V, I, Z or V^+, V^-, Γ ?

Q: *How do I choose **which** relationship to use when describing/analyzing transmission line activity? What if I make the **wrong** choice? How will I know **if** my analysis is correct?*

A: Remember, the two relationships are **equivalent**. There is **no** explicitly wrong or right choice—**both** will provide you with precisely the **same** correct answer!

For example, we know that the total voltage and current can be determined from knowledge wave representation:

$$V(z) = V^+(z) + V^+(z)$$
$$= V^+(z)\big(1 + \Gamma(z)\big)$$

$$I(z) = \frac{V^+(z) - V^+(z)}{Z_0}$$
$$= \frac{V^+(z)\big(1 - \Gamma(z)\big)}{Z_0}$$

Or explicitly using the wave solutions $V^{+}(z) = V_0^{+} e^{-j\beta z}$ and $V^{-}(z) = V_0^{-} e^{+j\beta z}$:

$$\begin{aligned} V(z) &= V_0^{+}\ e^{-j\beta z} + V_0^{-}\ e^{+j\beta z} \\ &= V_0^{+}\left(e^{-j\beta z} + \Gamma_0\ e^{+j\beta z}\right) \end{aligned}$$

$$\begin{aligned} I(z) &= \frac{V_0^{+}\ e^{-j\beta z} - V_0^{-}\ e^{+j\beta z}}{Z_0} \\ &= \frac{V_0^{+}\left(e^{-j\beta z} - \Gamma_0\ e^{+j\beta z}\right)}{Z_0} \end{aligned}$$

More importantly, we find that **line impedance** $Z(z) = V(z)/I(z)$ can be expressed as:

$$\begin{aligned} Z(z) &= Z_0\,\frac{V^{+}(z) + V^{+}(z)}{V^{+}(z) - V^{+}(z)} \\ &= Z_0\left(\frac{1+\Gamma(z)}{1-\Gamma(z)}\right) \end{aligned}$$

Look what happened—the line impedance can be **completely** and unambiguously expressed in terms of **reflection coefficient** $\Gamma(z)$!

More explicitly:

$$Z(z) = Z_0\,\frac{V_0^{+}\ e^{-j\beta z} + V_0^{-}\ e^{+j\beta z}}{V_0^{+}\ e^{-j\beta z} - V_0^{-}\ e^{+j\beta z}} = Z_0\,\frac{1+\Gamma_0\ e^{+j2\beta z}}{1-\Gamma_0\ e^{+j2\beta z}}$$

With a little algebra, we find likewise that the wave functions can be determined from $V(z), I(z)$ and $Z(z)$:

$$V^{+}(z) = \frac{V(z) + I(z)Z_0}{2} = \frac{V(z)}{Z(z)}\left(\frac{Z(z) + Z_0}{2}\right)$$

$$V^{-}(z) = \frac{V(z) - I(z)Z_0}{2} = \frac{V(z)}{Z(z)}\left(\frac{Z(z) - Z_0}{2}\right)$$

From this result we easily find that the reflection coefficient $\Gamma(z)$ can **likewise** be written directly in terms of **line impedance**:

$$\Gamma(z) = \frac{Z(z) - Z_0}{Z(z) + Z_0}$$

Thus, the values $\Gamma(z)$ and $Z(z)$ are **equivalent** parameters—if we know **one**, then we can directly determine the **other**!

Q: *So, if they are equivalent, why wouldn't I **always** use the current, voltage, line impedance representation? After all, I am more **familiar** and more confident those quantities. The **wave** representation sort of **scares** me!*

A: Perhaps I can **convince** you of the value of the **wave** representation.

Remember, the time-harmonic solution to the telegraphers equation simply boils down to **two complex constants**—V_0^+ and V_0^-. Once these complex values have been determined, we can describe **completely** the activity **all** points along our transmission line.

For the **wave representation** we find:

$$V^+(z) = V_0^+ \, e^{-j\beta z}$$

$$V^-(z) = V_0^+ \, e^{+j\beta z}$$

$$\Gamma(z) = \frac{V_0^-}{V_0^+} \, e^{+j2\beta z}$$

Note that the **magnitudes** of the complex functions are in fact **constants** (with respect to position *z*):

$$\left|V^{+}(z)\right| = \left|V_0^{+}\right|$$

$$\left|V^{-}(z)\right| = \left|V_0^{+}\right|$$

$$\left|\Gamma(z)\right| = \left|\frac{V_0^{-}}{V_0^{+}}\right|$$

While the **relative phase** of these complex functions are expressed as a **simple** linear relationship with respect to z:

$$\boldsymbol{arg}\left\{V^{+}(z)\right\} = -\beta z$$

$$\boldsymbol{arg}\left\{V^{-}(z)\right\} = +\beta z$$

$$\boldsymbol{arg}\left\{\Gamma(z)\right\} = +2\beta z$$

Now, **contrast** this with the complex current, voltage, impedance functions:

$$V(z) = V_0^+ \; e^{-j\beta z} + V_0^- \; e^{+j\beta z}$$

$$I(z) = \frac{V_0^+ \; e^{-j\beta z} - V_0^- \; e^{+j\beta z}}{Z_0}$$

$$Z(z) = Z_0 \frac{V_0^+ \; e^{-j\beta z} + V_0^- \; e^{+j\beta z}}{V_0^+ \; e^{-j\beta z} - V_0^- \; e^{+j\beta z}}$$

With magnitude:

$$|V(z)| = \left| V_0^+ \; e^{-j\beta z} + V_0^- \; e^{+j\beta z} \right| = ??$$

$$|I(z)| = \frac{\left| V_0^+ \; e^{-j\beta z} - V_0^- \; e^{+j\beta z} \right|}{Z_0} = ??$$

$$|Z(z)| = Z_0 \frac{\left| V_0^+ \; e^{-j\beta z} + V_0^- \; e^{+j\beta z} \right|}{\left| V_0^+ \; e^{-j\beta z} - V_0^- \; e^{+j\beta z} \right|} = ??$$

and phase:

$$\mathbf{arg}\{V(z)\} = \mathbf{arg}\left\{V_0^+ \; e^{-j\beta z} + V_0^- \; e^{+j\beta z}\right\} = ??$$

$$\mathbf{arg}\{I(z)\} = \mathbf{arg}\left\{V_0^+ \; e^{-j\beta z} - V_0^- \; e^{+j\beta z}\right\} = ??$$

$$\begin{aligned}\mathbf{arg}\{Z(z)\} &= \mathbf{arg}\left\{V_0^+ \; e^{-j\beta z} + V_0^- \; e^{+j\beta z}\right\} \\ &\quad -\mathbf{arg}\left\{V_0^+ \; e^{-j\beta z} - V_0^- \; e^{+j\beta z}\right\} \\ &= ??\end{aligned}$$

Q: *It appears to me that when attempting to describe the activity along a transmission line—as a function of **position** z—it is much **easier** and more **straightforward** to use the **wave** representation.*

*Is my insightful conclusion **correct** (nyuck, nyuck, nyuck)?*

A: Yes it is! However, this does **not** mean that we **never** determine $V(z)$, $I(z)$, or $Z(z)$; these quantities are still **fundamental** and very important—particularly at each **end** of the transmission line!