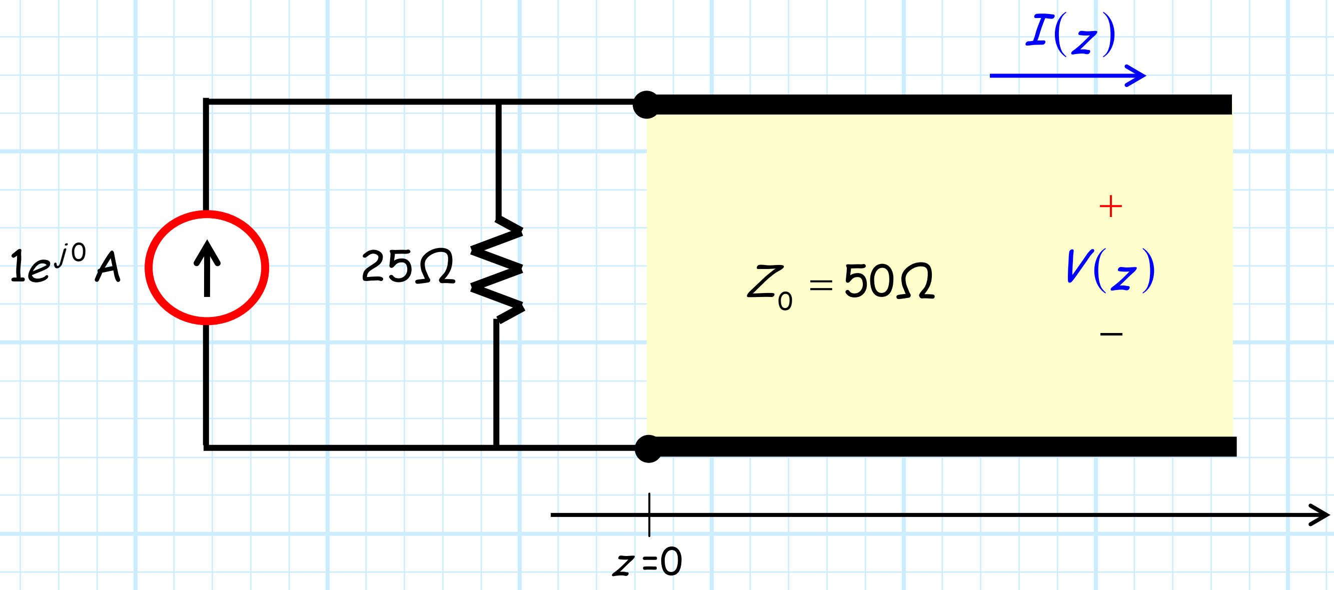
Example: Boundary Conditions and Sources

Consider the circuit below:



It is known that the **current** along the transmission line is:

$$I(z) = 0.4\, e^{-j\beta z} - B\, e^{+j\beta z} \quad A \quad \text{for } z > 0$$

where B is some unknown **complex** value.

Determine the **value of** B.

Hint: $B \neq -0.6$

Solution

Since the **line current** is:

$$I(z) = 0.4\, e^{-j\beta z} - Be^{+j\beta z} = I_0^+\, e^{-j\beta z} + I_0^-\, e^{+j\beta z}$$

we conclude from inspection that:

$$I_0^+ = 0.4 \quad \text{and} \quad I_0^- = -B$$

and since:

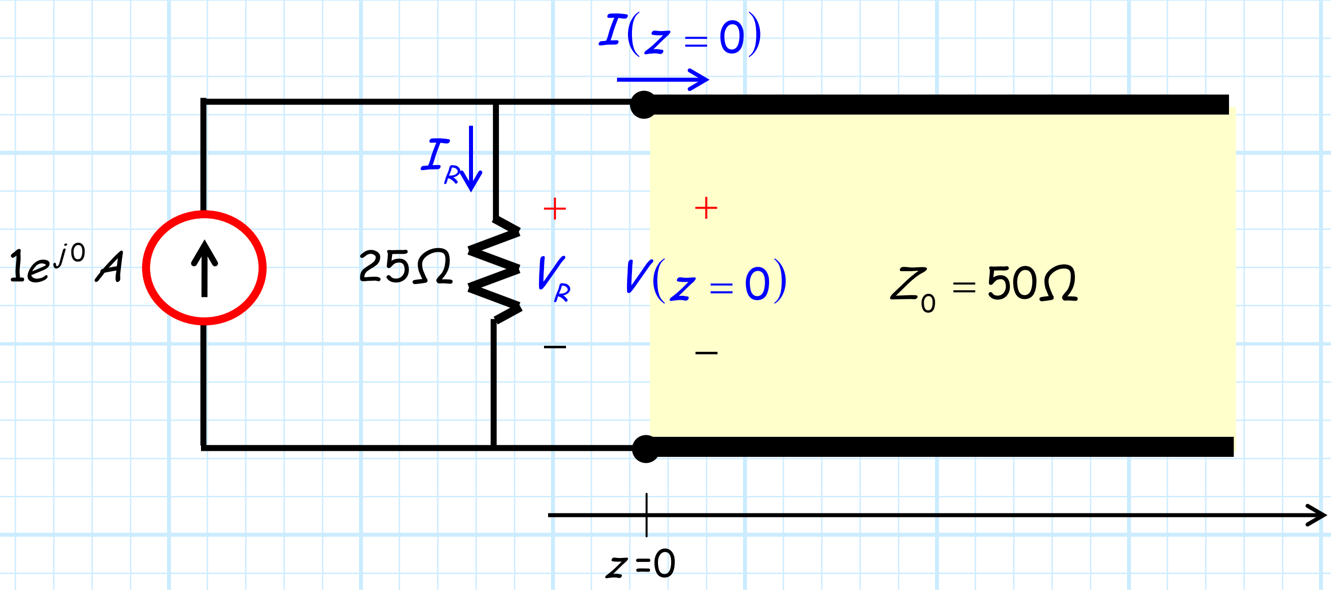
$$V_0^+ = Z_0\, I_0^+ \quad \text{and} \quad V_0^- = -Z_0\, I_0^-$$

we conclude:

$$V_0^+ = Z_0\, I_0^+ = 50(0.4) = 20.0 \quad \text{and} \quad V_0^- = -Z_0\, I_0^- = -50(-B) = 50B$$

Therefore, the voltage along this transmission line is:

$$\begin{aligned} V(z) &= V^+(z) + V^-(z) \\ &= V_0^+\, e^{-j\beta z} + V_0^-\, e^{+j\beta z} \\ &= 20\, e^{-j\beta z} + 50B\, e^{+j\beta z} \end{aligned}$$



Now, from **KCL** we find the **boundary condition** imposed by the source:

$$1.0 - I_R = I(z=0)$$

And from **KVL:**

$$V_R = V(z=0)$$

Thus from Ohm's Law we find:

$$25 = \frac{V_R}{I_R} = \frac{V(z=0)}{e^{j0} - I(z=0)}$$

Therefore:

$$V(z=0) = 25\left(e^{j0} - I(z=0)\right)$$

where:

$$\begin{aligned} I(z=0) &= 0.4\, e^{-j\beta(0)} - Be^{+j\beta(0)} \\ &= 0.4 - B \end{aligned}$$

and:

$$V(z=0) = 20\,e^{-j\beta(0)} + 50B\,e^{+j\beta(0)}$$
$$= 20 + 50B$$

Inserting this into the previous equation:

$$(20 + 50B) = 25\left(e^{j0} - (0.4 - B)\right)$$
$$= 25 - 0.4(25) + 25B$$
$$= 15 + 25B$$

One equation and **one** unknown! Solving for B:

$$\underline{\underline{B = -0.2}}$$