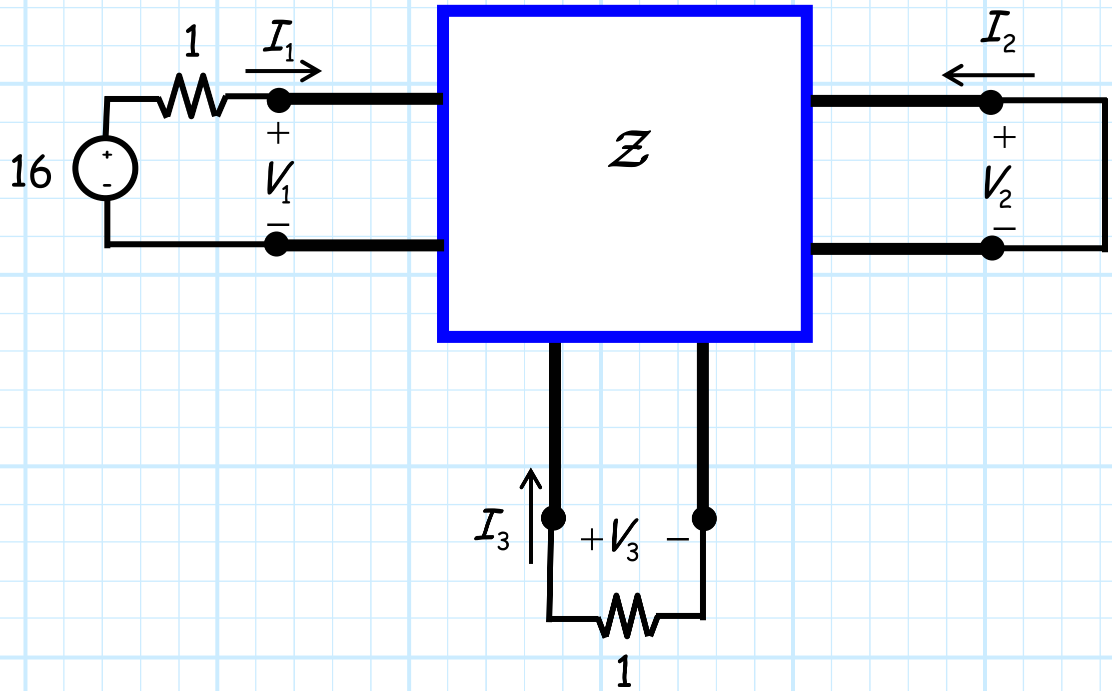
Example: Using the Impedance Matrix

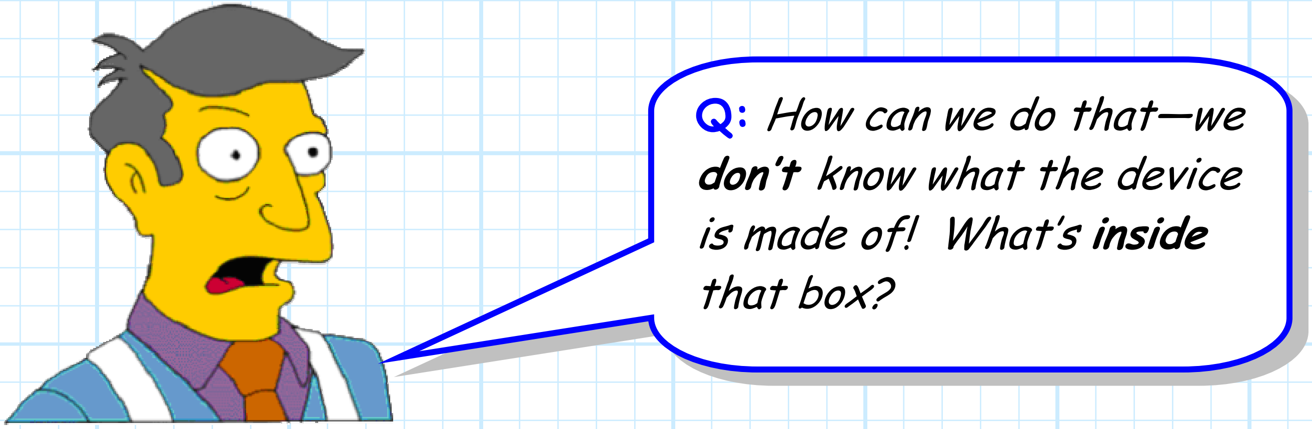
Consider the following circuit:



Where the 3-port **device** is characterized by the **impedance matrix**:

$$\mathcal{Z} = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 4 \\ 2 & 4 & 1 \end{bmatrix}$$

Let's now determine all port **voltages** V_1, V_2, V_3 and all **currents** I_1, I_2, I_3.



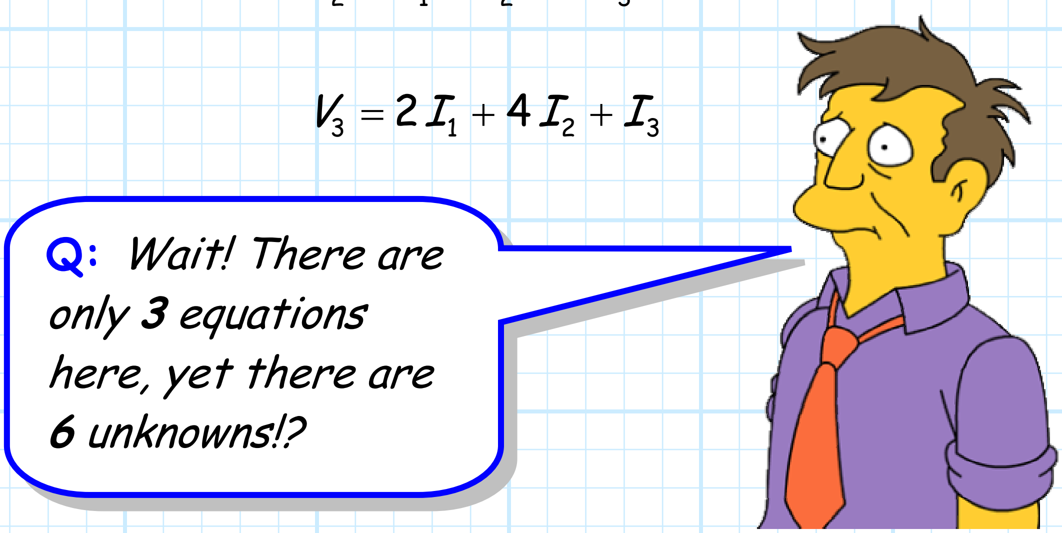
A: We **don't** need to know what's inside that box! We know its impedance matrix, and that **completely** characterizes the device (or, at least, characterizes it at **one** frequency).

Thus, we have enough information to solve this problem. From the impedance matrix we know:

$$V_1 = 2\,I_1 + I_2 + 2\,I_3$$

$$V_2 = I_1 + I_2 + 4\,I_3$$

$$V_3 = 2\,I_1 + 4\,I_2 + I_3$$



A: True! The impedance matrix describes the device in the box, but it does **not** describe the devices **attached** to it. We require **more** equations to describe them.

1. The **source** at port 1 is described by the equation:

$$V_1 = 16.0 - (1)\, I_1$$

2. The **short** circuit on port 2 means that:

$$V_2 = 0$$

3. While the **load** on port 3 leads to:

$$V_3 = -(1)\, I_3 \quad \text{(note the \textbf{minus sign}!)}$$

Now we have **6** equations and **6** unknowns! Combining equations, we find:

$$V_1 = 16 - I_1 = 2\, I_1 + I_2 + 2\, I_3$$
$$\therefore \quad 16 = 3\, I_1 + I_2 + 2\, I_3$$

$$V_2 = 0 = I_1 + I_2 + 4\, I_3$$
$$\therefore 0 = I_1 + I_2 + 4\, I_3$$

$$V_3 = -I_3 = 2\, I_1 + 4\, I_2 + I_3$$
$$\therefore \quad 0 = 2\, I_1 + 4\, I_2 + 2 I_3$$

Solving, we find (I'll let **you** do the algebraic details!):

$$I_1 = 7.0 \qquad I_2 = -3.0 \qquad I_3 = -1.0$$

$$V_1 = 9.0 \qquad V_2 = 0.0 \qquad V_3 = 1.0$$