

A Matched, Lossless Reciprocal 3-Port Network

Consider a 3-port device. Such a device would have a scattering matrix :

$$\mathcal{S} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

Assuming the device is passive and made of simple (isotropic) materials, the device will be **reciprocal**, so that:

$$S_{21} = S_{12} \quad S_{31} = S_{13} \quad S_{23} = S_{32}$$

Likewise, if it is **matched**, we know that:

$$S_{11} = S_{22} = S_{33} = 0$$

As a result, a **lossless, reciprocal** device would have a scattering matrix of the form:

$$\mathcal{S} = \begin{bmatrix} 0 & S_{21} & S_{31} \\ S_{21} & 0 & S_{32} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

Just **3** non-zero scattering parameters define the **entire** matrix!

Likewise, if we wish for this network to be **lossless**, the scattering matrix must be **unitary**, and therefore:

$$\begin{aligned} |S_{21}|^2 + |S_{31}|^2 &= 1 & S_{31}^* S_{32} &= 0 \\ |S_{21}|^2 + |S_{32}|^2 &= 1 & S_{21}^* S_{32} &= 0 \\ |S_{31}|^2 + |S_{32}|^2 &= 1 & S_{21}^* S_{31} &= 0 \end{aligned}$$

Since each complex value S is represented by **two real numbers** (i.e., real and imaginary parts), the equations above result in **9** real equations. The problem is, the 3 complex values S_{21} , S_{31} and S_{32} are represented by only **6** real unknowns.

We have **over constrained** our problem! There are **no solutions** to these equations!



*As unlikely as it might seem, this means that a matched, lossless, reciprocal 3-port device of **any kind** is a **physical impossibility!***

*You **can** make a lossless reciprocal 3-port device, **or** a matched reciprocal 3-port device, **or even** a matched, lossless (but non-reciprocal) 3-port network.*

*But try as you might, you **cannot** make a lossless, matched, **and** reciprocal three port component!*