Example: Using the Impedance Matrix

Consider the following circuit:

Where the 3-port device is characterized by the impedance matrix:

\[
\mathbf{Z} = \begin{bmatrix}
2 & 1 & 2 \\
1 & 1 & 4 \\
2 & 4 & 1 \\
\end{bmatrix}
\]

Let's now determine all port voltages \( V_1, V_2, V_3 \) and all currents \( I_1, I_2, I_3 \).
A: We don’t need to know what’s inside that box! We know its impedance matrix, and that completely characterizes the device (or, at least, characterizes it at one frequency).

Thus, we have enough information to solve this problem. From the impedance matrix we know:

\[
\begin{align*}
V_1 &= 2I_1 + I_2 + 2I_3 \\
V_2 &= I_1 + I_2 + 4I_3 \\
V_3 &= 2I_1 + 4I_2 + I_3
\end{align*}
\]

Q: How can we do that—we don’t know what the device is made of! What’s inside that box?

A: True! The impedance matrix describes the device in the box, but it does not describe the devices attached to it. We require more equations to describe them.
1. The source at port 1 is described by the equation:

\[ V_1 = 16.0 - (1) I_1 \]

2. The short circuit on port 2 means that:

\[ V_2 = 0 \]

3. While the load on port 3 leads to:

\[ V_3 = -(1) I_3 \] (note the minus sign!)

Now we have 6 equations and 6 unknowns! Combining equations, we find:

\[ V_1 = 16 - I_1 = 2 I_1 + I_2 + 2 I_3 \]
\[ \therefore 16 = 3 I_1 + I_2 + 2 I_3 \]

\[ V_2 = 0 = I_1 + I_2 + 4 I_3 \]
\[ \therefore 0 = I_1 + I_2 + 4 I_3 \]

\[ V_3 = -I_3 = 2 I_1 + 4 I_2 + I_3 \]
\[ \therefore 0 = 2 I_1 + 4 I_2 + 2 I_3 \]

Solving, we find (I'll let you do the algebraic details!):

\[ I_1 = 7.0 \quad I_2 = -3.0 \quad I_3 = -1.0 \]
\[ V_1 = 9.0 \quad V_2 = 0.0 \quad V_3 = 1.0 \]