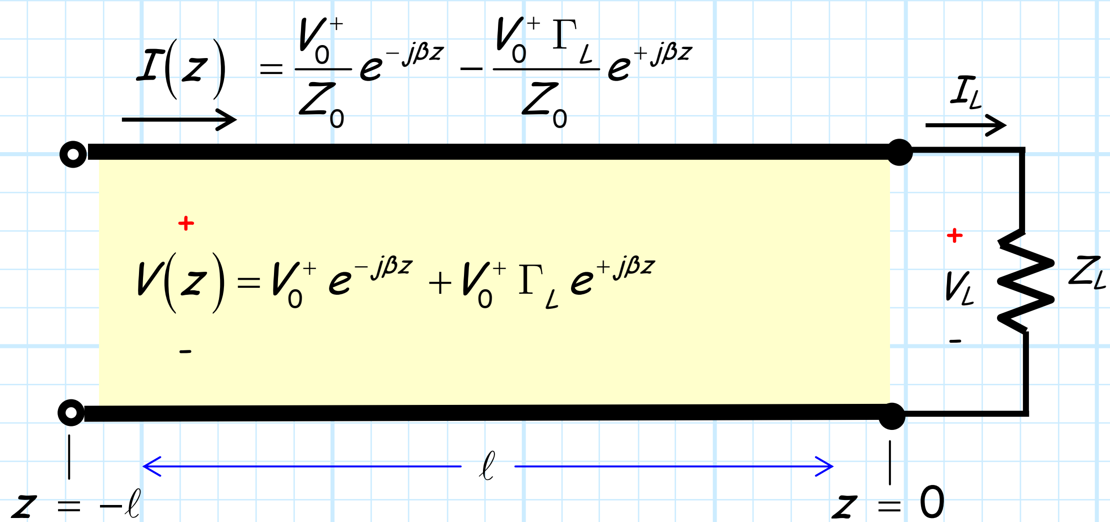
# Incident, Reflected, and Absorbed Power

We have discovered that **two waves propagate** along a transmission line, one in each direction ($V^{+}(z)$ and $V^{-}(z)$).




The result is that electromagnetic **energy** flows along the transmission line at a given **rate** (i.e., **power**).

# The Powers that Be

**Q:** *At what **rate** does **energy** flow along a transmission line, and where does that power **go**?*

**A:** We can answer that question by determining the power **absorbed** by the **load**!

You of course recall that the **time-averaged** power (a **real** value!) absorbed by a **complex** impedance $Z_L$ is:

$$P_{abs} = \frac{1}{2}\text{Re}\left\{V_L\, I_L^*\right\} = \frac{\left|V_L\right|^2}{2}\text{Re}\left\{\frac{1}{Z_L^*}\right\} = \frac{\left|I_L\right|^2}{2}\text{Re}\left\{Z_L\right\}$$

Of course, the **load** voltage and current is simply the voltage an current at the **end** of the transmission line (at $z=0$).

# This happy result

A **happy** result is that we can then use our **transmission line theory** to determine this absorbed power:

$$
\begin{aligned}
P_{abs} &= \frac{1}{2}\mathrm{Re}\left\{V_L\, I_L^{\,*}\right\} \\
&= \frac{1}{2}\mathrm{Re}\left\{V(z=0)\, I(z=0)^{*}\right\} \\
&= \frac{1}{2\,Z_0}\mathrm{Re}\left\{\left(V_0^{+}\left[e^{-j\beta 0} + \Gamma_L e^{+j\beta 0}\right]\right)\left(V_0^{+}\left[e^{-j\beta 0} - \Gamma_L e^{+j\beta 0}\right]\right)^{*}\right\} \\
&= \frac{\left|V_0^{+}\right|^2}{2\,Z_0}\mathrm{Re}\left\{1 - \left(\Gamma_L^{*} - \Gamma_L\right) - \left|\Gamma_L\right|^2\right\} \\
&= \frac{\left|V_0^{+}\right|^2}{2\,Z_0}\left(1 - \left|\Gamma_L\right|^2\right)
\end{aligned}
$$

# Incident Power

The **significance** of this result can be seen by **rewriting** the expression as:

$$P_{abs} = \frac{\left|V_0^+\right|^2}{2\,Z_0}\left(1-\left|\Gamma_L\right|^2\right) = \frac{\left|V_0^+\right|^2}{2\,Z_0} - \frac{\left|V_0^+\Gamma_L\right|^2}{2\,Z_0} = \frac{\left|V_0^+\right|^2}{2\,Z_0} - \frac{\left|V_0^-\right|^2}{2\,Z_0}$$

The **two terms** in above expression have a very definite **physical meaning**.

The **first** term is the time-averaged **power of the wave** propagating along the transmission line **toward the load**.

We say that this wave is **incident** on the load:

$$P_{inc} = P^+ = \frac{\left|V_0^+\right|^2}{2Z_0}$$

# Reflected Power

Likewise, the second term of the $P_{abs}$ equation describes the **power of the wave** moving in the other direction (**away from the load**).

We refer to this as the wave **reflected** from the load:

$$P_{ref} = P^{-} = \frac{|V_0^-|^2}{2Z_0} = \frac{|\Gamma_L|^2 |V_0^+|^2}{2Z_0} = |\Gamma_L|^2 P_{inc}$$

# Energy is Conserved

Thus, the power **absorbed** by the load (i.e., the power **delivered to** the load) is simply:

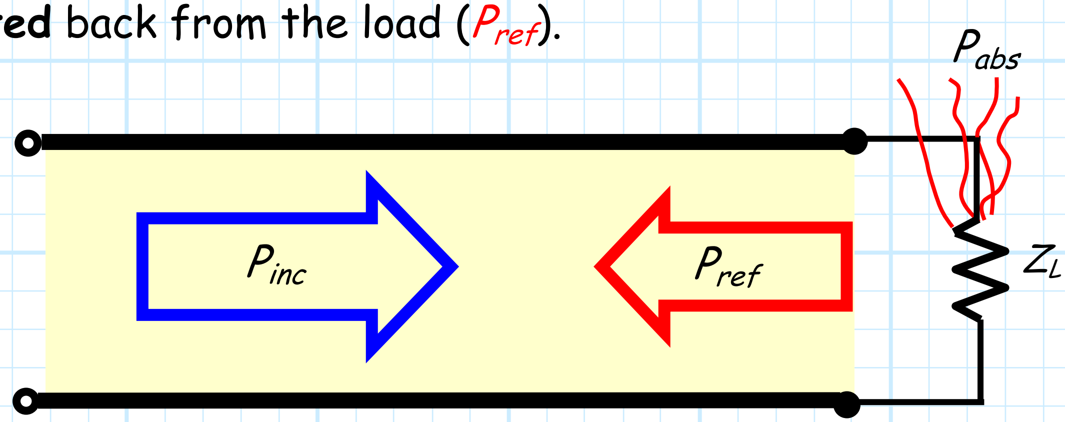
$$P_{abs} = P_{inc} - P_{ref}$$

or, rearranging, we find:

$$P_{inc} = P_{abs} + P_{ref}$$

This equation is simply an expression of the **conservation of energy** !

It says that power flowing **toward** the load ($P_{inc}$) is either **absorbed** by the load ($P_{abs}$) or **reflected** back from the load ($P_{ref}$).




Now let's consider some **special cases**, and the **power** that results.

# Special Case #1: $|\Gamma|^2=1$

For this case, we find that the load absorbs **no power**!

$$P_{abs} = P_{inc}\left(1-|\Gamma_L|^2\right) = P_{inc}(1-1) = 0$$

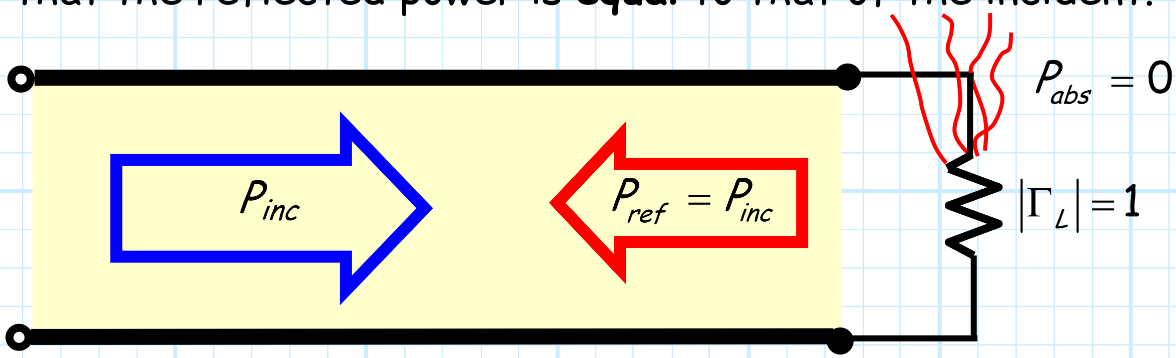
Likewise, we find that the reflected power is **equal** to the incident:

$$P_{ref} = |\Gamma_L|^2 P_{inc} = (1) P_{inc} = P_{inc}$$

Note these two results are completely **consistent**—by conservation of energy, if one is true the other **must** also be:

$$P_{inc} = P_{abs} + P_{ref} = 0 + P_{ref} = P_{ref}$$

In this case, **no power** is absorbed by the load. **All** of the incident power is **reflected**, so that the reflected power is **equal** to that of the incident.



# Special Case #2: $|\Gamma|^2=0$

For this case, we find that there is **no reflected power**!

$$P_{ref} = |\Gamma_L|^2 P_{inc} = (0) P_{inc} = 0$$

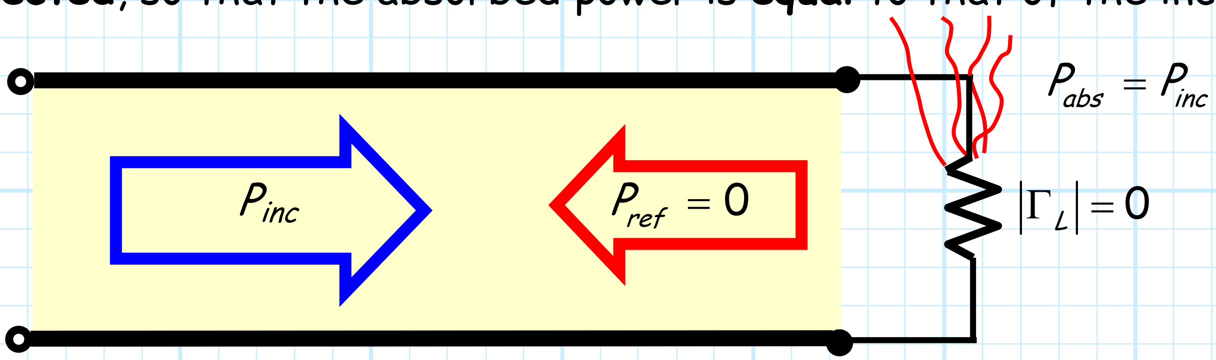
Likewise, we find that the absorbed power is **equal** to the incident:

$$P_{abs} = P_{inc}\left(1-|\Gamma_L|^2\right) = P_{inc}\left(1-0\right) = P_{inc}$$

Note these two results are completely **consistent**—by conservation of energy, if one is true the other **must** also be:

$$P_{inc} = P_{abs} + P_{ref} = P_{abs} + 0 = P_{abs}$$

In this case, **all** the incident power is absorbed by the load. **None** of the incident power is **reflected**, so that the absorbed power is **equal** to that of the incident.



# Case #3: $0<|\Gamma|^2<1$

For this case, we find that the **reflected** power is greater than zero, but **less** than the incident power.

$$0 < P_{ref} = |\Gamma_L|^2 P_{inc} < P_{inc}$$

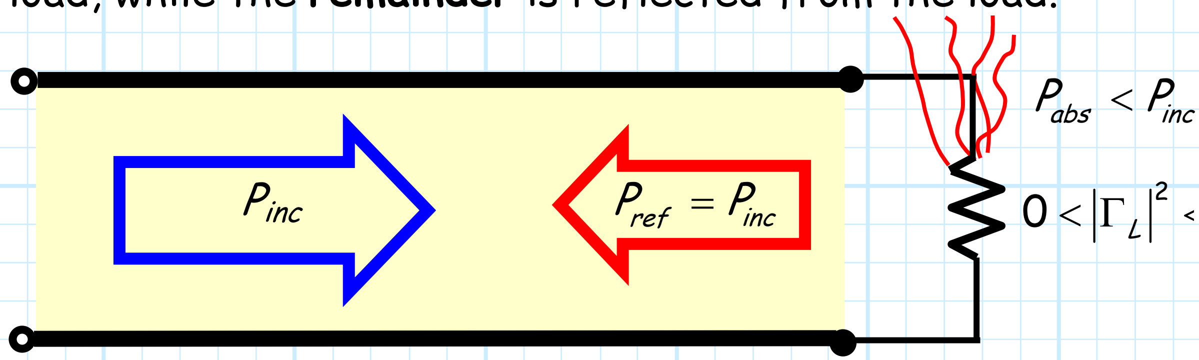
Likewise, we find that the **absorbed** power is **also** greater than zero, but **less** than the incident power.

$$0 < P_{abs} = P_{inc}\left(1 - |\Gamma_L|^2\right) < P_{inc}$$

Note these two results are completely **consistent**—by conservation of energy, if one is true the other **must** also be:

$$0 < P_{ref} = P_{inc} - P_{abs} < P_{inc} \qquad \text{and} \qquad 0 < P_{abs} = P_{inc} - P_{ref} < P_{inc}$$

In this case, the incident power is **divided**. **Some** of the incident power is absorbed by the load, while the **remainder** is reflected from the load.



# Case #4: $|\Gamma|^2>1$

For this case, we find that the **reflected** power is **greater** than the **incident** power.

$$0 < P_{ref} = |\Gamma_L|^2 P_{inc} < P_{inc}$$

**Q:** *Yikes! What's up with that?*

*This result does **not** seem at all consistent with your conservation of energy argument.*

*How can the reflected power be **larger** than the incident?*

**A:** Quite insightful!

It is indeed a result quite **askew** with our conservation of energy analysis.

To see why, let's determine the **absorbed** power for this case.

$$P_{abs} = P_{inc}\left(1-|\Gamma_L|^2\right) < 0$$

The power absorbed by the load is **negative**!

# Case #4 – the load cannot be passive

This result actually has a **physical interpretation**.

A negative absorbed power indicates that the load is not absorbing power at all—it is instead **producing** power!

This makes sense if **you think** about it.

The power flowing **away** from the load (the reflected power) can be larger than the power flowing **toward** the load (the incident power) **only** if the load itself is **creating** this extra power.

The load in this case would not be a power **sink**, it would be a power **source**.

**Q:** *But how could a **passive** load be a power source?*

**A:** It **can't**.

A **passive** device cannot produce power.

# Passive loads

Thus, we have come to an **important conclusion**!

The reflection coefficient of any and all passive loads **must** have a **magnitude** that is **less than one**.

$$|\Gamma_L| \leq 1 \quad \text{for all passive loads}$$

**Q:** *Can $|\Gamma_L|$ every be **greater** than one?*

**A:** Sure, if the "load" is an **active** device.

In other words, the load must have some **external power** source connected to it.

**Q:** *What about the case where $|\Gamma_L| < 0$, shouldn't we examine **that** situation as well?*

**A:** That would be just plain **silly**; do **you** see why?