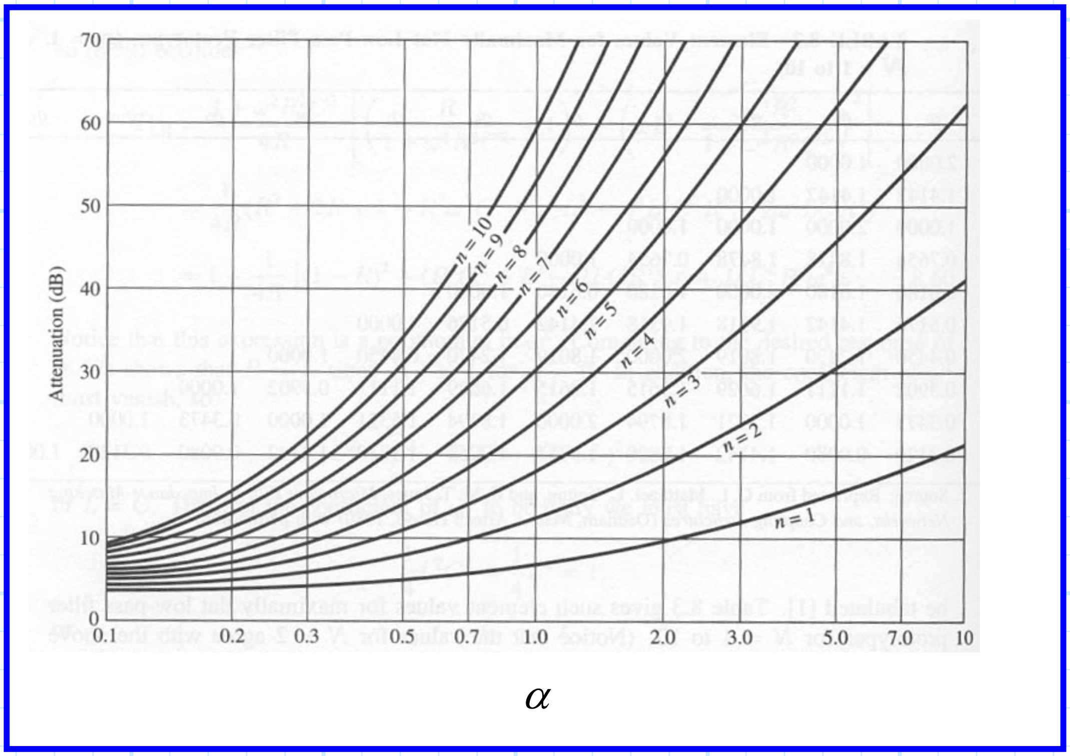
Filter Design Worksheet

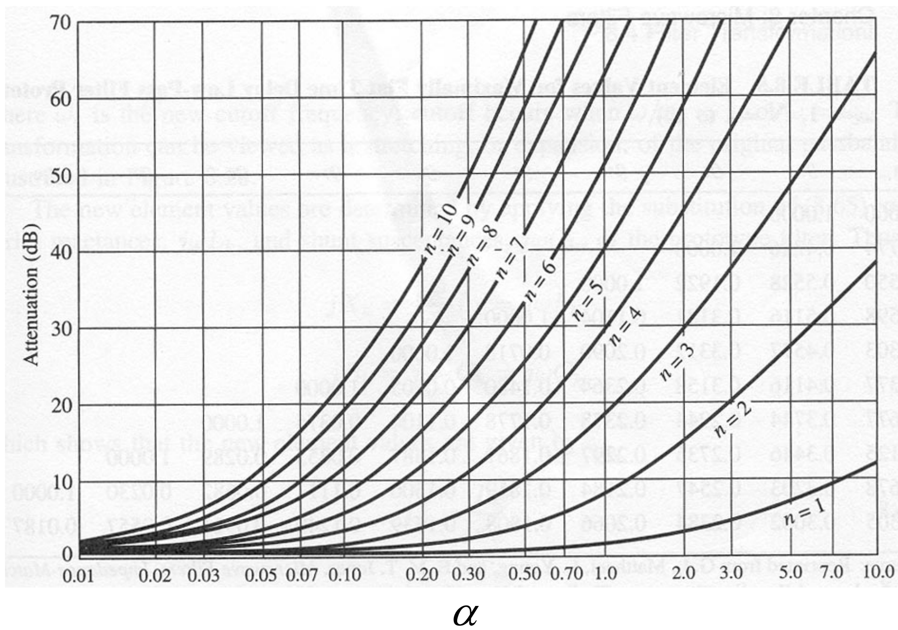
Q: *Given the order of a Butterworth or Chebychev bandpass filter, what will my stop band attenuation be? Or stated another way, what should the order of my filter be, in order to achieve a desired amount of stop-band attenuation* ($-10\log_{10} T(\omega)$) *??*

A: Consult the **normalized attenuation charts** (They're in your book)!

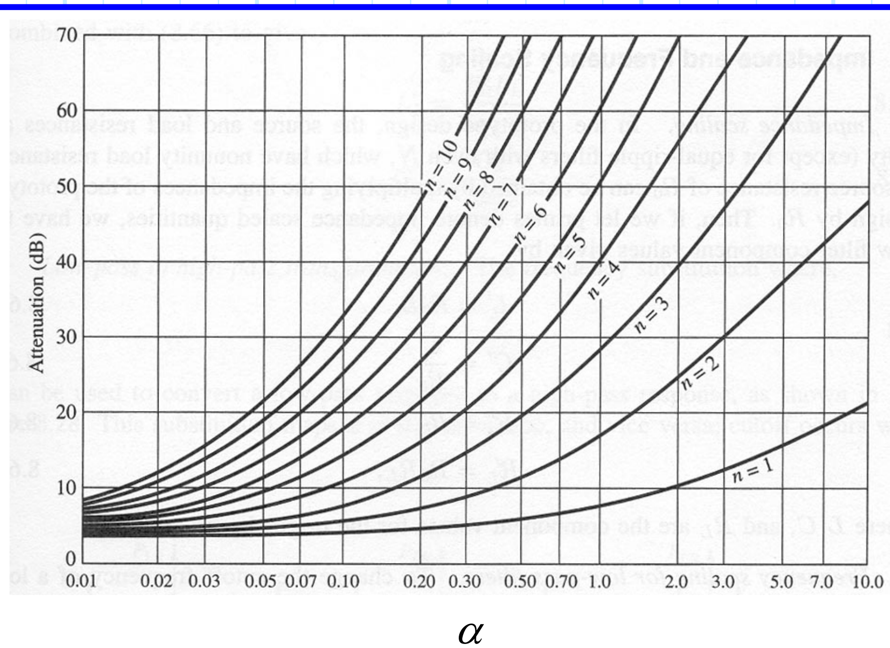
For example, the normalized attenuation chart for a **Butterworth** filter is:



While the normalized attenuation chart for a **Chebychev** with **0.5 dB** of passband ripple is:



And the normalized attenuation chart for a **Chebychev** with **3.0 dB** of passband ripple is:



Q: *Great, how the heck do I use **these** ??*

A: The variable α is a **normalized** frequency variable. The plots show attenuation versus frequency for a filter of **order** *n.*

Say we have a **bandpass filter**, whose (3 dB) passband extends from f_1 to f_2 ($f_2 > f_1$). The bandwidth of this filter would therefore be $f_2 - f_1$.

Using these values, we can define a **normalized frequency** α as:

$$\alpha = \left| \frac{1}{\Delta} \left(\frac{f}{f_0} - \frac{f_0}{f} \right) \right| - 1$$

where:

$$f_0 = \sqrt{f_1 f_2}$$

$$\Delta = \frac{f_2 - f_1}{f_0}$$

Thus, given a frequency f, we can calculate a value α.

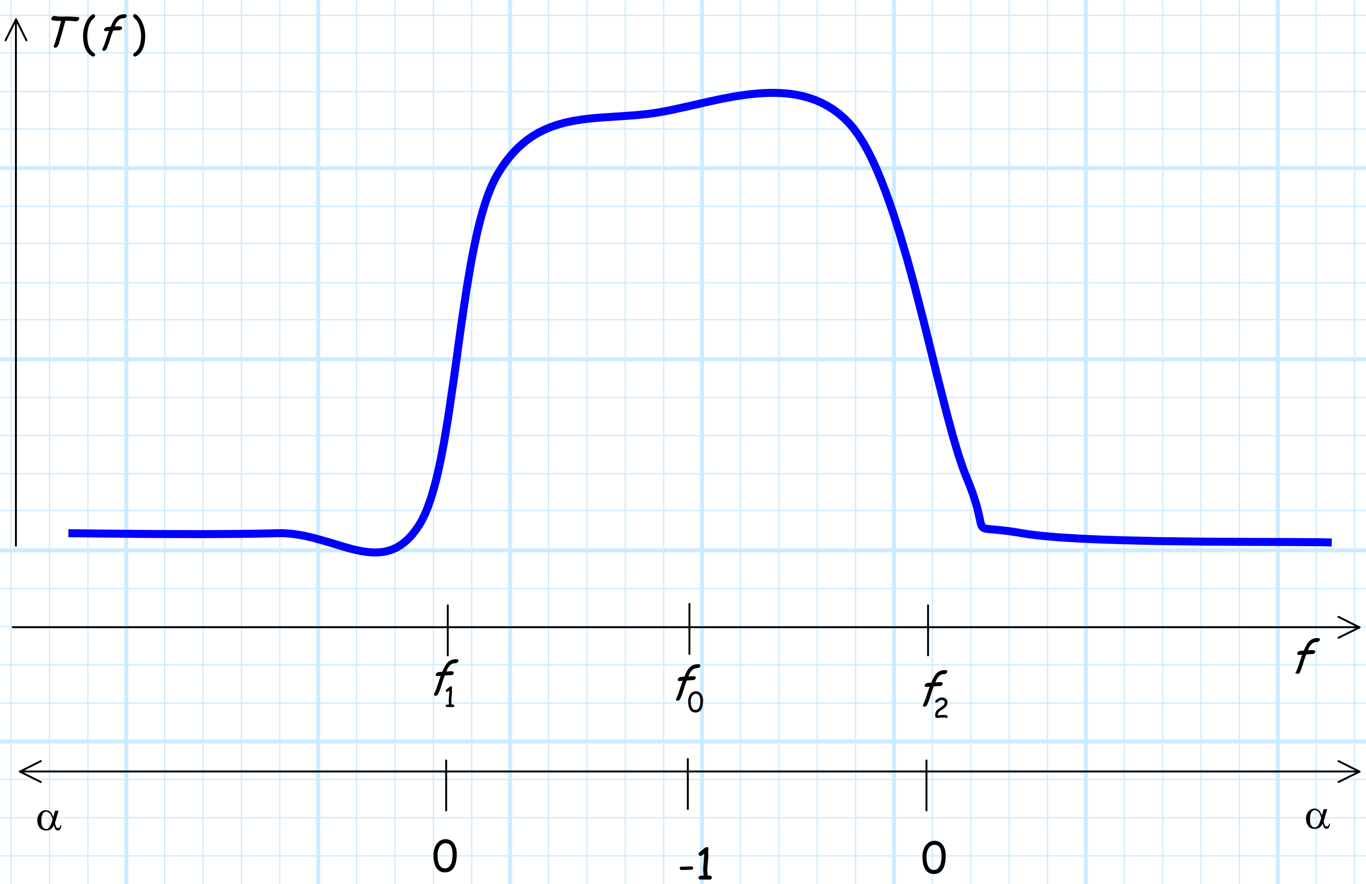
* It turns out that all frequencies f **outside** the pass band of the filter will have **positive** values of α, while frequencies **within** the pass band will result in **negative** values of α.

* Accordingly, if $f = f_1$ or $f = f_2$, the value of α will be **zero** (try it!).

* As a result, the attenuation charts *give* answers for **positive** values of α only, corresponding to frequencies in the **stop band**.

* In other words, the attenuation charts provide information about the stop band **attenuation** only. Note as α gets **larger**, the attenuation for all filter orders **increases**.

* This makes since, as an increasing α corresponds to a frequency f either greater than f_2 and increasing, or a frequency f less than f_1 and decreasing.



For **example**, consider a Butterworth bandpass filter whose passband extends from 1 GHz to 4 GHz.

Therefore, $f_1 = 1$ GHz and $f_2 = 4$ GHz, resulting in $f_0 = 2$ GHz and $\Delta = 1.5$.

Q1: *By how **much** is a 500 MHz signal attenuated if the filter has order n=6 ?*

For $f = 0.5$ GHz:

$$\begin{aligned}\alpha &= \left|\frac{1}{\Delta}\left(\frac{f}{f_0} - \frac{f_0}{f}\right)\right| - 1 \\ &= \left|\frac{1}{1.5}\left(\frac{0.5}{2.0} - \frac{2.0}{0.5}\right)\right| - 1 \\ &= 1.5\end{aligned}$$

It appears from the **attenuation chart** that this filter attenuates a 500 MHz signal approximately 50 dB.

Q2: *What should the filter order n be, if we need to attenuate signals at 6.6 GHz by at least 40 dB?*

For f = 8 GHz:

$$\begin{aligned}\alpha &= \left|\frac{1}{\Delta}\left(\frac{f}{f_0} - \frac{f_0}{f}\right)\right| - 1 \\ &= \left|\frac{1}{1.5}\left(\frac{6.6}{2.0} - \frac{2.0}{6.6}\right)\right| - 1 \\ &= 1.0\end{aligned}$$

Again from the chart, we find at α = 1.0, a filter with order *n* =7 (or higher) will attenuate a 6.6 GHz signal by more than 40 dB.

Now **you** too can determine filter attenuation and /or order. I hope you've been **paying attention** !!