A Matched, Lossless Reciprocal 3-Port Network

Consider a 3-port device. Such a device would have a scattering matrix :

$$\mathcal{S} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

Assuming the device is passive and made of simple (isotropic) materials, the device will be **reciprocal,** so that:

$$S_{21} = S_{12} \qquad S_{31} = S_{13} \qquad S_{23} = S_{32}$$

Likewise, if it is **matched,** we know that:

$$S_{11} = S_{22} = S_{33} = 0$$

As a result, a **lossless**, **reciprocal** device would have a scattering matrix of the form:

$$\mathcal{S} = \begin{bmatrix} 0 & S_{21} & S_{31} \\ S_{21} & 0 & S_{32} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

Just **3** non-zero scattering parameters define the **entire** matrix!

Likewise, if we wish for this network to be **lossless**, the scattering matrix must be **unitary**, and therefore:

$$
\begin{aligned}
|S_{21}|^2 + |S_{31}|^2 &= 1 \qquad & S_{31}^* S_{32} &= 0 \\
|S_{21}|^2 + |S_{32}|^2 &= 1 \qquad & S_{21}^* S_{32} &= 0 \\
|S_{31}|^2 + |S_{32}|^2 &= 1 \qquad & S_{21}^* S_{31} &= 0
\end{aligned}
$$

Since each complex value S is represented by **two real numbers** (i.e., real and imaginary parts), the equations above result in **9** real equations. The problem is, the 3 complex values S_{21}, S_{31} and S_{32} are represented by only **6** real unknowns.

We have **over constrained** our problem ! There are **no solutions** to these equations !