V, I, Z or V^+, V^-, Γ ?

Q: *How do I choose **which** relationship to use when describing/analyzing transmission line activity? What if I make the **wrong** choice? How will I know **if** my analysis is correct?*

A: Remember, the two relationships are **equivalent**.

There is **no** explicitly wrong or right choice—**both** will provide you with precisely the **same** correct answer!

For example, we know that the total voltage and current can be determined from knowledge wave representation:

$$\begin{aligned} V(z) &= V^+(z) + V^-(z) \\ &= V^+(z)\left(1 + \Gamma(z)\right) \end{aligned}$$

$$\begin{aligned} I(z) &= \frac{V^+(z) - V^-(z)}{Z_0} \\ &= \frac{V^+(z)\left(1 - \Gamma(z)\right)}{Z_0} \end{aligned}$$

A direct mapping from Z to Γ

More importantly, we find that **line impedance** $Z(z) = V(z)/I(z)$ can be expressed as:

$$Z(z) = Z_0 \frac{V^+(z) + V^-(z)}{V^+(z) - V^-(z)} = Z_0 \left(\frac{1 + \Gamma(z)}{1 - \Gamma(z)} \right)$$

→ **Look** what happened—the line impedance can be **completely** and unambiguously expressed in terms of **reflection coefficient** $\Gamma(z)$!

And a mapping from Γ to Z

With a little **algebra**, we find likewise that the wave functions can be determined from $V(z), I(z)$ and $Z(z)$:

$$V^{+}(z) = \frac{V(z) + I(z)Z_0}{2} = \frac{V(z)}{Z(z)}\left(\frac{Z(z) + Z_0}{2}\right)$$

$$V^{-}(z) = \frac{V(z) - I(z)Z_0}{2} = \frac{V(z)}{Z(z)}\left(\frac{Z(z) - Z_0}{2}\right)$$

From this result we easily find that the reflection coefficient $\Gamma(z)$ can **likewise** be written directly in terms of **line impedance**:

$$\Gamma(z) = \frac{Z(z) - Z_0}{Z(z) + Z_0}$$

The two representations are equivalent!

Thus, the values $\Gamma(z)$ and $Z(z)$ are **equivalent** parameters—if we know **one**, then we can directly determine the **other**—each is dependent on transmission line parameters (*L,C,R,G*) **only**!

Q: *So, if they are equivalent, why wouldn't I **always** use the current, voltage, line impedance representation?*

*After all, I am more **familiar** and more confident those quantities.*

*The **wave** representation sort of **scares** me!*

A: Perhaps I can **convince** you of the value of the **wave** representation.

Remember, the time-harmonic solution to the telegraphers equation simply boils down to **two complex constants**—V_0^+ and V_0^-.

Once these complex values have been determined, we can describe **completely** the activity **all** points along our transmission line.

Look how simple this is!

For the **wave representation** we find:

$$V^+(z) = V_0^+ \, e^{-j\beta z} \qquad V^-(z) = V_0^+ \, e^{+j\beta z} \qquad \Gamma(z) = \frac{V_0^-}{V_0^+} e^{+j2\beta z}$$

Note that the **magnitudes** of the complex functions are in fact **constants** (with respect to position z):

$$\left|V^+(z)\right| = \left|V_0^+\right| \qquad \left|V^-(z)\right| = \left|V_0^+\right| \qquad \left|\Gamma(z)\right| = \left|\frac{V_0^-}{V_0^+}\right|$$

While the **relative phase** of these complex functions are expressed as a **simple** linear relationship with respect to z:

$$\arg\{V^+(z)\} = -\beta z \qquad \arg\{V^-(z)\} = +\beta z \qquad \arg\{\Gamma(z)\} = +2\beta z$$

Yuck!

Now, **contrast** this with the complex **current**, **voltage**, **impedance** functions:

$$V(z) = V_0^+ \; e^{-j\beta z} + V_0^- \; e^{+j\beta z}$$

$$I(z) = \frac{V_0^+ \; e^{-j\beta z} - V_0^- \; e^{+j\beta z}}{Z_0}$$

$$Z(z) = Z_0 \frac{V_0^+ \; e^{-j\beta z} + V_0^- \; e^{+j\beta z}}{V_0^+ \; e^{-j\beta z} - V_0^- \; e^{+j\beta z}}$$

With magnitudes:

$$|V(z)| = \left| V_0^+ \; e^{-j\beta z} + V_0^- \; e^{+j\beta z} \right| = ??$$

$$|I(z)| = \frac{\left| V_0^+ \; e^{-j\beta z} - V_0^- \; e^{+j\beta z} \right|}{Z_0} = ??$$

$$|Z(z)| = Z_0 \frac{\left| V_0^+ \; e^{-j\beta z} + V_0^- \; e^{+j\beta z} \right|}{\left| V_0^+ \; e^{-j\beta z} - V_0^- \; e^{+j\beta z} \right|} = ??$$

V^+, V^-, Γ is much simpler

And likewise phase:

$$\arg\{V(z)\} = \arg\{V_0^+ \, e^{-j\beta z} + V_0^- \, e^{+j\beta z}\} = ??$$

$$\arg\{I(z)\} = \arg\{V_0^+ \, e^{-j\beta z} - V_0^- \, e^{+j\beta z}\} = ??$$

$$\begin{aligned}\arg\{Z(z)\} &= \arg\{V_0^+ \, e^{-j\beta z} + V_0^- \, e^{+j\beta z}\} \\ &\quad -\arg\{V_0^+ \, e^{-j\beta z} - V_0^- \, e^{+j\beta z}\} \\ &= ??\end{aligned}$$

Q: *It appears to me that when attempting to describe the activity along a transmission line—as a function of* ***position*** *z—it is much* ***easier*** *and more* ***straightforward*** *to use the* ***wave*** *representation(nyuck, nyuck, nyuck).*

A: That's right! However, this does **not** mean that we **never** determine $V(z)$, $I(z)$, or $Z(z)$; these quantities are still **fundamental** and very important—particularly at each **end** of the transmission line!