$$

Step 3: Determine **trans-admittance** Y_{11} and Y_{21} .

$$Y_{11} = \frac{I_1}{V_1} = \frac{3}{2R}$$

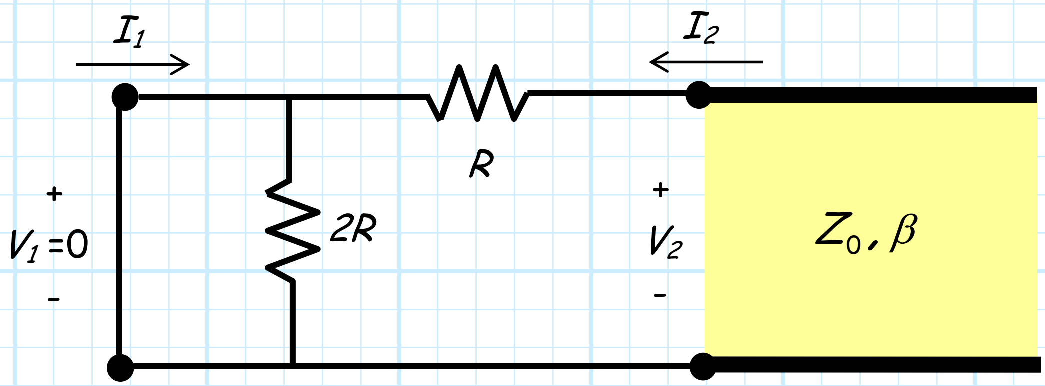
$$Y_{21} = \frac{I_2}{V_1} = -\frac{1}{R}$$

Note that Y_{21} is **real**—but **negative**!

This is **still** a valid physical result, **although** you will find that the **diagonal** terms of an impedance or admittance matrix (e.g., Y_{22}, Z_{11}, Y_{44}) will **always** have a real component that is **positive**.

To find the **other two** trans-admittance parameters, we must **move** the short and then **repeat** each of our previous steps!

Step 1: Place a **short** at port 1.



Step 2: Determine **currents** I_1 and I_2 .

Note that **after** a short was placed at port **1**, resistor $2R$ has **zero** voltage across it—and thus **zero current** through it!

Likewise, from KVL we find that the **voltage** across resistor R is equal to V_2.

Finally, we see from KCL that $I_1 = I_2$.

The current I_2 thus:

$$I_2 = \frac{V_2}{R}$$

and thus:

$$I_1 = -\frac{V_2}{R}$$

Step 3: Determine **trans-admittance** Y_{12} and Y_{22} .

$$Y_{12} = \frac{I_1}{V_2} = -\frac{1}{R}$$

$$Y_{22} = \frac{I_2}{V_2} = \frac{1}{R}$$

The **admittance** matrix of this two-port device is therefore:

$$\mathcal{Y} = \frac{1}{R}\begin{bmatrix} 1.5 & -1 \\ -1 & 1 \end{bmatrix}$$

Note this device (as **you** may have suspected) is **lossy** and **reciprocal**.

Q: *What about the **impedance** matrix? How can we **determine** that?*

A: **One** way is simply determine the **inverse** of the admittance matrix above.

$$\begin{aligned} \mathcal{Z} &= \mathcal{Y}^{-1} \\ &= R\begin{bmatrix} 1.5 & -1 \\ -1 & 1 \end{bmatrix}^{-1} \\ &= R\begin{bmatrix} 2 & 2 \\ 2 & 3 \end{bmatrix} \end{aligned}$$

A: **Another** way to determine the impedance matrix is simply to apply the **definition** of trans-impedance to **directly** determine the elements of the impedance matrix—**similar** to how we just determined the admittance matrix!

Specifically, follow these **steps**:

Step 1: Place an **open** at port 2 (or 1)

Step 2: Determine **voltages** V_1 and V_2 .

Step 3: Determine trans-**impedance** Z_{11} and Z_{21} (or Z_{12} and Z_{22}).

You try this procedure on the circuit of this example, and make sure **you** get the **same** result for $\mathcal{Z}$ as we determined on the previous page (from matrix inversion)—after all, **you** want to do **well** on my long, scary, evil **exam**!