

Kuroda's Identities

We find that **Kuroda's Identities** can be very useful in making the implementation of Richard's transformations more **practicable**.

Kuroda's Identities essentially provide a list of **equivalent** two port networks. By equivalent, we mean that they have **precisely** the same scattering/impedance/admittance/transmission matrices.

In other words, we can **replace** one two-port network with its equivalent in a circuit, and the behavior and characteristics (e.g., its scattering matrix) of the circuit will **not** change!

Q: *Why would we want to do this?*

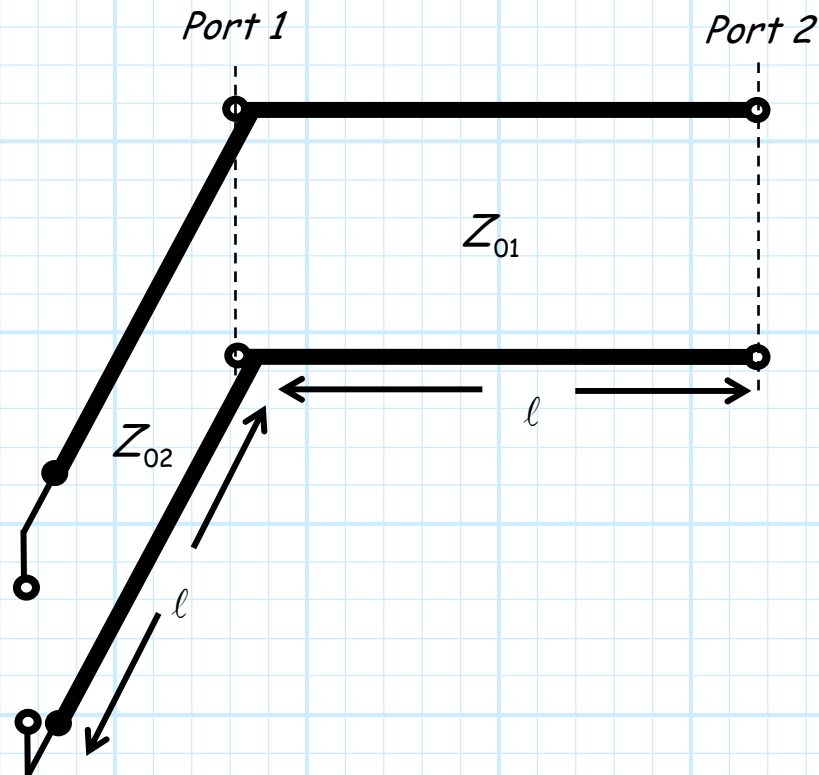
A: Because one of the equivalent may be more **practical** to implement!

For example, we can use Kuroda's Identities to:

- 1) Physically **separate** transmission line stubs.
- 2) Transform series stubs into **shunt** stubs.
- 3) Change impractical **characteristic impedances** into more realizable ones.

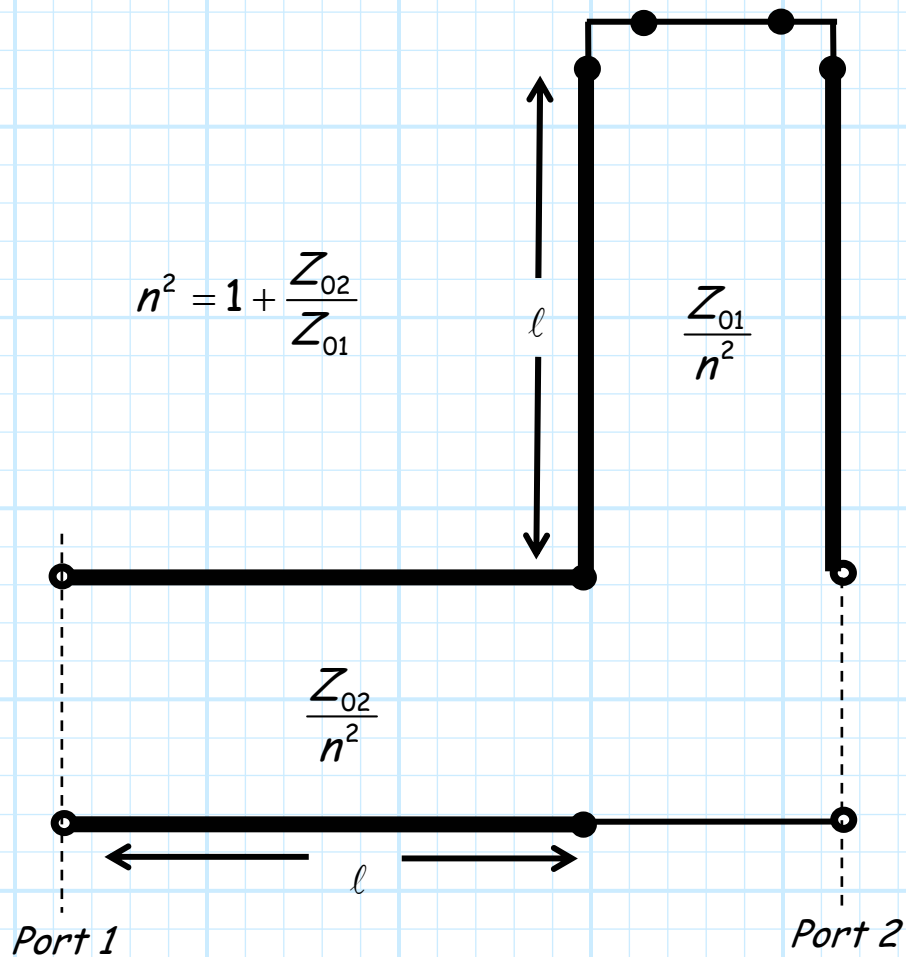
Four Kuroda's identities are provided in a very **ambiguous** and confusing table (Table 8.7) in your **book**. We will find the **first two** identities to be the most useful.

Consider the following two-port network, constructed with a length of transmission line, and an **open-circuit shunt stub**:



Note that the **length** of the stub and the transmission line are **identical**, but the characteristic **impedance** of each are **different**.

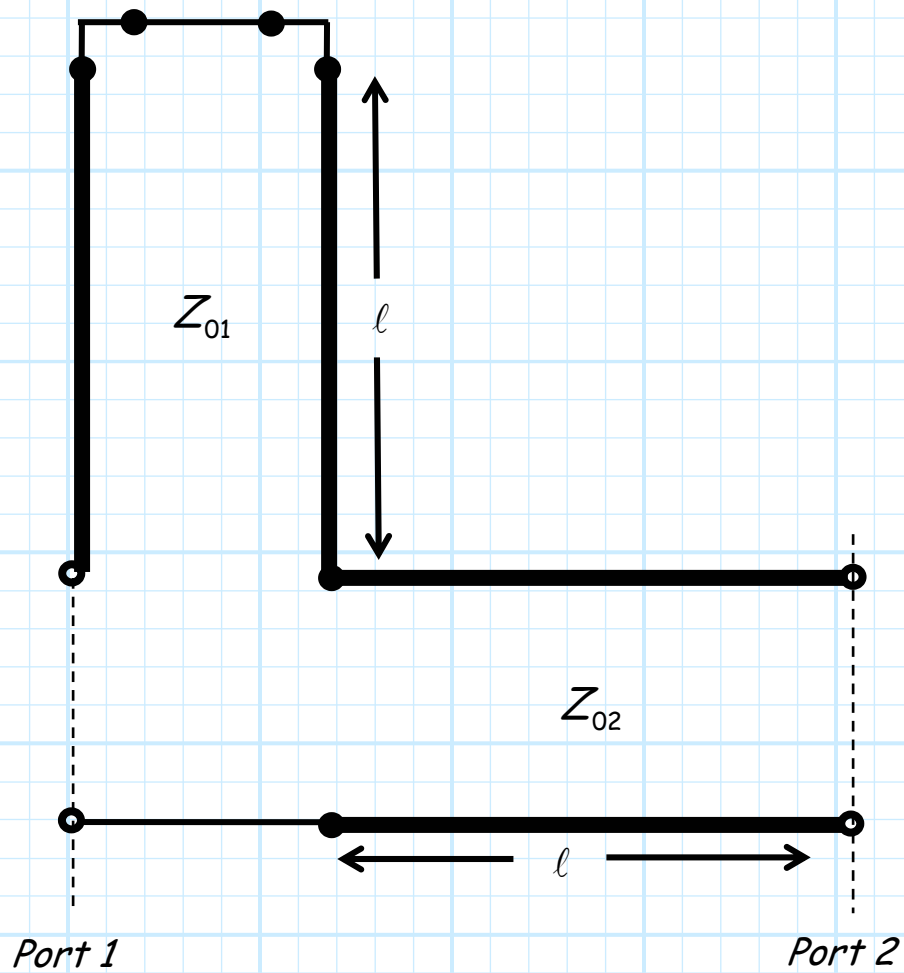
The **first Kuroda identity** states that the two-port network above is **precisely** the same two-port network as **this** one:



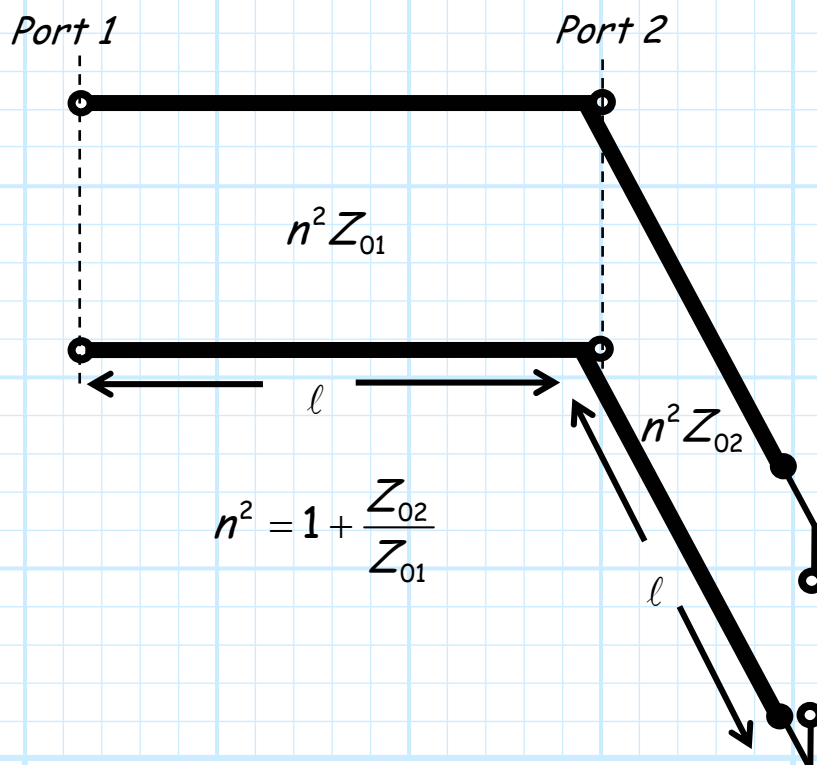
Thus, we can **replace** the first structure in some circuit with the one above, and the behavior that circuit will **not change** in the least!

Note this equivalent circuit uses a **short-circuited series stub**.

The **second** of Kuroda's Identities states that this two port network:

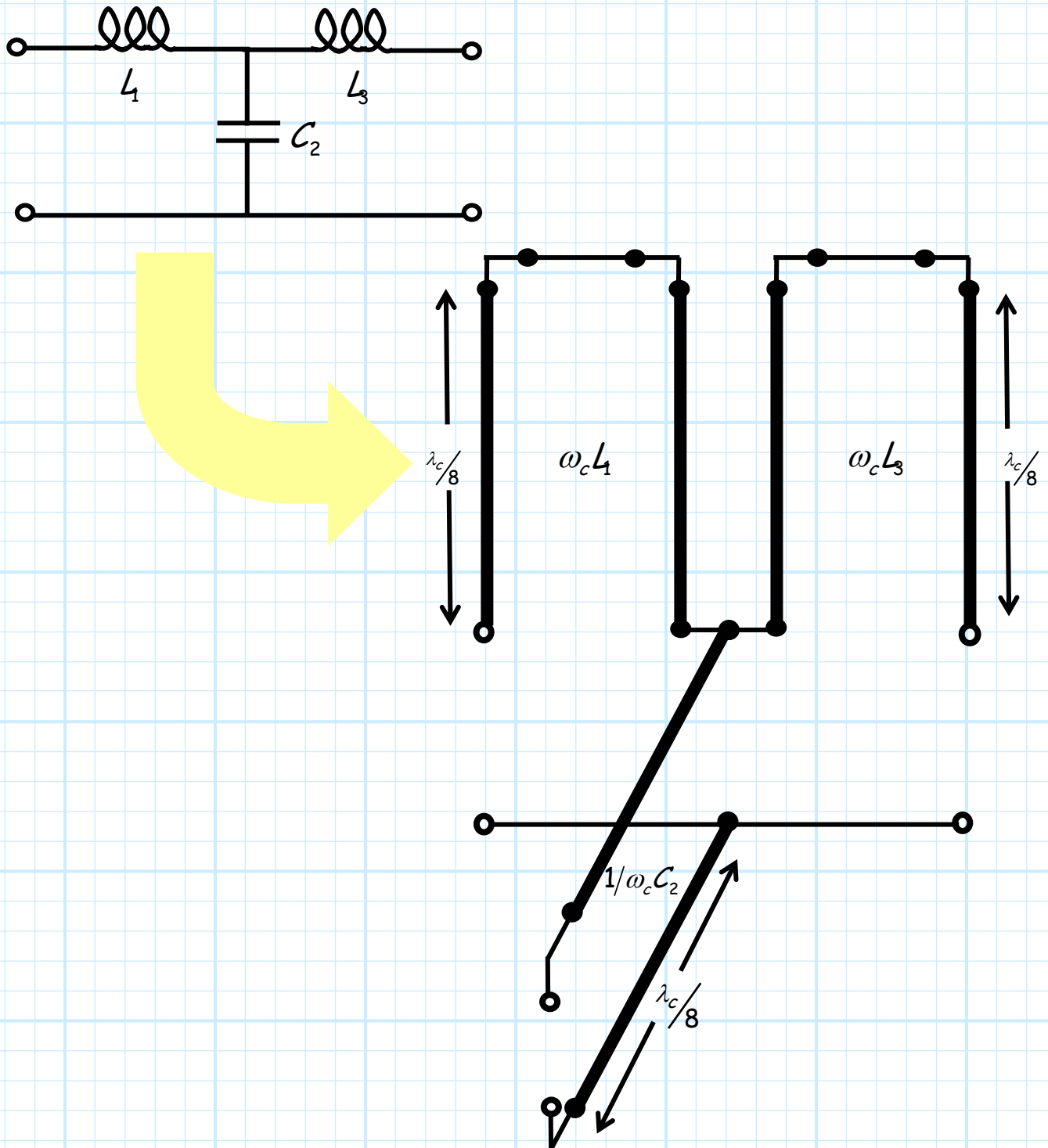


Is precisely identical to this two-port network:



With regard to **Richard's Transformation**, these identities are useful when we replace the series inductors with **shorted stubs**.

To see **why** this is useful when implementing a **lowpass filter** with distributed elements, consider this third order filter example, realized using Richard's Transformations:

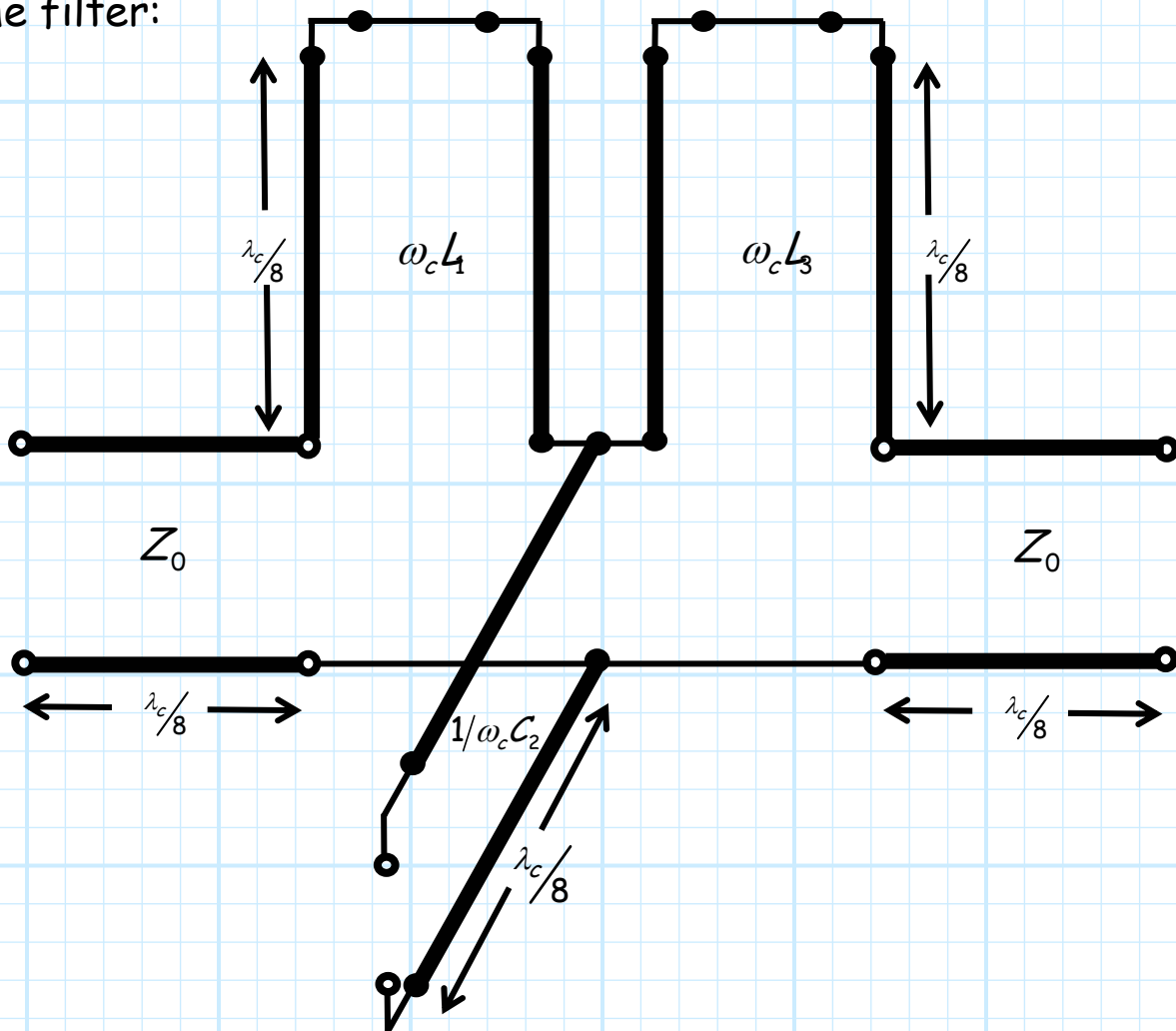


Note that we have a few **problems** in terms of implementing this design!

First of all the stubs are ideally **infinitely close** to each other—how do we build that? We could physically **separate** them, but this would introduce some **transmission line length** between them that would **mess up** our filter response!

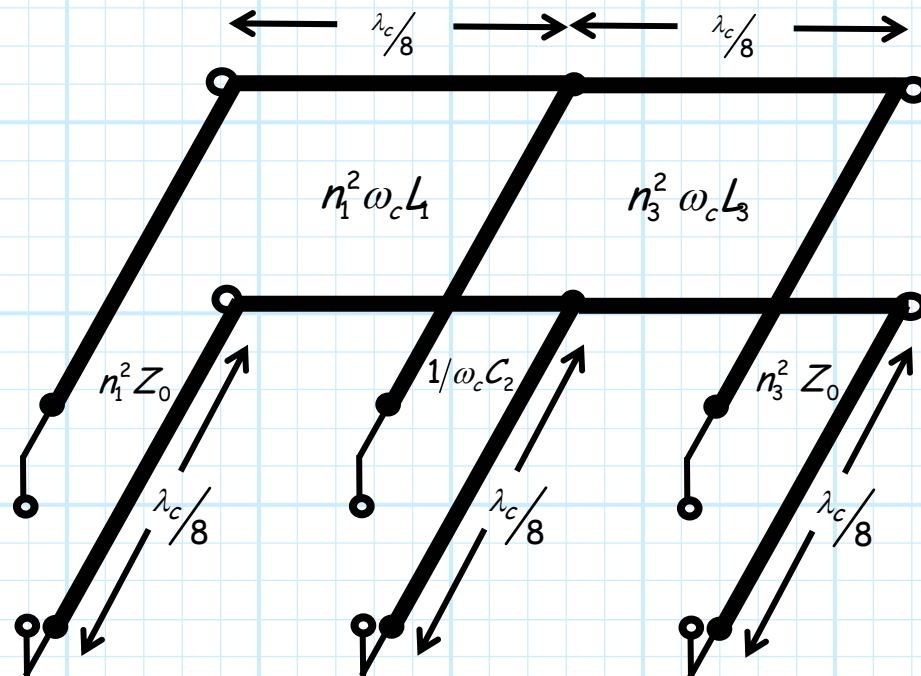
Secondly, **series** stubs are difficult to construct in microstrip/stripline—we like **shunt** stubs **much better**!

To solve these problems, we first **add** a short length of transmission line (Z_0 and $\ell = \lambda_c/8$) to the **beginning** and **end** of the filter:



Note adding these lengths only results in a **phase shift** in the filter response—the transmission and reflection functions will remain **unchanged**.

Now we can use the second of **Kuroda's Identities** to replace the **series stubs** with **shunts**:



where:

$$n_1^2 = 1 + \frac{Z_0}{\omega_c L_1}$$

$$n_3^2 = 1 + \frac{Z_0}{\omega_c L_3}$$

Now **this** is a realizable filter! Note the **three stubs** are separated, and they are all **shunt stubs**.

Note that a specific **numerical** example (example 8.5) of this procedure is given on pp. 409-411 of **your book**.