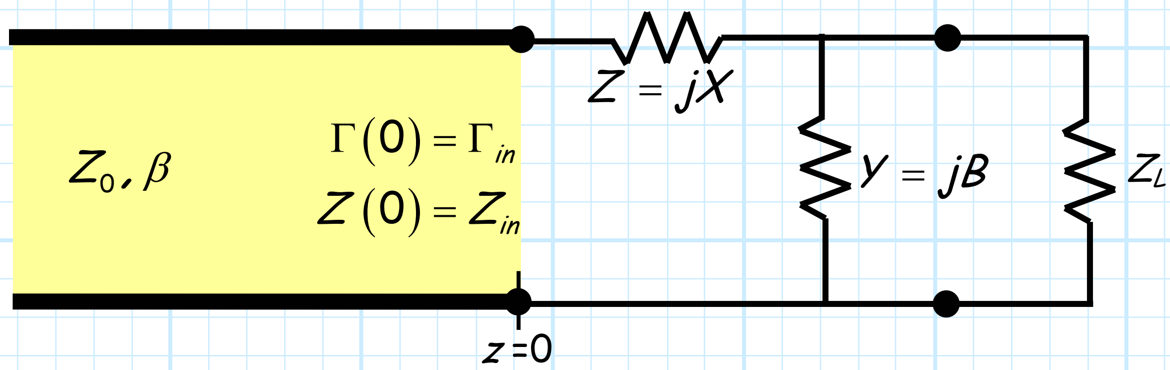
L-Network Analysis

Consider the **first** matching L-network, which we shall denote as matching **network (A)**:



Note that this matching network consists of just **two** lumped elements, which must be **purely reactive**—in other words, a **capacitor** and an **inductor**!

To make $\Gamma_{in} = 0$, the **input impedance** of the network must be:

$$Z_{in} = Z_0$$

Note that using **basic** circuit analysis we find that this input impedance is:

$$Z_{in} = jX + \frac{\left(\frac{1}{jB}\right) Z_L}{\frac{1}{jB} + Z_L} = jX + \frac{Z_L}{1 + jBZ_L}$$

Note that a **matched** network, with $Z_{in} = Z_0$, means that:

$$\text{Re}\{Z_{in}\} = Z_0 \qquad \textbf{AND} \qquad \text{Im}\{Z_{in}\} = 0$$

Note that there are **two** equations.

This works out well, since we have **two** unknowns (B and X)!

Essentially, the L-network matching network can be viewed as consisting of **two distinct parts**, each attempting to satisfy a specific requirement.

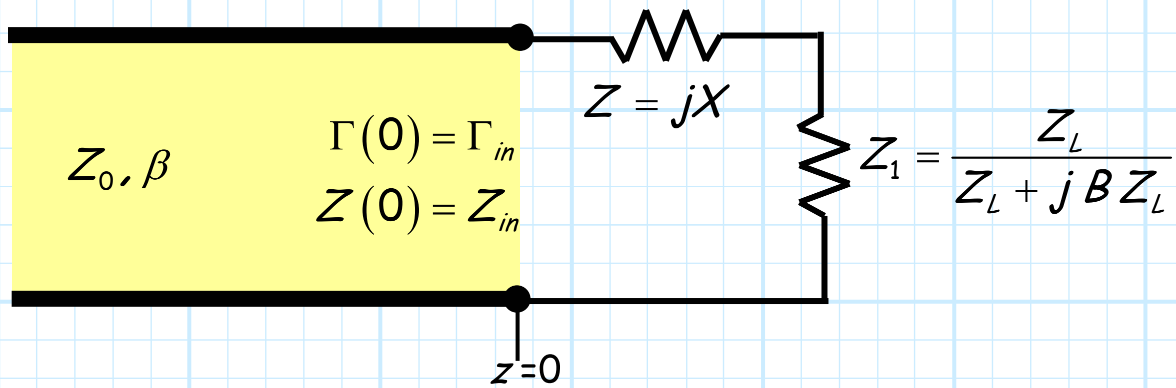
Part 1: Selecting $Y = jB$

Since the shunt element Y and Z_L are in **parallel**, we can combine them into one element that we shall call Y_1:

$$Y_1 \doteq Y + \frac{1}{Z_L} = jB + Y_L$$

The impedance of this element is therefore:

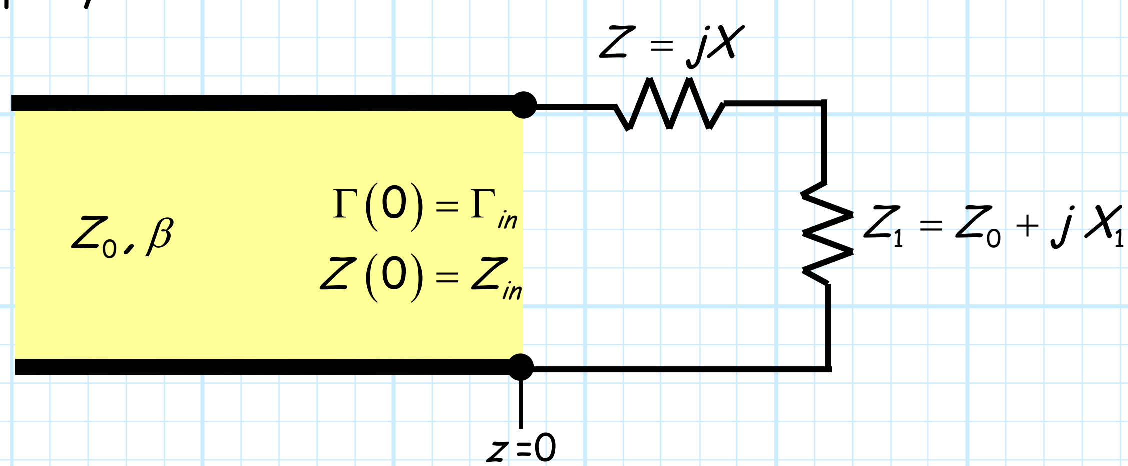
$$Z_1 = \frac{1}{Y_1} = \frac{1}{jB + Y_L} = \frac{Z_L}{Z_L + j\,B\,Z_L}$$



To achieve a perfect match, we must set the value of susceptance B such that:

$$Re\{Z_1\} = Re\left\{\frac{Z_L}{Z_L + j B Z_L}\right\} = Z_0$$

Thus, if B is properly selected:



Hopefully, the second part of the matching is now very obvious to **you**!

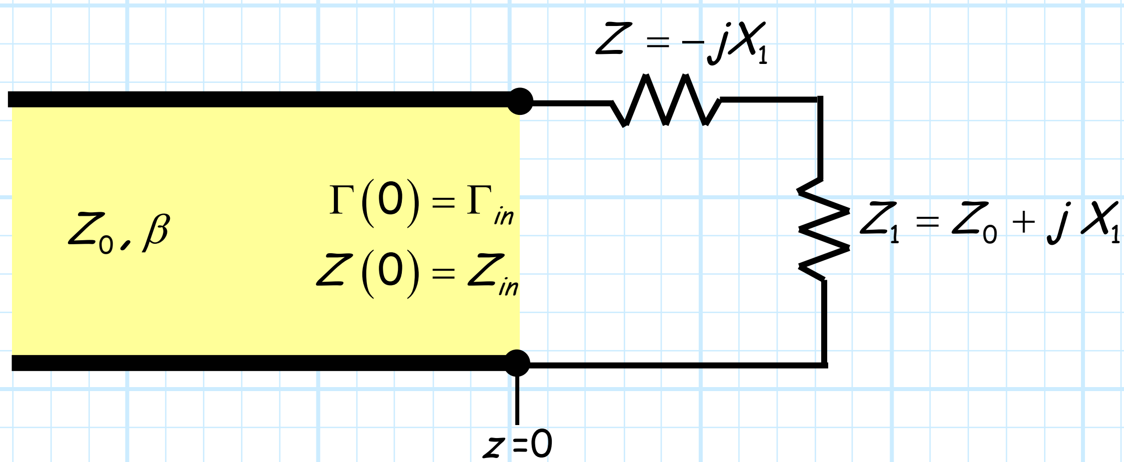
Part 2: Selecting $Z = jX$

Note that the impedance $Z_1 = Z_L \parallel 1/jB$ has the ideal real value of Z_0. However, it likewise posses an **annoying** imaginary part of:

$$X_1 = Im\{Z_1\} = Im\left\{\frac{Z_L}{Z_L + j\,B\,Z_L}\right\}$$

However, this imaginary component is easily removed by setting the **series** element $Z = j\,X$ to its equal but **opposite** value! I.E.,:

$$X = -X_1 = -Im\left\{\frac{Z_L}{Z_L + j\,B\,Z_L}\right\}$$



Thus, we find that:

$$
\begin{aligned}
Z_{in} &= Z + Z_1 \\
&= -jX_1 + Z_0 + jX_1 \\
&= Z_0
\end{aligned}
$$

We have created a **perfect match**!

Going through this complex algebra, we can solve for the **required** values X and B to **satisfy** these two equations—to create a **matched** network!

$$B = \frac{X_L \pm \sqrt{R_L/Z_0}\,\sqrt{R_L^2 + X_L^2 - Z_0 R_L}}{R_L^2 + X_L^2}$$

and,

$$X = \frac{1}{B} + \frac{X_L\, Z_0}{R_L} - \frac{Z_0}{B\, R_L}$$

where $Z_L = R_L + jX_L$.

Note:

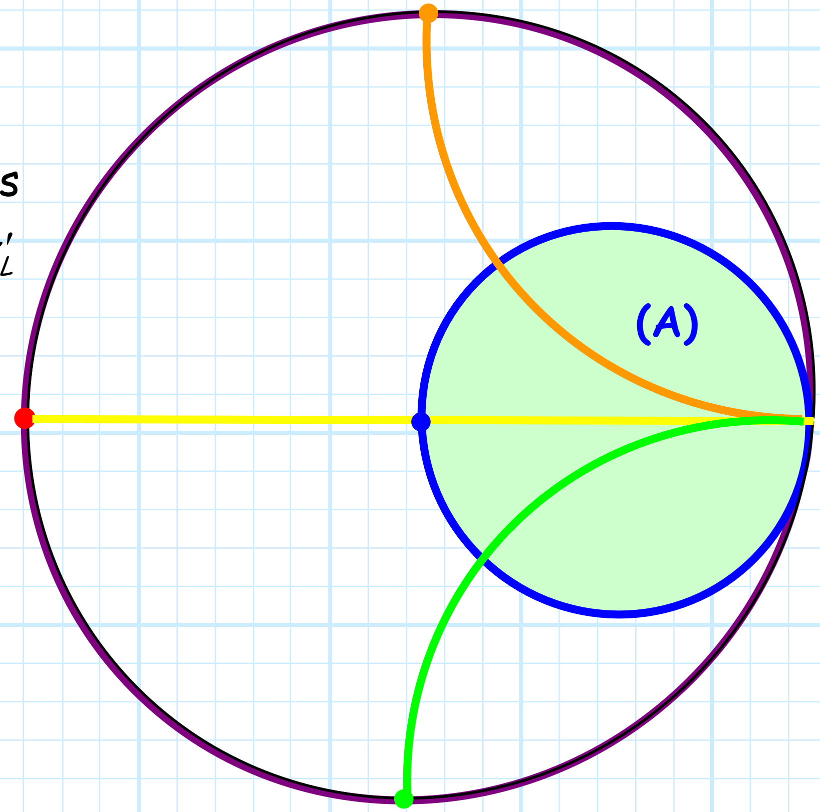
1) Because of the $\pm$, there are **two** solutions for B (and thus X).

2) For jB to be purely imaginary (i.e., reactive), B must be **real**. From the term:

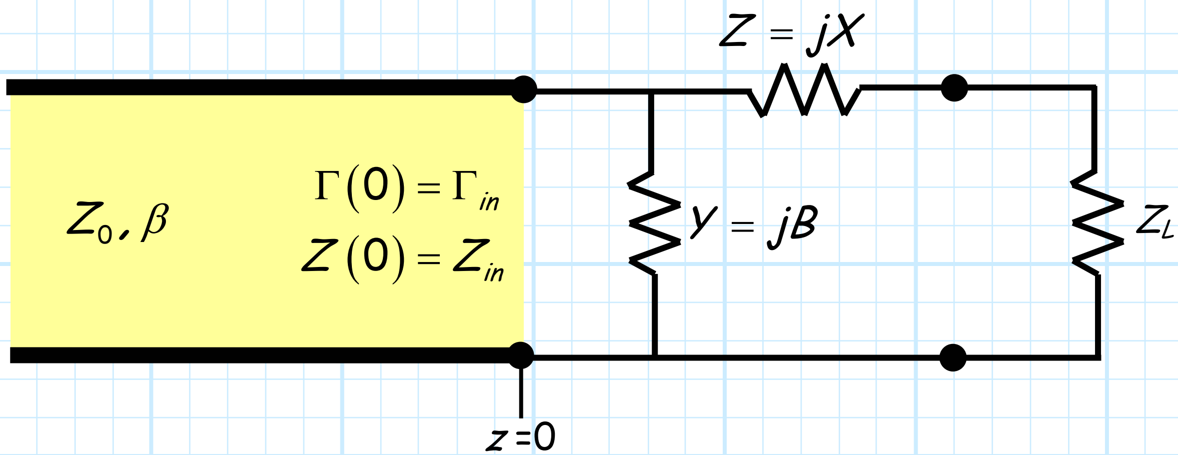
$$\sqrt{R_L^2 + X_L^2 - Z_0 R_L}$$

in the expression for B, we note that R_L **must** be greater than Z_0 ($R_L > Z_0$) to insure that B and thus X is real.

In other words, this matching network can only be used when $R_L > Z_0$. Notice that this condition means that the normalized load z_L' lies **inside** the $r=1$ circle on the Smith Chart!



Now let's consider the **second** of the two L-networks, which we shall call **network (B)**. Note it **also** is formed with just two lumped elements.



To make $\Gamma_{in} = 0$, the **input admittance** of the network must be:

$$Y_{in} = Y_0$$

Note from circuit theory that the input **admittance** for this network is:

$$Y_{in} = jB + \frac{1}{jX + Z_L}$$

Therefore a **matched** network, with $Y_{in} = Y_0$, is described as:

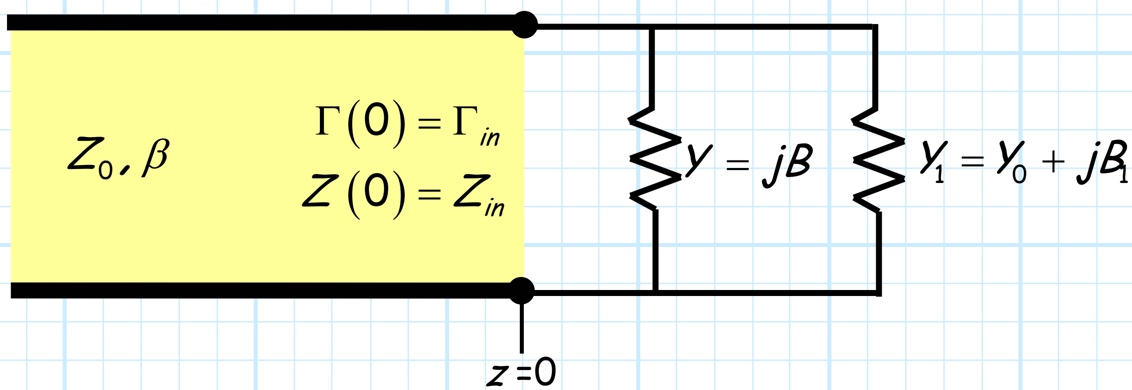
$$\text{Re}\{Y_{in}\} = Y_0 \qquad \textbf{AND} \qquad \text{Im}\{Y_{in}\} = 0$$

For this design, we set the value of $Z = jX$ such that the admittance Y_1:

$$Y_1 \doteq \frac{1}{Z + Z_L} = \frac{1}{jX + Z_L}$$

has a real part equal to Y_0:

$$Y_0 = Re\{Y\}_1 = Re\left\{\frac{1}{jX + Z_L}\right\}$$



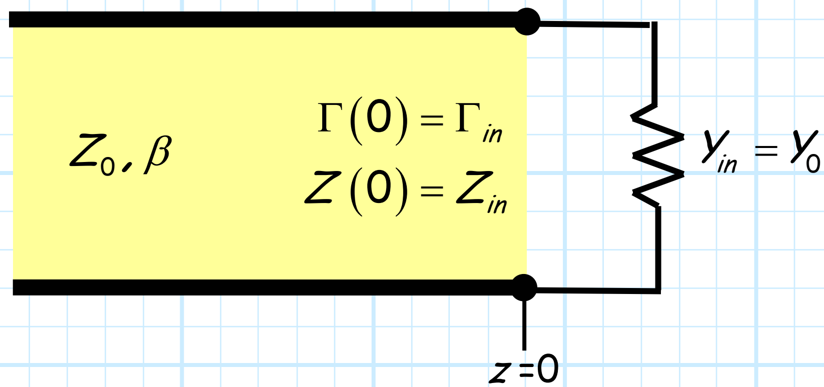
Now, it is evident that a perfect match will occur if the shunt element $Y = jB$ is set to "cancel" the reactive component of Y_1:

$$B = -Im\{Y\}_1 = -Im\left\{\frac{1}{jX + Z_L}\right\}$$

So that we find:

$$Y_{in} = Y + Y_1 = -jB_1 + \left(Y_0 + jB_1\right) = Y_0$$

A **perfect match**!



With these two equations, we can directly solve for the **required** values X and B for a **matched** network:

$$X = \pm\sqrt{R_L\left(Z_0 - R_L\right)} - X_L$$

and,

$$B = \pm\frac{\sqrt{\left(Z_0 - R_L\right)/R_L}}{Z_0}$$

where $Z_L = R_L + jX_L$.

Note:

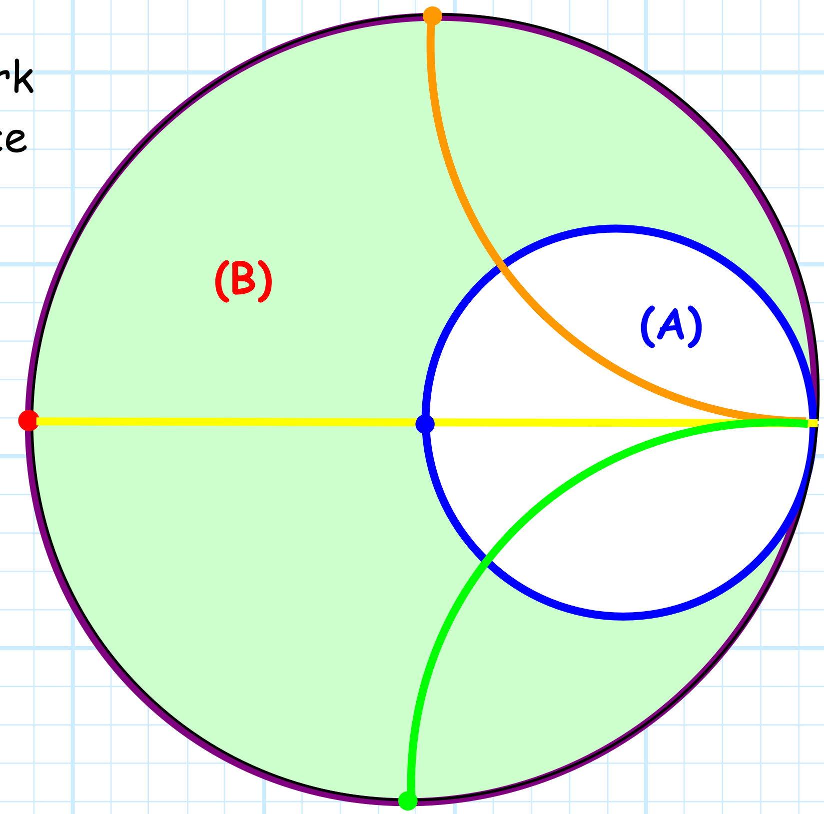
1) Because of the $\pm$, there are **two** solutions for B (and thus X).

2) For jB and jX to be purely imaginary (i.e., reactive), B and X must be **real**. We note from the term:

$$\left(Z_0 - R_L\right)$$

that R_L must be **less** than Z_0 ($R_L < Z_0$) to insure that B and thus X are real.

In other words, this matching network can **only** be used when $R_L < Z_0$. Notice that this condition means that the normalized load z_L' lies **outside** the $r = 1$ circle on the Smith Chart!



Once the values of X and B are found, we can determine the required values of inductance L and/or capacitance C, **for the signal frequency** ω_0**!**

Recall that:

$$X = \begin{cases} \omega_0 L & \text{if} \quad X > 0 \\ \dfrac{-1}{\omega_0 C} & \text{if} \quad X < 0 \end{cases}$$

and that:

$$B = \begin{cases} \omega_0 C & \text{if} \quad B > 0 \\ \dfrac{-1}{\omega_0 L} & \text{if} \quad B < 0 \end{cases}$$

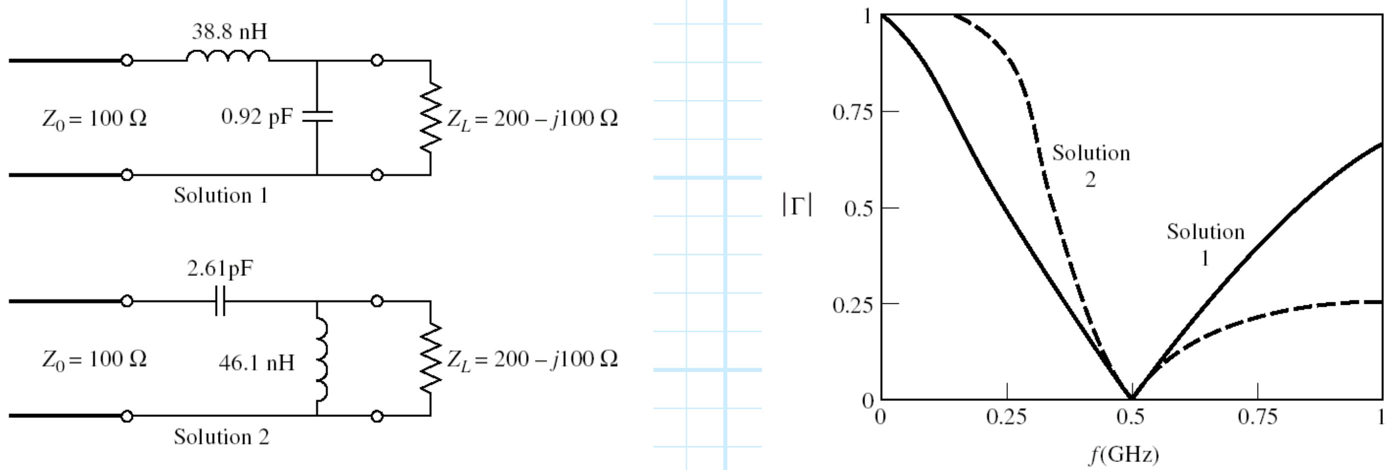
Make sure that **you** see and know why these equations are true.

As a result, we see that the reactance or susceptance of the elements of our L-network will have the proper values for matching at precisely **one** and **only** one frequency!

→ And this frequency **better** be the signal frequency ω_0!

If the signal frequency **changes** from this design frequency, the reactance and susceptance of the matching network inductors and capacitors will likewise change. The circuit will **no longer** be matched.

→ This matching network has a **narrow bandwidth!**



An L-Network Design Example

One other problem; it becomes **very** difficult to build quality **lumped** elements with useful values past 1 or 2 GHz. Thus, L-Network solutions are generally applicable only in the **RF region** (i.e., < 2GHz).