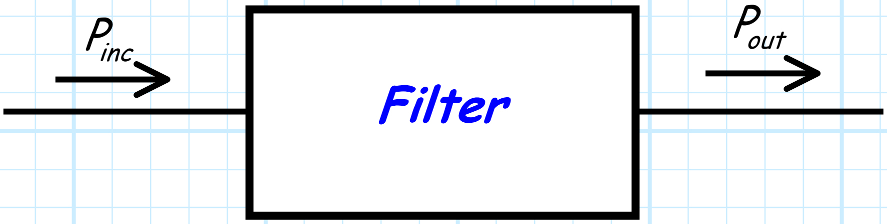
Filters

A RF/microwave **filter** is (typically) a passive, reciprocal, 2-port linear device.



If port 2 of this device is terminated in a **matched** load, then we can relate the incident and output power as:

$$P_{out} = |S_{21}|^2 \, P_{inc}$$

We define this power transmission through a filter in terms of the **power transmission coefficient T**:

$$\mathbf{T} \doteq \frac{P_{out}}{P_{inc}} = |S_{21}|^2$$

Since microwave filters are typically **passive**, we find that:

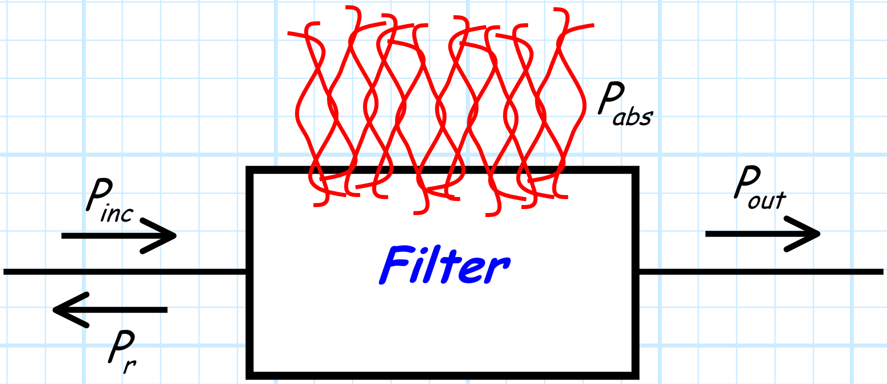
$$0 \leq \mathbf{T} \leq 1$$

in other words, $P_{out} \leq P_{inc}$.

Q: *What happens to the "missing" power $P_{inc} - P_{out}$?*

A: **Two** possibilities: the power is either **absorbed** (P_{abs}) by the filter (converted to heat), or is **reflected** (P_r) at the input port.

I.E.:



Thus, by conservation of energy:

$$P_{inc} = P_r + P_{abs} + P_{out}$$

Now **ideally,** a microwave filter is lossless, therefore $P_{abs} = 0$ and:

$$P_{inc} = P_r + P_{out}$$

which **alternatively** can be written as:

$$\frac{P_{inc}}{P_{inc}} = \frac{P_r + P_{out}}{P_{inc}}$$

$$1 = \frac{P_r}{P_{inc}} + \frac{P_{out}}{P_{inc}}$$

Recall that $P_{out}/P_{inc} = \mathbf{T}$, and we can likewise **define** P_r/P_{inc} as the **power reflection coefficient**:

$$\mathbf{\Gamma} \doteq \frac{P_r}{P_{inc}} = |S_{11}|^2$$

We again emphasize that the filter output port is terminated in a **matched** load.

Thus, we can conclude that for a **lossless** filter:

$$1 = \mathbf{\Gamma} + \mathbf{T}$$

Which is simply **another** way of saying for a lossless device that $1 = |S_{11}|^2 + |S_{21}|^2$.

Now, **here's** the important part!

For a microwave **filter**, the coefficients $\mathbf{\Gamma}$ and $\mathbf{T}$ are **functions of frequency**! I.E.,:

$$\mathbf{\Gamma}(\omega) \quad \text{and} \quad \mathbf{T}(\omega)$$

The **behavior** of a microwave filter is described by these **functions**!

We find that for most signal frequencies ω_s, these functions will have a value equal to one of **two** different **approximate** values.

Either:

$$\Gamma(\omega=\omega_s)\approx 0 \qquad \text{and} \qquad T(\omega=\omega_s)\approx 1$$

or

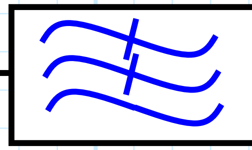
$$\Gamma(\omega=\omega_s)\approx 1 \qquad \text{and} \qquad T(\omega=\omega_s)\approx 0$$

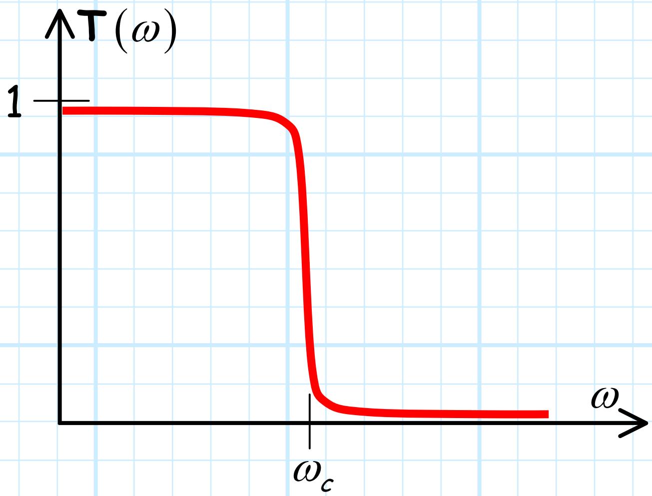
In the **first** case, the signal frequency ω_s is said to lie in the **pass-band** of the filter. Almost all of the incident signal power will **pass through** the filter.

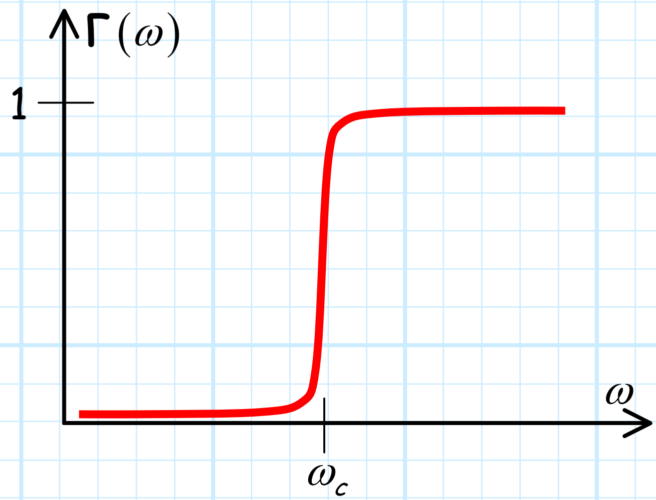
In the **second** case, the signal frequency ω_s is said to lie in the **stop-band** of the filter. Almost all of the incident signal power will be **reflected** at the input—almost no power will appear at the filter output.

Consider then these **four types** of functions of $\Gamma(\omega)$ and $T(\omega)$:

1. Low-Pass Filter







Note for this filter:

$$T(\omega) = \begin{cases} \approx 1 & \omega < \omega_c \\ \approx 0 & \omega > \omega_c \end{cases} \qquad \Gamma(\omega) = \begin{cases} \approx 0 & \omega < \omega_c \\ \approx 1 & \omega > \omega_c \end{cases}$$

This filter is a **low-pass** type, as it **"passes"** signals with frequencies **less** than ω_c, while **"rejecting"** signals at frequencies **greater** than ω_c.

Q: *This frequency ω_c seems to be very important! What is it?*

A: Frequency ω_c is a filter parameter known as the **cutoff frequency**; a value that **approximately** defines the frequency region where the filter pass-band **transitions** into the filter stop band.

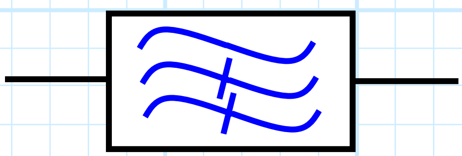
According, this frequency is defined as the frequency where the power **transmission** coefficient is equal to ½:

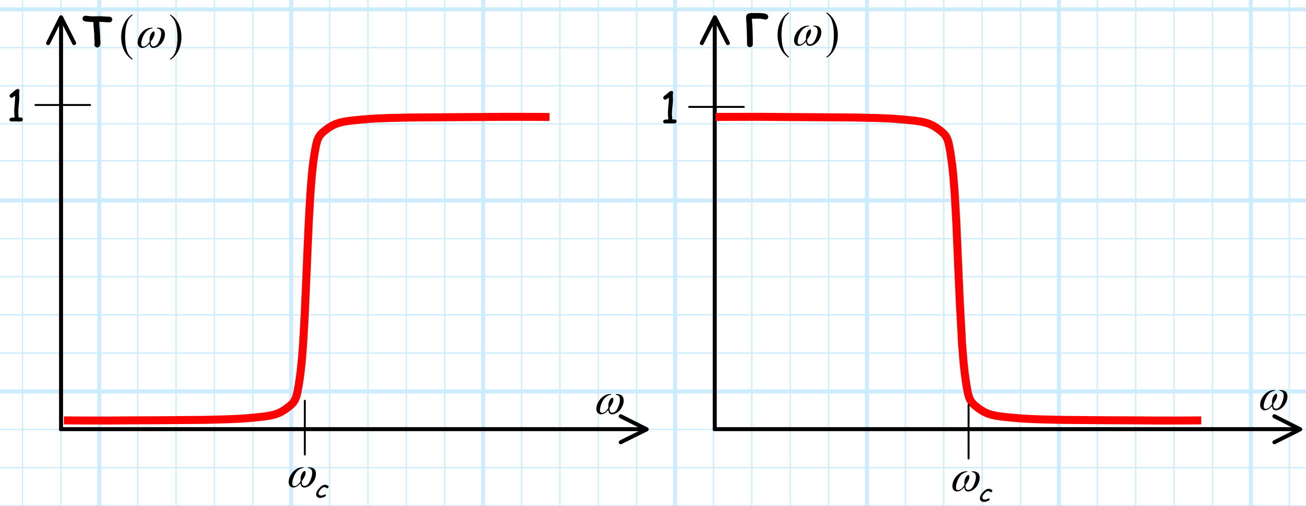
$$\mathrm{T}(\omega = \omega_c) = 0.5$$

Note for a lossless filter, the cutoff frequency is **likewise** the value where the power **reflection** coefficient is ½:

$$\Gamma(\omega = \omega_c) = 0.5$$

2. High-Pass Filter



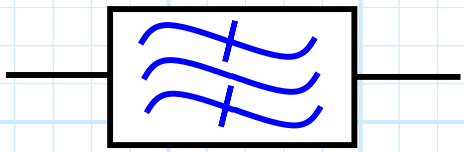


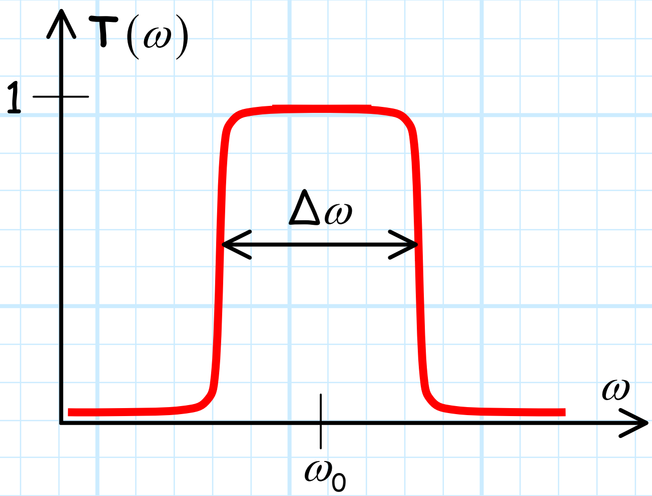
Note for this filter:

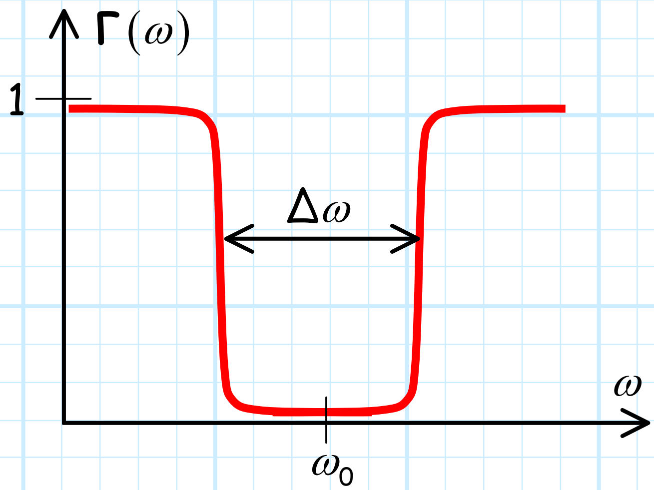
$$\mathsf{T}(\omega)=\begin{cases}\approx 0 & \omega<\omega_c\\ \approx 1 & \omega>\omega_c\end{cases}\qquad \Gamma(\omega)=\begin{cases}\approx 1 & \omega<\omega_c\\ \approx 0 & \omega>\omega_c\end{cases}$$

This filter is a **high-pass** type, as it **"passes"** signals with frequencies **greater** than ω_c, while **"rejecting"** signals at frequencies **less** than ω_c.

3. Band-Pass Filter







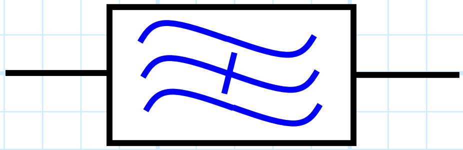
Note for this filter:

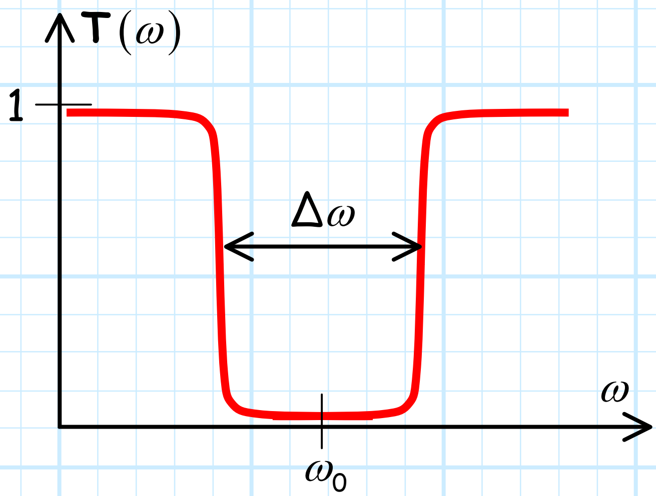
$$\mathsf{T}(\omega)=\begin{cases}\approx 1 & |\omega-\omega_0|<\Delta\omega/2\\ \approx 0 & |\omega-\omega_0|>\Delta\omega/2\end{cases}\qquad \Gamma(\omega)=\begin{cases}\approx 0 & |\omega-\omega_0|<\Delta\omega/2\\ \approx 1 & |\omega-\omega_0|<\Delta\omega/2\end{cases}$$

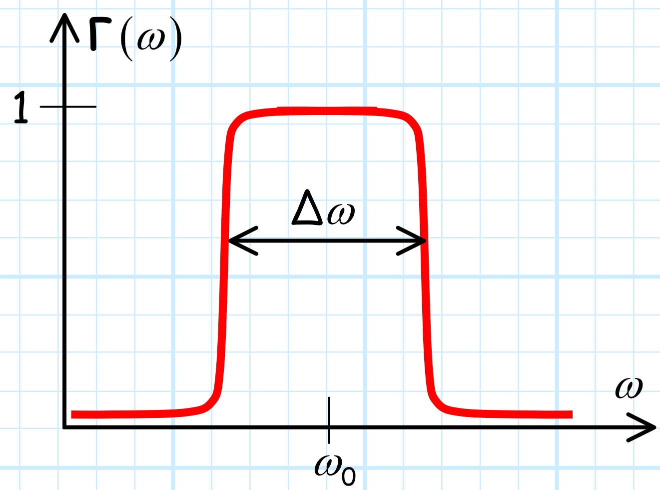
This filter is a **band-pass** type, as it **"passes"** signals within a frequency bandwidth $\Delta\omega$, while **"rejecting"** signals at all frequencies **outside this bandwidth**.

In addition to filter bandwidth $\Delta\omega$, a fundamental parameter of bandpass filters is ω_0, which defines the **center frequency** of the filter bandwidth.

3. Band-Stop Filter







Note for this filter:

$$\mathbf{T}(\omega) = \begin{cases} \approx 0 & |\omega - \omega_0| < \Delta\omega/2 \\ \approx 1 & |\omega - \omega_0| > \Delta\omega/2 \end{cases} \qquad \Gamma(\omega) = \begin{cases} \approx 1 & |\omega - \omega_0| < \Delta\omega/2 \\ \approx 0 & |\omega - \omega_0| < \Delta\omega/2 \end{cases}$$

This filter is a **band-stop** type, as it **"rejects"** signals within a frequency bandwidth $\Delta\omega$, while **"passing"** signals at all frequencies **outside this bandwidth**.