A Matched, Lossless, Reciprocal 3-Port Network

Consider a **3-port** device. Such a device would have a scattering matrix :

$$\mathcal{S} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

Assuming the device is passive and made of simple (isotropic) materials, the device will be **reciprocal**, so that:

$$S_{21} = S_{12} \qquad S_{31} = S_{13} \qquad S_{23} = S_{32}$$

Likewise, if it is **matched**, we know that $S_{11} = S_{22} = S_{33} = 0$.

As a result, a **lossless**, **reciprocal** device would have a scattering matrix of the form:

$$\mathcal{S} = \begin{bmatrix} 0 & S_{21} & S_{31} \\ S_{21} & 0 & S_{32} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

Just **3** non-zero scattering parameters define the **entire** matrix!

Likewise, if we wish for this network to be **lossless**, the scattering matrix must be **unitary**, and therefore:

$$|S_{21}|^2 + |S_{31}|^2 = 1 \qquad S_{31}^* S_{32} = 0$$

$$|S_{21}|^2 + |S_{32}|^2 = 1 \qquad S_{21}^* S_{32} = 0$$

$$|S_{31}|^2 + |S_{32}|^2 = 1 \qquad S_{21}^* S_{31} = 0$$

Since each complex value S is represented by **two real numbers** (i.e., real and imaginary parts), the equations above result in **9** real equations. The problem is, the 3 complex values S_{21}, S_{31} and S_{32} are represented by only **6** real unknowns.

We have **over constrained** our problem ! There are **no solutions** to these equations !

*As unlikely as it might seem, this means that a matched, lossless, reciprocal **3-port** device of **any** kind is a **physical impossibility**!*

*You **can** make a lossless reciprocal 3-port device, **or** a matched reciprocal 3-port device, **or even** a matched, lossless (but non-reciprocal) 3-port network.*

*But try as you might, you **cannot** make a lossless, matched, **and** reciprocal three port component!*