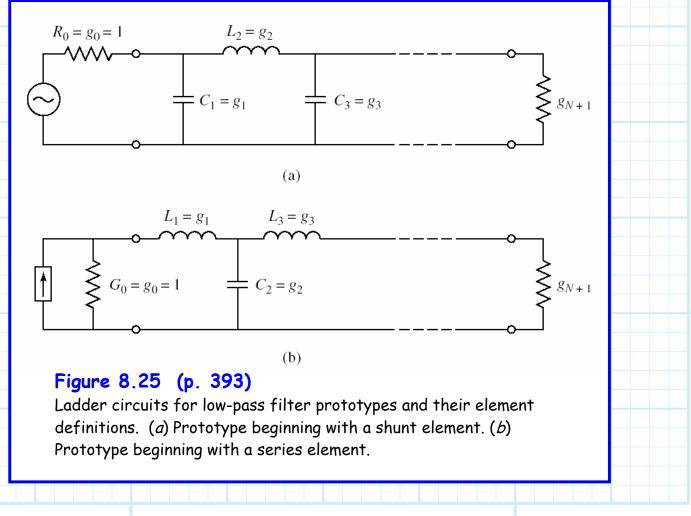
Filter Realizations Using Lumped Elements

Our first filter circuit will be "realized" with lumped elements.

Lumped elements—we mean inductors L and capacitors C!

Since each of these elements are (ideally) perfectly **reactive**, the resulting filter will be **lossless** (ideally).

We will first consider two configurations of a ladder circuit:



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Note that these two structures provide a **low-pass** filter response (evaluate the circuits at $\omega = 0$ and $\omega = \infty$!).

Moreover, these structures have N different **reactive** elements (i.e., N degrees of design freedom) and thus can be used to realize an **N-order** power loss ratio.

For example, consider the **Butterworth** power loss ratio function:

$$P_{LR}(\omega) = 1 + \left(\frac{\omega}{\omega_c}\right)^2$$

Recall this is a **low-pass** function, as $P_{LR} = 1$ at $\omega = 0$, and $P_{LR} = \infty$ at $\omega = \infty$. Note also that at $\omega = \omega_c$:

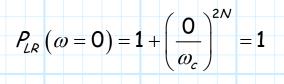
$$P_{LR}\left(\omega=\omega_{c}\right)=1+\left(\frac{\omega_{c}}{\omega_{c}}\right)^{2N}=1+1^{2N}=2$$

Meaning that:

$$\Gamma(\omega = \omega_c) = T(\omega = \omega_c) = \frac{1}{2}$$

In other words, ω_c defines the 3dB bandwidth of the low-pass filter.

Likewise, we find that this Butterworth function is **maximally** flat at $\omega = 0$:



and:

$$\frac{d^n P_{LR}(\omega)}{d\omega^n} \bigg|_{\omega=0} = 0 \quad \text{for all } n$$

Now, we can determine the function $P_{LR}(\omega)$ for a lumped element ladder circuit of Nelements using our knowledge of **complex circuit theory**.

Then, we can **equate** the resulting polynomial to the **maximally flat** function above. In this manner, we can determine the appropriate **values** of all inductors L and capacitors C!

An **example** of this method is given on pages 392 and 393 of your book. In this case, the filter is very **simple**—just **one** inductor and **one** capacitor. However, as the book shows, finding the solution requires quite a bit **complex algebra**!

Fortunately, your book likewise provides **tables** of complete Butterworth and Chebychev Low-Pass **solutions** (up to order 10) for the ladder circuits of figure 8.25—**no** complex algebra required!

			The second second	1000		2010					
Ν	g 1	<i>g</i> ₂	<i>g</i> ₃	g_4	g 5	g 6	g 7	g_8	g 9	g 10	<i>g</i> ₁₁
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						also get a
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					200.000
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000		inter în		
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000
		Networks,	and Coupli	ng Structure	es (Dedham	, Mass.: Art	ech House,	, 1980) with	permission	1.	
		1000		1	0.5.3	D Dimela	2			100	
Ν	g_1	<i>g</i> ₂	<i>g</i> ₃	g 4	0.5 d g ₅	B Ripple	<i>g</i> 7	g_8	<i>8</i> 9	g 10	g 11
N 1	<i>g</i> 1 0.6986	g ₂ 1.0000	<i>g</i> ₃	g 4			<i>g</i> 7	g 8	89	B 10	<i>g</i> ₁₁
	0.6986 1.4029	1.0000 0.7071	1.9841	Take .			87	g 8	89	g 10	<u>g11</u>
1	0.6986 1.4029 1.5963	1.0000 0.7071 1.0967	1.9841 1.5963	1.0000	<i>g</i> 5		<i>g</i> 7	g 8	89	g 10	<u>g11</u>
1 2 3 4	0.6986 1.4029 1.5963 1.6703	1.0000 0.7071 1.0967 1.1926	1.9841 1.5963 2.3661	1.0000 0.8419	<i>g</i> ₅ 1.9841	<i>g</i> ₆	<i>8</i> 7	g ₈	89	<i>8</i> 10	<u>g11</u>
1 2 3 4 5	0.6986 1.4029 1.5963 1.6703 1.7058	1.0000 0.7071 1.0967 1.1926 1.2296	1.9841 1.5963 2.3661 2.5408	1.0000 0.8419 1.2296	<i>g</i> ₅ 1.9841 1.7058	g ₆ 1.0000	Junea Maga	<i>g</i> 8	89	810	<u>g11</u>
1 2 3 4 5 6	0.6986 1.4029 1.5963 1.6703 1.7058 1.7254	1.0000 0.7071 1.0967 1.1926 1.2296 1.2479	1.9841 1.5963 2.3661 2.5408 2.6064	1.0000 0.8419 1.2296 1.3137	<i>g</i> ₅ 1.9841 1.7058 2.4758	g ₆ 1.0000 0.8696	1.9841	1,000	<i>8</i> 9	<i>8</i> 10	<u>g11</u>
1 2 3 4 5 6 7	0.6986 1.4029 1.5963 1.6703 1.7058 1.7254 1.7372	1.0000 0.7071 1.0967 1.1926 1.2296 1.2479 1.2583	1.9841 1.5963 2.3661 2.5408 2.6064 2.6381	1.0000 0.8419 1.2296 1.3137 1.3444	<i>g</i> ₅ 1.9841 1.7058 2.4758 2.6381	<i>g</i> ₆ 1.0000 0.8696 1.2583	1.9841 1.7372	1.0000	2.000 1.0000 1.2011	<i>g</i> 10	<u>g11</u>
1 2 3 4 5 6 7 8	0.6986 1.4029 1.5963 1.6703 1.7058 1.7254 1.7372 1.7451	1.0000 0.7071 1.0967 1.1926 1.2296 1.2296 1.2479 1.2583 1.2647	1.9841 1.5963 2.3661 2.5408 2.6064 2.6381 2.6564	1.0000 0.8419 1.2296 1.3137 1.3444 1.3590	<i>g</i> ₅ 1.9841 1.7058 2.4758 2.6381 2.6964	<i>g</i> ₆ 1.0000 0.8696 1.2583 1.3389	1.9841 1.7372 2.5093	1.0000 0.8796	1.9841		<u>g11</u>
1 2 3 4 5 6 7 8 9	0.6986 1.4029 1.5963 1.6703 1.7058 1.7254 1.7254 1.7372 1.7451 1.7504	1.0000 0.7071 1.0967 1.1926 1.2296 1.2479 1.2583 1.2647 1.2690	1.9841 1.5963 2.3661 2.5408 2.6064 2.6381 2.6564 2.6564 2.6678	1.0000 0.8419 1.2296 1.3137 1.3444 1.3590 1.3673	<i>g</i> ₅ 1.9841 1.7058 2.4758 2.6381 2.6964 2.7239	<i>g</i> ₆ 1.0000 0.8696 1.2583 1.3389 1.3673	1.9841 1.7372 2.5093 2.6678	1.0000 0.8796 1.2690	1.9841 1.7504	1.0000	
1 2 3 4 5 6 7 8 9	0.6986 1.4029 1.5963 1.6703 1.7058 1.7254 1.7372 1.7451	1.0000 0.7071 1.0967 1.1926 1.2296 1.2296 1.2479 1.2583 1.2647	1.9841 1.5963 2.3661 2.5408 2.6064 2.6381 2.6564	1.0000 0.8419 1.2296 1.3137 1.3444 1.3590	<i>g</i> ₅ 1.9841 1.7058 2.4758 2.6381 2.6964 2.7239 2.7392	86 1.0000 0.8696 1.2583 1.3389 1.3673 1.3806	1.9841 1.7372 2.5093	1.0000 0.8796	1.9841		<u>g11</u> 1.9841
1 2 3 4 5 6 7 8 9 10	0.6986 1.4029 1.5963 1.6703 1.7058 1.7254 1.7254 1.7372 1.7451 1.7504 1.7543	1.0000 0.7071 1.0967 1.1926 1.2296 1.2479 1.2583 1.2647 1.2690 1.2721	1.9841 1.5963 2.3661 2.5408 2.6064 2.6381 2.6564 2.6564 2.6678 2.6754	1.0000 0.8419 1.2296 1.3137 1.3444 1.3590 1.3673 1.3725	<i>g</i> 5 1.9841 1.7058 2.4758 2.6381 2.6964 2.7239 2.7392 3.0 d	 g6 1.0000 0.8696 1.2583 1.3389 1.3673 1.3806 B Ripple 	1.9841 1.7372 2.5093 2.6678 2.7231	1.0000 0.8796 1.2690 1.3485	1.9841 1.7504 2.5239	1.0000 0.8842	1.9841
1 2 3 4 5 6 7 8 9	0.6986 1.4029 1.5963 1.6703 1.7058 1.7254 1.7372 1.7451 1.7504 1.7543 <i>8</i> 1	1.0000 0.7071 1.0967 1.1926 1.2296 1.2479 1.2583 1.2647 1.2690 1.2721	1.9841 1.5963 2.3661 2.5408 2.6064 2.6381 2.6564 2.6564 2.6678	1.0000 0.8419 1.2296 1.3137 1.3444 1.3590 1.3673	<i>g</i> ₅ 1.9841 1.7058 2.4758 2.6381 2.6964 2.7239 2.7392	86 1.0000 0.8696 1.2583 1.3389 1.3673 1.3806	1.9841 1.7372 2.5093 2.6678	1.0000 0.8796 1.2690	1.9841 1.7504	1.0000	
1 2 3 4 5 6 7 8 9 10 <i>N</i> 1	0.6986 1.4029 1.5963 1.6703 1.7058 1.7254 1.7254 1.7372 1.7451 1.7504 1.7543 <i>g</i> 1 1.9953	1.0000 0.7071 1.0967 1.1926 1.2296 1.2479 1.2583 1.2647 1.2690 1.2721 <i>g</i> ₂ 1.0000	1.9841 1.5963 2.3661 2.5408 2.6064 2.6381 2.6564 2.6564 2.6678 2.6754 <i>8</i> 3	1.0000 0.8419 1.2296 1.3137 1.3444 1.3590 1.3673 1.3725	<i>g</i> 5 1.9841 1.7058 2.4758 2.6381 2.6964 2.7239 2.7392 3.0 d	 g6 1.0000 0.8696 1.2583 1.3389 1.3673 1.3806 B Ripple 	1.9841 1.7372 2.5093 2.6678 2.7231	1.0000 0.8796 1.2690 1.3485	1.9841 1.7504 2.5239	1.0000 0.8842	1.9841
1 2 3 4 5 6 7 8 9 10 <i>N</i> 1 2	0.6986 1.4029 1.5963 1.6703 1.7058 1.7254 1.7254 1.7372 1.7451 1.7504 1.7543 <i>g</i> 1 1.9953 3.1013	1.0000 0.7071 1.0967 1.1926 1.2296 1.2479 1.2583 1.2647 1.2690 1.2721 <u>82</u> 1.0000 0.5339	1.9841 1.5963 2.3661 2.5408 2.6064 2.6381 2.6564 2.6678 2.6754 <i>g</i> ₃ 5.8095	1.0000 0.8419 1.2296 1.3137 1.3444 1.3590 1.3673 1.3725 <i>8</i> 4	<i>g</i> 5 1.9841 1.7058 2.4758 2.6381 2.6964 2.7239 2.7392 3.0 d	 g6 1.0000 0.8696 1.2583 1.3389 1.3673 1.3806 B Ripple 	1.9841 1.7372 2.5093 2.6678 2.7231	1.0000 0.8796 1.2690 1.3485	1.9841 1.7504 2.5239	1.0000 0.8842	1.9841
1 2 3 4 5 6 7 8 9 10 <i>N</i> 1 2 3	0.6986 1.4029 1.5963 1.6703 1.7058 1.7254 1.7372 1.7451 1.7504 1.7543 <i>g</i> 1 1.9953 3.1013 3.3487	$\begin{array}{c} 1.0000\\ 0.7071\\ 1.0967\\ 1.1926\\ 1.2296\\ 1.2479\\ 1.2583\\ 1.2647\\ 1.2690\\ 1.2721\\ \hline g_2\\ \hline g_2\\ 1.0000\\ 0.5339\\ 0.7117\\ \end{array}$	1.9841 1.5963 2.3661 2.5408 2.6064 2.6381 2.6564 2.6678 2.6754 <i>g</i> ₃ 5.8095 3.3487	1.0000 0.8419 1.2296 1.3137 1.3444 1.3590 1.3673 1.3725 <i>8</i> 4 1.0000	85 1.9841 1.7058 2.4758 2.6381 2.6964 2.7239 2.7392 3.0 d 85	 g6 1.0000 0.8696 1.2583 1.3389 1.3673 1.3806 B Ripple 	1.9841 1.7372 2.5093 2.6678 2.7231	1.0000 0.8796 1.2690 1.3485	1.9841 1.7504 2.5239 <i>8</i> 9	1.0000 0.8842 g10	1.9841
1 2 3 4 5 6 7 8 9 10 <i>N</i> 1 2 3 4	0.6986 1.4029 1.5963 1.6703 1.7058 1.7254 1.7372 1.7451 1.7504 1.7543 <i>g</i> 1 1.9953 3.1013 3.3487 3.4389	1.0000 0.7071 1.0967 1.1926 1.2296 1.2479 1.2583 1.2647 1.2690 1.2721 <i>g</i> ₂ 1.0000 0.5339 0.7117 0.7483	1.9841 1.5963 2.3661 2.5408 2.6064 2.6381 2.6564 2.6678 2.6754 <i>8</i> 3 5.8095 3.3487 4.3471	1.0000 0.8419 1.2296 1.3137 1.3444 1.3590 1.3673 1.3725 <i>8</i> 4 1.0000 0.5920	85 1.9841 1.7058 2.4758 2.6381 2.6964 2.7239 2.7392 3.0 d 85 5.8095	86 1.0000 0.8696 1.2583 1.3389 1.3673 1.3806 B Ripple 86	1.9841 1.7372 2.5093 2.6678 2.7231	1.0000 0.8796 1.2690 1.3485	1.9841 1.7504 2.5239	1.0000 0.8842 g10	1.9841
1 2 3 4 5 6 7 8 9 10 <i>N</i> 1 2 3 4 5	0.6986 1.4029 1.5963 1.6703 1.7058 1.7254 1.7372 1.7451 1.7504 1.7543 <i>g</i> ₁ 1.9953 3.1013 3.3487 3.4389 3.4817	1.0000 0.7071 1.0967 1.1926 1.2296 1.2479 1.2583 1.2647 1.2690 1.2721 <i>g</i> ₂ 1.0000 0.5339 0.7117 0.7483 0.7618	1.9841 1.5963 2.3661 2.5408 2.6064 2.6381 2.6564 2.6678 2.6754 <i>g</i> ₃ 5.8095 3.3487 4.3471 4.5381	1.0000 0.8419 1.2296 1.3137 1.3444 1.3590 1.3673 1.3725 <i>g</i> 4 1.0000 0.5920 0.7618	85 1.9841 1.7058 2.4758 2.6381 2.6964 2.7239 2.7392 3.0 d 85 5.8095 3.4817	86 1.0000 0.8696 1.2583 1.3899 1.3673 1.3806 B Ripple 86 1.0000	1.9841 1.7372 2.5093 2.6678 2.7231 <i>g</i> 7	1.0000 0.8796 1.2690 1.3485	1.9841 1.7504 2.5239 <i>8</i> 9	1.0000 0.8842 g10	1.9841
1 2 3 4 5 6 7 8 9 10 <i>N</i> 1 2 3 4 5 6	0.6986 1.4029 1.5963 1.6703 1.7058 1.7254 1.7254 1.7451 1.7504 1.7543 <i>g</i> 1 1.9953 3.1013 3.3487 3.4389 3.4817 3.5045	1.0000 0.7071 1.0967 1.1926 1.2296 1.2479 1.2583 1.2647 1.2690 1.2721 g ₂ 1.0000 0.5339 0.7117 0.7483 0.7618 0.7685	1.9841 1.5963 2.3661 2.5408 2.6064 2.6381 2.6564 2.6678 2.6754 <i>g</i> ₃ 5.8095 3.3487 4.3471 4.5381 4.6061	1.0000 0.8419 1.2296 1.3137 1.3444 1.3590 1.3673 1.3725 <i>g</i> 4 1.0000 0.5920 0.7618 0.7929	85 1.9841 1.7058 2.4758 2.6381 2.6964 2.7239 2.7392 3.0 d 85 5.8095 3.4817 4.4641	<i>g</i> ₆ 1.0000 0.8696 1.2583 1.3389 1.3673 1.3806 B Ripple <i>g</i> ₆ 1.0000 0.6033	1.9841 1.7372 2.5093 2.6678 2.7231 <i>g</i> 7 5.8095	1.0000 0.8796 1.2690 1.3485 g ₈	1.9841 1.7504 2.5239 <i>8</i> 9	1.0000 0.8842 g10	1.9841
1 2 3 4 5 6 7 8 9 10 <i>N</i> 1 2 3 4 5 6 7	0.6986 1.4029 1.5963 1.6703 1.7058 1.7254 1.7254 1.7451 1.7504 1.7543 <i>g</i> ₁ 1.9953 3.1013 3.3487 3.4389 3.4817 3.5045 3.5182	1.0000 0.7071 1.0967 1.1926 1.2296 1.2479 1.2583 1.2647 1.2690 1.2721 <i>g</i> ₂ 1.0000 0.5339 0.7117 0.7483 0.7618 0.7685 0.7723	1.9841 1.5963 2.3661 2.5408 2.6064 2.6381 2.6564 2.6564 2.6678 2.6754 <i>g</i> ₃ 5.8095 3.3487 4.3471 4.5381 4.6061 4.6386	1.0000 0.8419 1.2296 1.3137 1.3444 1.3590 1.3673 1.3725 <i>8</i> 4 1.0000 0.5920 0.7618 0.7929 0.8039	85 1.9841 1.7058 2.4758 2.6381 2.6964 2.7239 2.7392 3.0 d 85 5.8095 3.4817 4.4641 4.6386	<i>g</i> ₆ 1.0000 0.8696 1.2583 1.3389 1.3673 1.3806 B Ripple <i>g</i> ₆ 1.0000 0.6033 0.7723	1.9841 1.7372 2.5093 2.6678 2.7231 <i>g</i> 7 5.8095 3.5182	1.0000 0.8796 1.2690 1.3485 g ₈ 1.0000	1.9841 1.7504 2.5239 <i>8</i> 9	1.0000 0.8842 g10	1.9841
1 2 3 4 5 6 7 8 9 10 <i>N</i> 1 2 3 4 5 6 7 8	$\begin{array}{c} 0.6986\\ 1.4029\\ 1.5963\\ 1.6703\\ 1.7058\\ 1.7254\\ 1.7372\\ 1.7451\\ 1.7504\\ 1.7504\\ 1.7543\\ \hline g_1\\ 1.9953\\ 3.1013\\ 3.3487\\ 3.4389\\ 3.4817\\ 3.5045\\ 3.5182\\ 3.5277\\ \end{array}$	$\begin{array}{c} 1.0000\\ 0.7071\\ 1.0967\\ 1.1926\\ 1.2296\\ 1.2479\\ 1.2583\\ 1.2647\\ 1.2690\\ 1.2721\\ \hline \\ g_2\\ \hline \\ 1.0000\\ 0.5339\\ 0.7117\\ 0.7483\\ 0.7618\\ 0.7685\\ 0.7723\\ 0.7745\\ \hline \end{array}$	1.9841 1.5963 2.3661 2.5408 2.6064 2.6381 2.6564 2.6678 2.6754 <i>g</i> ₃ 5.8095 3.3487 4.3471 4.5381 4.6061 4.6386 4.6575	1.0000 0.8419 1.2296 1.3137 1.3444 1.3590 1.3673 1.3725 <i>8</i> 4 1.0000 0.5920 0.7618 0.7929 0.8039 0.8089	85 1.9841 1.7058 2.4758 2.6381 2.6964 2.7239 2.7392 3.0 d 85 5.8095 3.4817 4.4641 4.6386 4.6990	86 1.0000 0.8696 1.2583 1.389 1.3673 1.3806 B Ripple 86 1.0000 0.6033 0.7723 0.8018	1.9841 1.7372 2.5093 2.6678 2.7231 <i>g</i> 7 5.8095 3.5182 4.4990	1.0000 0.8796 1.2690 1.3485 g ₈ 1.0000 0.6073	1.9841 1.7504 2.5239 <i>8</i> 9 5.8095	1.0000 0.8842 g10	1.9841
1 2 3 4 5 6 7 8 9 10 <i>N</i> 1 2 3 4 5 6 7	0.6986 1.4029 1.5963 1.6703 1.7058 1.7254 1.7254 1.7451 1.7504 1.7543 <i>g</i> ₁ 1.9953 3.1013 3.3487 3.4389 3.4817 3.5045 3.5182	1.0000 0.7071 1.0967 1.1926 1.2296 1.2479 1.2583 1.2647 1.2690 1.2721 <i>g</i> ₂ 1.0000 0.5339 0.7117 0.7483 0.7618 0.7685 0.7723	1.9841 1.5963 2.3661 2.5408 2.6064 2.6381 2.6564 2.6564 2.6678 2.6754 <i>g</i> ₃ 5.8095 3.3487 4.3471 4.5381 4.6061 4.6386	1.0000 0.8419 1.2296 1.3137 1.3444 1.3590 1.3673 1.3725 <i>8</i> 4 1.0000 0.5920 0.7618 0.7929 0.8039	85 1.9841 1.7058 2.4758 2.6381 2.6964 2.7239 2.7392 3.0 d 85 5.8095 3.4817 4.4641 4.6386	<i>g</i> ₆ 1.0000 0.8696 1.2583 1.3389 1.3673 1.3806 B Ripple <i>g</i> ₆ 1.0000 0.6033 0.7723	1.9841 1.7372 2.5093 2.6678 2.7231 <i>g</i> 7 5.8095 3.5182	1.0000 0.8796 1.2690 1.3485 g ₈ 1.0000	1.9841 1.7504 2.5239 <i>8</i> 9	1.0000 0.8842 g10	1.9841

Q: What?! What the heck do these values g_n mean?

A: We can use the values g_n to find the values of inductors and capacitors required for a given **cutoff frequency** ω_c and source resistance R_s (Z_0).

Specifically, we use the values of g_n to find ladder circuit inductor and capacitor values as:

$$\mathcal{L}_n = \mathcal{G}_n \left(\frac{\mathcal{R}_s}{\omega_c} \right) \qquad \qquad \mathcal{C}_n = \mathcal{G}_n \left(\frac{1}{\mathcal{R}_s \, \omega_c} \right)$$

where $n = 1, 2, \cdots, N$

Likewise, the value g_{N+1} describes the load impedance. Specifically, we find that if the last reactive element (i.e., g_N) of the ladder circuit is a shunt capacitor, then:

$$R_L = g_{N+1} R_s$$

Whereas, if the last reactive element (i.e., g_N) of the ladder circuit is a series inductor, then:

$$R_{L} = \frac{R_{s}}{q_{N+1}}$$

Note however for the **Butterworth** solutions (in Table 8.3) we find that $g_{N+1} = 1$ always, and therefore:

$$R_L = R_s$$

regardless of the last element.

Moreover, we note (in Table 8.4) that this (i.e., $g_{N+1} = 1$) is likewise true for the Chebychev solutions—provided that N is odd!

Thus, since we typically desire a filter where:

$$R_{L}=R_{s}=Z_{0}$$

We can use **any** order of **Butterworth** filter, or an **odd** order of **Chebychev**.

> In other words, avoid even order Chebychev filters!