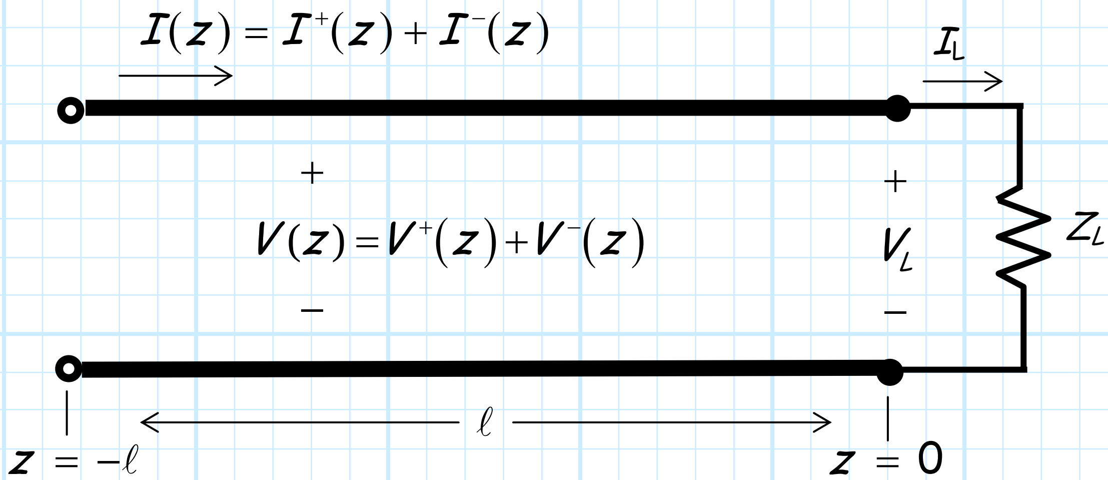
Incident, Reflected, and Absorbed Power

We have discovered that **two waves propagate** along a transmission line, one in each direction ($V^+(z)$ and $V^-(z)$).



The result is that electromagnetic energy flows along the transmission line at a given rate (i.e., **power**).

Q: *How much power flows along a transmission line, and where does that power go?*

A: We can answer that question by determining the power **absorbed** by the **load**!

You of course recall that the **time-averaged** power (a **real** value!) absorbed by a **complex** impedance Z_L is:

$$P_{abs} = \frac{1}{2}\text{Re}\left\{V_L\, I_L^{\,*}\right\}$$

Of course, the **load** voltage and current is simply the voltage an current at the **end** of the transmission line (at $z = 0$). A **happy** result is that we can then use our **transmission line theory** to determine this absorbed power:

$$\begin{aligned} P_{abs} &= \frac{1}{2}\text{Re}\left\{V_L\, I_L^{\,*}\right\} \\ &= \frac{1}{2}\text{Re}\left\{V(z=0)\, I(z=0)^*\right\} \\ &= \frac{1}{2\,Z_0}\text{Re}\left\{\left(V_0^+\left[e^{-j\beta 0} + \Gamma_0\, e^{+j\beta 0}\right]\right)\left(V_0^+\left[e^{-j\beta 0} - \Gamma_0\, e^{+j\beta 0}\right]\right)^*\right\} \\ &= \frac{\left|V_0^+\right|^2}{2\,Z_0}\text{Re}\left\{1 - \left(\Gamma_0^{\,*} - \Gamma_0\right) - \left|\Gamma_0\right|^2\right\} \\ &= \frac{\left|V_0^+\right|^2}{2\,Z_0}\left(1 - \left|\Gamma_0\right|^2\right) \end{aligned}$$

The significance of this result can be seen by **rewriting** the expression as:

$$P_{abs} = \frac{\left|V_0^+\right|^2}{2\,Z_0}\left(1 - \left|\Gamma_0\right|^2\right) = \frac{\left|V_0^+\right|^2}{2\,Z_0} - \frac{\left|V_0^+\Gamma_0\right|^2}{2\,Z_0} = \frac{\left|V_0^+\right|^2}{2\,Z_0} - \frac{\left|V_0^-\right|^2}{2\,Z_0}$$

The two terms in above expression have a very definite **physical meaning**. The **first** term is the time-averaged **power of the wave** propagating along the transmission line **toward the load**.

We say that this wave is **incident** on the load:

$$P_{inc} = P_+ = \frac{|V_0^+|^2}{2Z_0}$$

Likewise, the second term of the P_{abs} equation describes the **power of the wave** moving in the other direction (**away from the load**). We refer to this as the wave **reflected** from the load:

$$P_{ref} = P_- = \frac{|V_0^-|^2}{2Z_0} = \frac{|\Gamma_L V_0^+|^2}{2Z_0} = |\Gamma_L|^2 \frac{|V_0^+|^2}{2Z_0} = |\Gamma_L|^2 P_{inc}$$

Thus, the power **absorbed** by the load (i.e., the power **delivered to** the load) is simply:

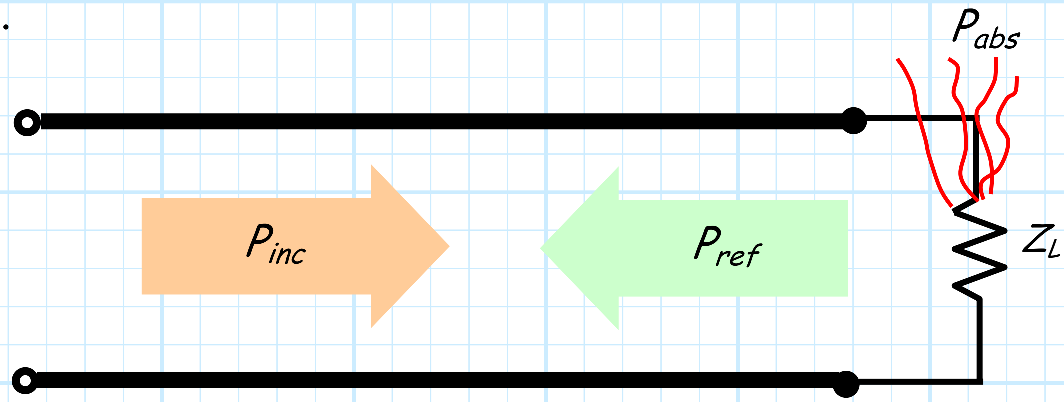
$$P_{abs} = \frac{|V_0^+|^2}{2Z_0} - \frac{|V_0^-|^2}{2Z_0} = P_{inc} - P_{ref}$$

or, rearranging, we find:

$$P_{inc} = P_{abs} + P_{ref}$$

This equation is simply an expression of the **conservation of energy** !

It says that power flowing **toward** the load (P_{inc}) is either **absorbed** by the load (P_{abs}) or **reflected** back from the load (P_{ref}).



Now let's consider some special cases:

1. $|\Gamma_L|^2 = 1$

For this case, we find that the load absorbs **no power**!

$$P_{abs} = P_{inc}\left(1-|\Gamma_0|^2\right) = P_{inc}(1-1) = 0$$

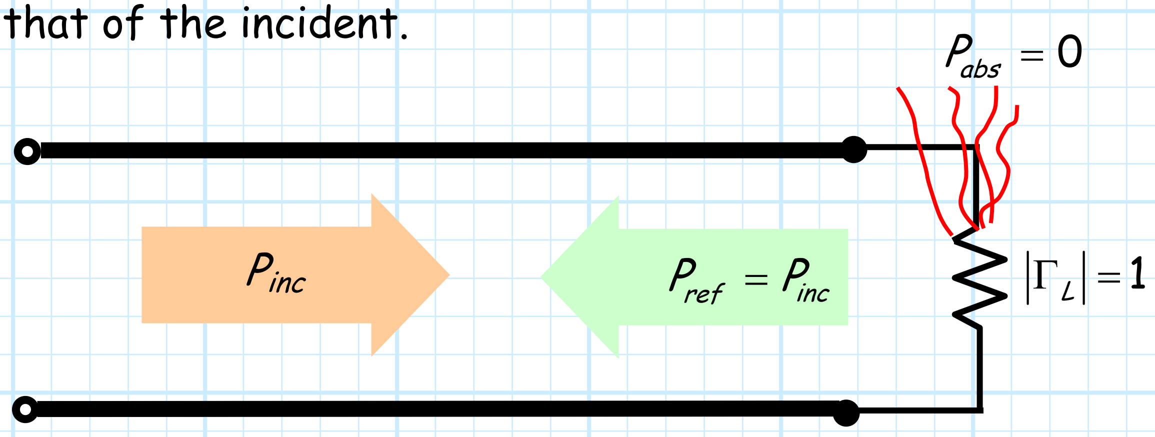
Likewise, we find that the reflected power is **equal** to the incident:

$$P_{ref} = |\Gamma_L|^2 P_{inc} = (1)P_{inc} = P_{inc}$$

Note these two results are completely **consistent**—by conservation of energy, if one is true the other **must** also be:

$$P_{inc} = P_{abs} + P_{ref} = 0 + P_{ref} = P_{ref}$$

In this case, **no power** is absorbed by the load. **All** of the incident power is **reflected**, so that the reflected power is **equal** to that of the incident.



2. $|\Gamma_L| = 0$

For this case, we find that there is **no reflected power**!

$$P_{ref} = |\Gamma_L|^2 P_{inc} = (0) P_{inc} = 0$$

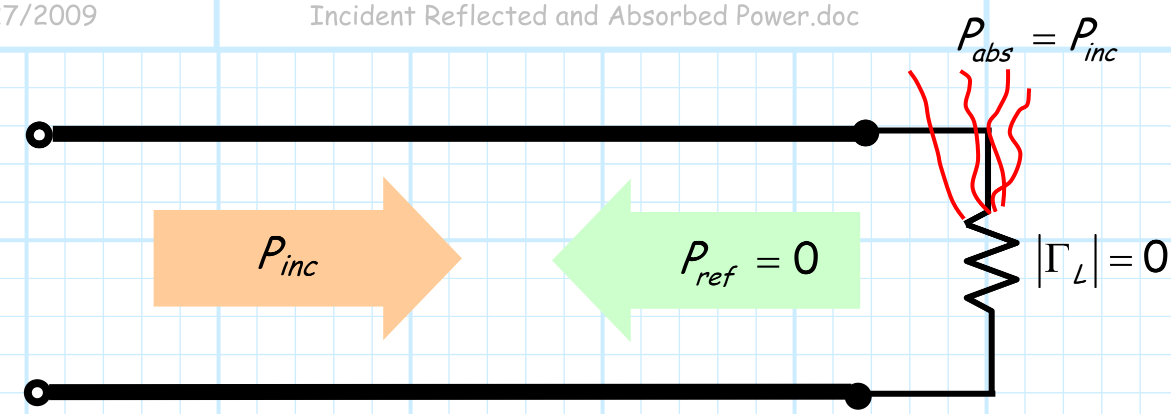
Likewise, we find that the absorbed power is **equal** to the incident:

$$P_{abs} = P_{inc}\left(1 - |\Gamma_0|^2\right) = P_{inc}(1-0) = P_{inc}$$

Note these two results are completely **consistent**—by conservation of energy, if one is true the other **must** also be:

$$P_{inc} = P_{abs} + P_{ref} = P_{abs} + 0 = P_{abs}$$

In this case, **all** the incident power is absorbed by the load. **None** of the incident power is **reflected**, so that the absorbed power is **equal** to that of the incident.



3. $0 < |\Gamma_L| < 1$

For this case, we find that the **reflected** power is greater than zero, but **less** than the incident power.

$$0 < P_{ref} = |\Gamma_L|^2 P_{inc} < P_{inc}$$

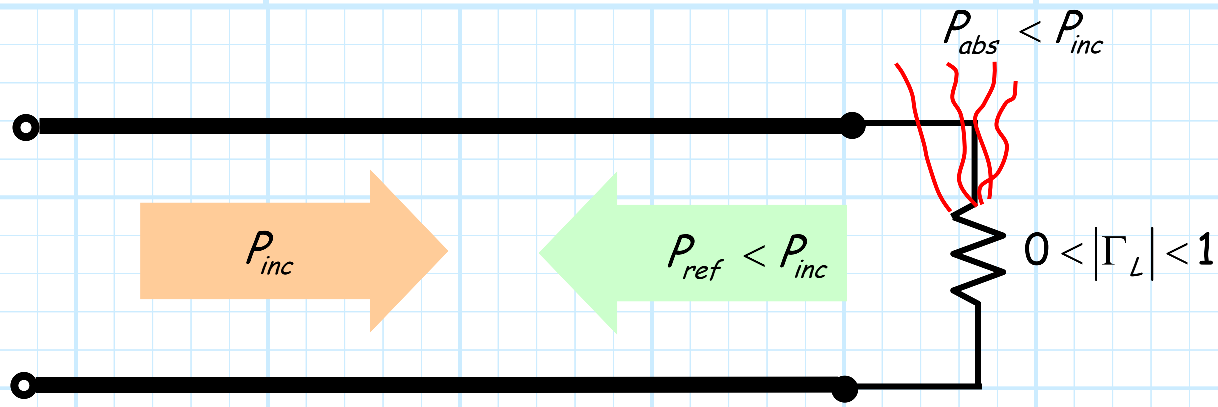
Likewise, we find that the **absorbed** power is **also** greater than zero, but **less** than the incident power.

$$0 < P_{abs} = P_{inc}\left(1 - |\Gamma_0|^2\right) < P_{inc}$$

Note these two results are completely **consistent**—by conservation of energy, if one is true the other **must** also be:

$$0 < P_{ref} = P_{inc} - P_{abs} < P_{inc} \quad \text{and} \quad 0 < P_{abs} = P_{inc} - P_{ref} < P_{inc}$$

In this case, the incident power is divided. Some of the incident power is absorbed by the load, while the remainder is reflected from the load.



4. $|\Gamma_L| > 1$

For this case, we find that the **reflected** power is **greater** than the **incident** power.

$$0 < P_{ref} = |\Gamma_L|^2 P_{inc} < P_{inc}$$

Q: *Yikes! What's up with that? This result does **not** seem at all consistent with your conservation of energy argument. How can the reflected power be **larger** than the incident?*

A: Quite insightful! It is indeed a result quite **askew** with our conservation of energy analysis. To see why, let's determine the **absorbed** power for this case.

$$P_{abs} = P_{inc}\left(1 - |\Gamma_L|^2\right) < 0$$

The power absorbed by the load is **negative**!

This result actually has a **physical interpretation**. A negative absorbed power indicates that the load is not absorbing power at all—it is instead **producing** power!

This makes sense if **you think** about it. The power flowing away from the load (the reflected power) can be larger than the power flowing toward the load (the incident power) **only** if the load itself is creating this extra power. The load is not a power **sink**, it is a power **source**.

Q: *But how could a **passive** load be a power source?*

A: It **can't**. A passive device cannot produce power. Thus, we have come to an important conclusion. The reflection coefficient of any and all passive loads **must** have a **magnitude** that is **less than one**.

$$|\Gamma_L| \leq 1 \quad \text{for all passive loads}$$

Q: *Can $|\Gamma_L|$ every be **greater** than one?*

A: Sure, if the "load" is an **active** device. In other words, the load must have some **external power** source connected to it.

Q: *What about the case where $|\Gamma_L| < 0$, shouldn't we examine **that** situation as well?*

A: That would be just plain **silly**; do **you** see why?