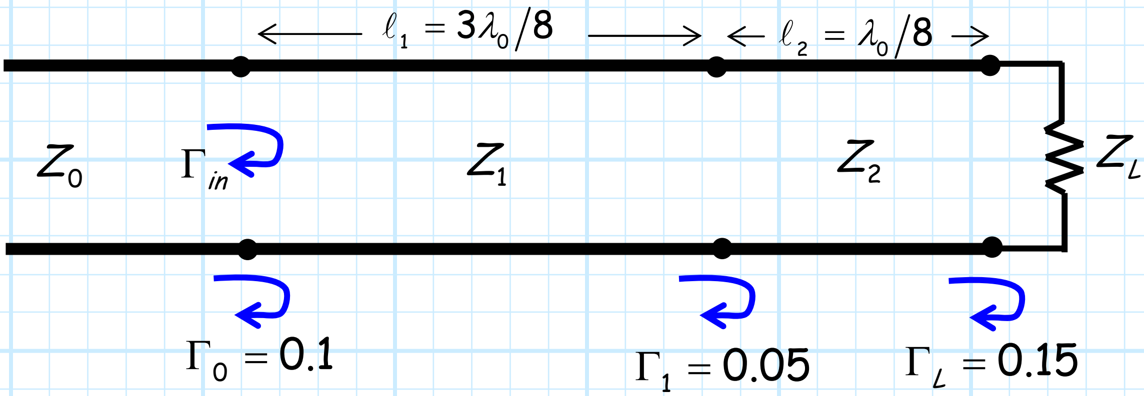
Example: The Theory of Small Reflections

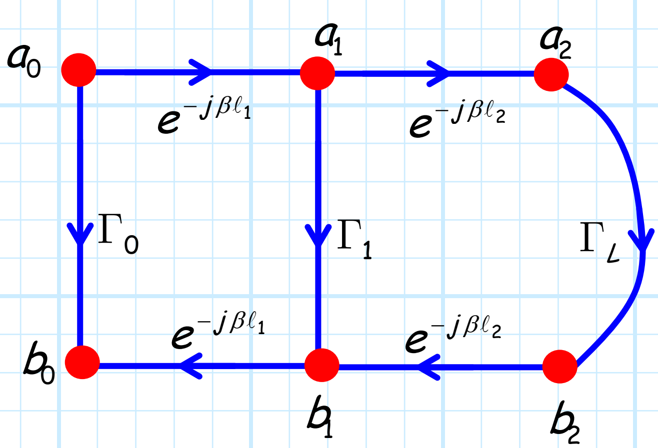
Use the **theory of small reflections** to determine a **numeric** value for the **input** reflection coefficient Γ_{in}, at the design frequency ω_0.



Note that the transmission line sections have **different lengths**!

Solution

Applying the theory of small reflections, the **approximate signal flow graph** of the structure becomes:

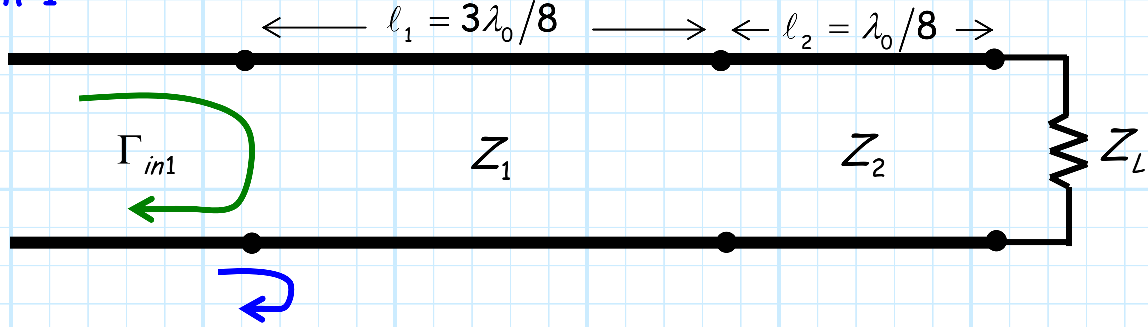


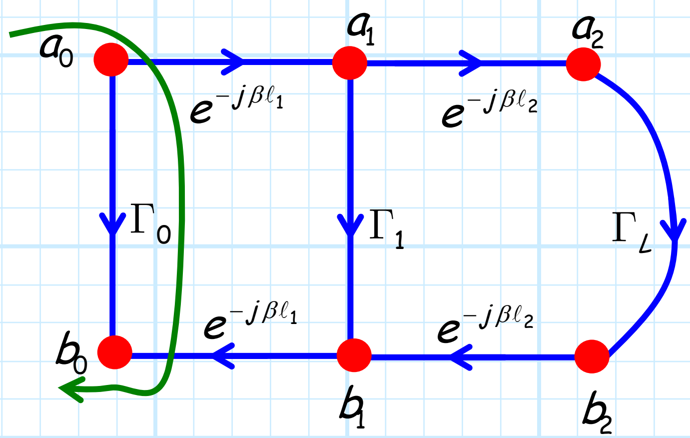
$$e^{-j\beta\ell_1} = e^{-j\left(\frac{2\pi}{\lambda}\frac{3\lambda}{8}\right)} = e^{-j\left(\frac{3\pi}{4}\right)}$$

$$e^{-j\beta\ell_2} = e^{-j\left(\frac{2\pi}{\lambda}\frac{\lambda}{8}\right)} = e^{-j\left(\frac{\pi}{4}\right)}$$

Note there are three **direct** propagation paths:

Path 1

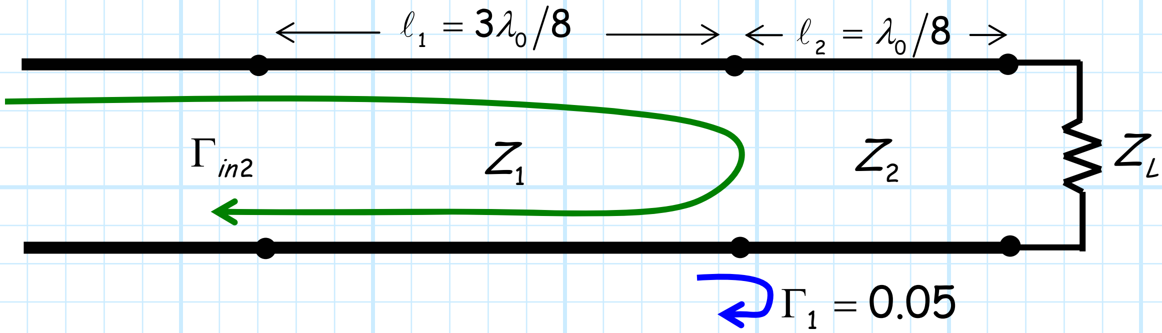




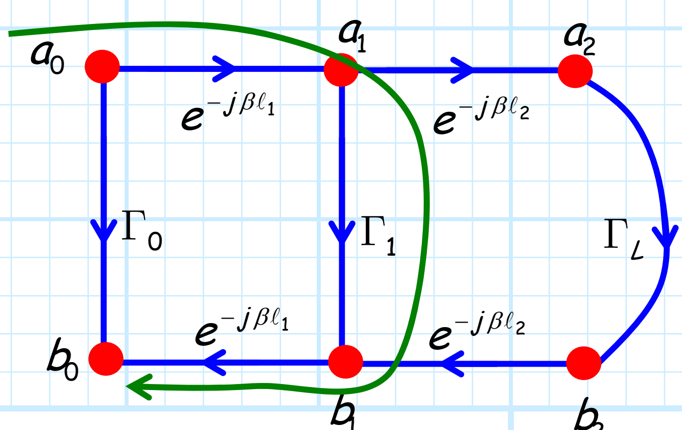
$$\therefore \quad p_1 = \Gamma_0 = 0.1$$

Path 2

This path includes propagation **down** and **back** a transmission line length ℓ_1 !

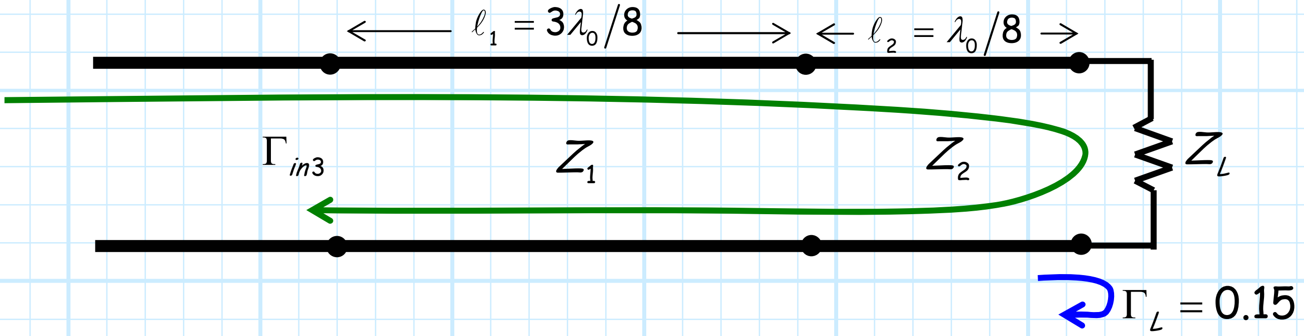


$$\begin{aligned} p_2 &= e^{-j\beta\ell_1}\,\Gamma_1\,e^{-j\beta\ell_1} \\ &= e^{-j^{3\pi}/_4}0.05e^{-j^{3\pi}/_4} \\ &= e^{-j^{3\pi}/_2}\,0.05 \\ &= +j0.05 \end{aligned}$$

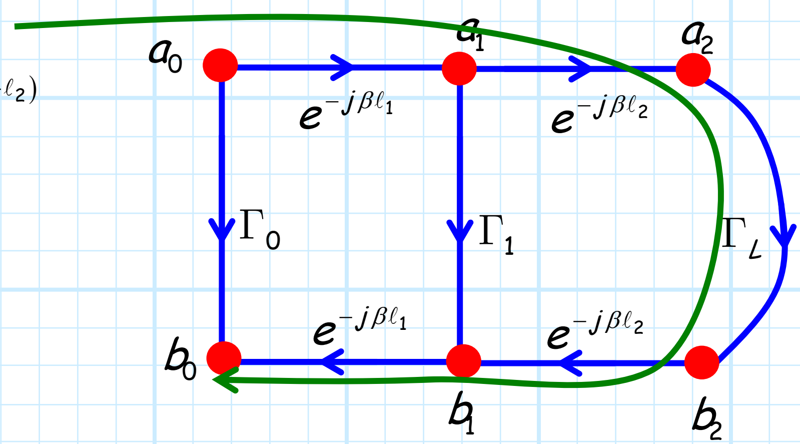


Path 3

This path includes propagation **down** and **back** transmission line lengths of $\ell_1 + \ell_2$!



$$
\begin{aligned}
p_3 &= e^{-j\beta(\ell_1+\ell_2)}\,\Gamma_L\, e^{-j\beta(\ell_1+\ell_2)} \\
&= e^{-j\pi}\ 0.15\ e^{-j\pi} \\
&= e^{-j2\pi}\ 0.15 \\
&= 0.15
\end{aligned}
$$



Thus, using the **theory of small reflections** we can determine approximately the input reflection coefficient:

$$
\begin{aligned}
\Gamma_{in} &= \frac{b_0}{a_0} \\
&= p_1 + p_2 + p_3 \\
&= 0.1 + j0.05 + 0.15 \\
&= \underline{\underline{0.25 + j0.05}}
\end{aligned}
$$