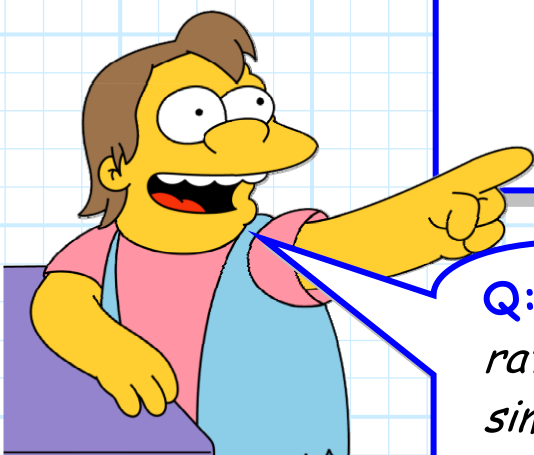
Line Impedance

Now let's define **line impedance** $Z(z)$, a **complex** function which is simply the ratio of the complex line **voltage** and complex line **current**:

$$Z(z) = \frac{V(z)}{I(z)}$$



Q: *Hey! I know what this is! The ratio of the voltage to current is simply the* ***characteristic impedance*** *Z_0, right ???*

A: NO! The line impedance $Z(z)$ is (generally speaking) **NOT** the transmission line **characteristic** impedance Z_0 !!!

→ It is **unfathomably important** that you understand this!!!!

To see why, recall that:

$$V(z) = V^+(z) + V^-(z)$$

And that:

$$I(z) = \frac{V^+(z) - V^-(z)}{Z_0}$$

Therefore:

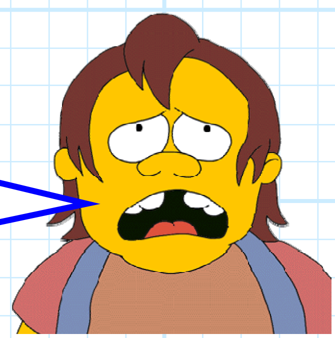
$$Z(z) = \frac{V(z)}{I(z)} = Z_0 \left(\frac{V^+(z) + V^-(z)}{V^+(z) - V^-(z)} \right) \neq Z_0$$

Or, more specifically, we can write:

$$Z(z) = Z_0 \left(\frac{V_0^+ \, e^{-j\beta z} + V_0^- \, e^{+j\beta z}}{V_0^+ e^{-j\beta z} - V_0^- e^{+j\beta z}} \right)$$

Q: *I'm confused! Isn't:*

$$V^+(z) / I^+(z) = Z_0 \text{ ???}$$



A: Yes! That **is** true! The ratio of the voltage to current for **each** of the two propagating waves is $\pm Z_0$. However, the ratio of the **sum** of the two voltages to the **sum** of the two currents is **not** equal to Z_0 (generally speaking)!

This is actually confirmed by the equation above. Say that $V^-(z) = 0$, so that only **one** wave ($V^+(z)$) is propagating on the line.

In this case, the ratio of the **total** voltage to the total current is simply the ratio of the voltage and current of the **one** remaining wave—the **characteristic impedance** Z_0!

$$Z(z)=\frac{V(z)}{I(z)}=Z_0\left(\frac{V^+(z)}{V^+(z)}\right)=\frac{V^+(z)}{I^+(z)}=Z_0 \quad \text{(when } V^-(z)=0\text{)}$$

Q: *So, it appears to me that characteristic impedance Z_0 is a **transmission line parameter**, depending **only** on the transmission line values L and C.*

*Whereas **line impedance** is $Z(z)$ depends the magnitude and phase of the two propagating waves $V^+(z)$ and $V^-(z)$ --values that depend **not only** on the transmission line, but also on the two things **attached** to either **end** of the transmission line!*

Right !?

A: **Exactly!** Moreover, note that characteristic impedance Z_0 is simply a **number**, whereas line impedance $Z(z)$ is a **function** of position (z) on the transmission line.