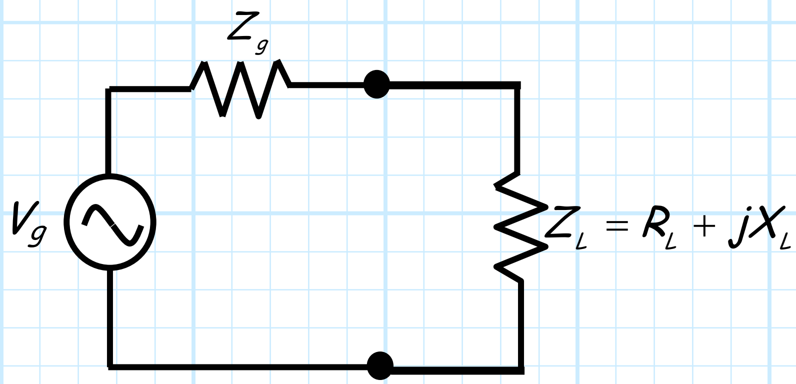
Matching Networks

Consider again the problem where a **passive load** is attached to an **active source**:



The load will **absorb power**—power that is **delivered** to it by the **source**.

$$\begin{aligned} P_L &= \frac{1}{2} Re\left\{V_L I_L^*\right\} \\ &= \frac{1}{2} Re\left\{\left(V_g \frac{Z_L}{Z_g + Z_L}\right)\left(\frac{V_g}{Z_g + Z_L}\right)^*\right\} \\ &= \frac{1}{2}\left|V_g\right|^2 \frac{Re\left\{Z_L\right\}}{\left|Z_g + Z_L\right|^2} \\ &= \frac{1}{2}\left|V_g\right|^2 \frac{R_L}{\left|Z_g + Z_L\right|^2} \end{aligned}$$

Recall that the power delivered to the load will be **maximized** (for a given V_g and Z_g) **if** the load impedance is equal to the **complex conjugate** of the source impedance ($Z_L = Z_g^*$).

We call this maximum power the **available power** P_{avl} **of the source**—it is, after all, the **largest** amount of power that the source can **ever** deliver!

$$\begin{aligned} P_L^{max} &\doteq P_{avl} \\ &= \frac{1}{2}\left|V_g\right|^2 \frac{R_g}{\left|Z_g + Z_g^*\right|^2} \\ &= \frac{1}{2}\left|V_g\right|^2 \frac{R_g}{\left|2R_g\right|^2} \\ &= \frac{\left|V_g\right|^2}{8\,R_g} \end{aligned}$$

* Note the available power of the **source** is dependent on **source** parameters **only** (i.e., V_g and R_g). This makes sense! Do **you** see why?

* Thus, we can say that to "take full advantage" of all the available power of the source, we must to make the load impedance the complex conjugate of the source impedance.

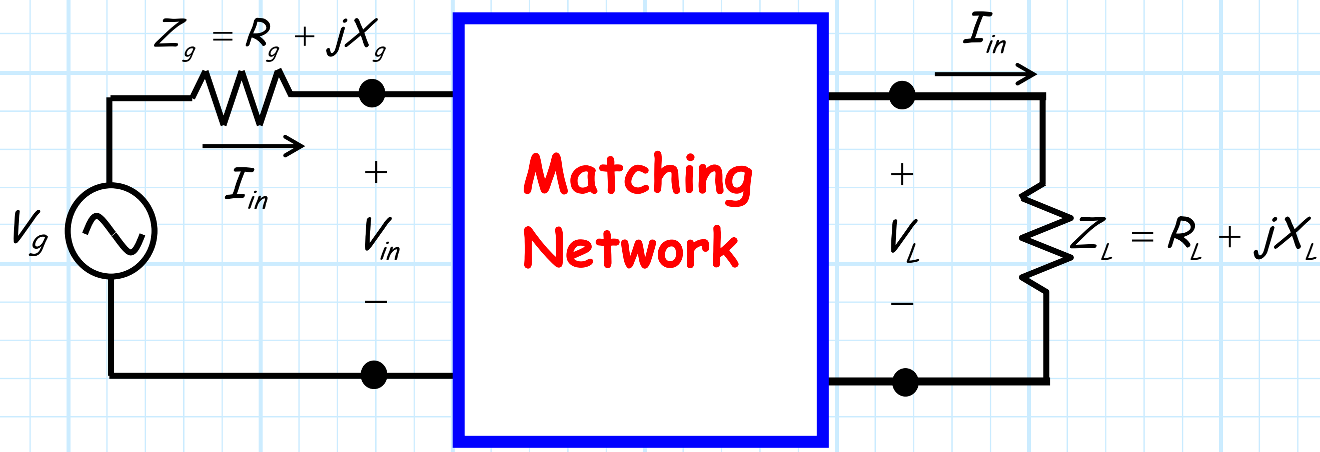
* Otherwise, the power delivered to the load will be less than power made available by the source! In other "words":

$$P_L \leq P_{avl}$$

Q: *But, you said that the load impedance typically models the input impedance of some useful device. We* ***don't*** *typically get to "select" or adjust this impedance—it* ***is*** *what it* ***is****. Must we then simply* ***accept*** *the fact that the delivered power will be less than the available power?*

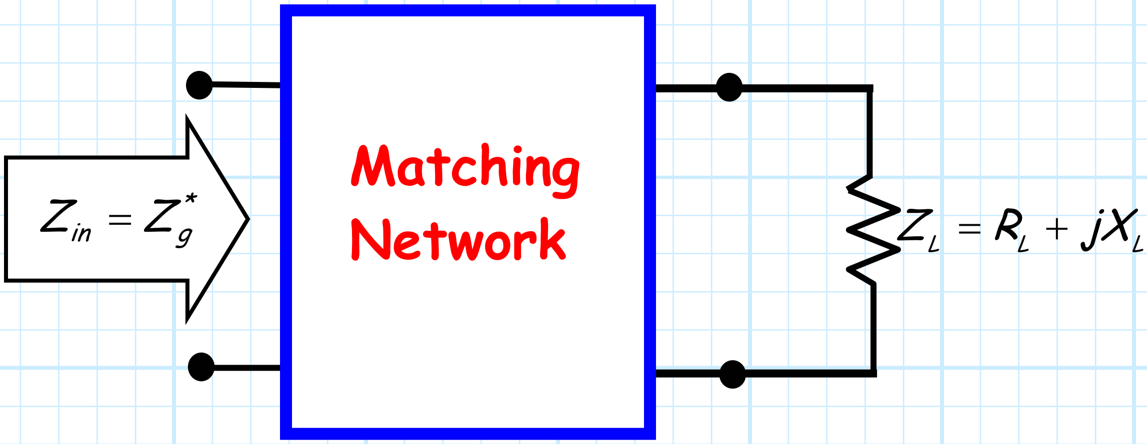
A: NO! We can in fact modify our circuit such that all available source power is delivered to the load—**without** in any way **altering** the impedance value of that **load**!

To accomplish this, we must insert a **matching network** between the source and the load:



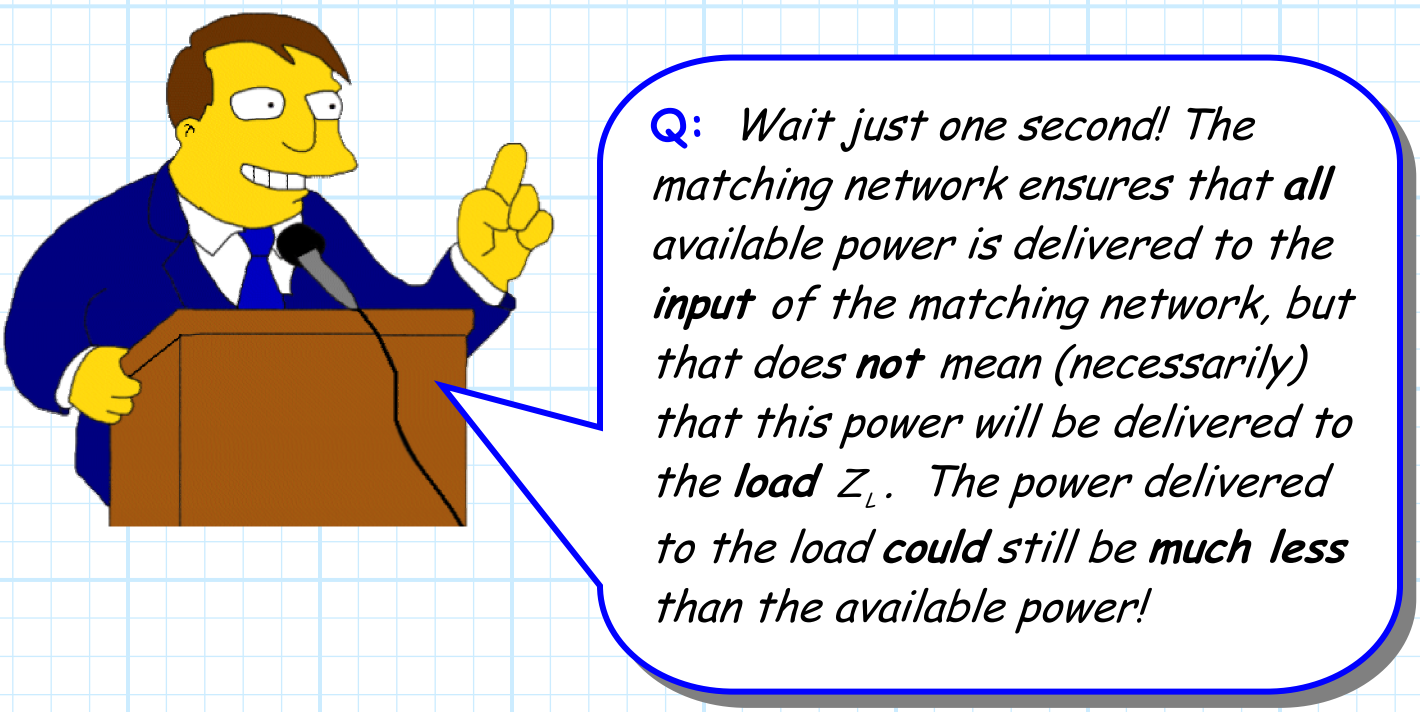
The sole purpose of this matching network is to "**transform**" the load impedance into an input impedance that **is conjugate matched** to the source! I.E.:

$$Z_{in} = \frac{V_{in}}{I_{in}} = Z_g^*$$



Because of this, **all** available source power is delivered to the input of the matching network (i.e., delivered to Z_{in}):

$$P_{in} = P_{avl}$$



A: True! To ensure that the **available power** delivered to the input of the matching network is **entirely** delivered to the **load**, we must construct our matching network such that it **cannot absorb any power**—the matching network must be **lossless**!

We must construct our matching network entirely with **reactive elements**!

Examples of reactive elements include inductors, capacitors, transformers, as well as lengths of **lossless transmission lines**.

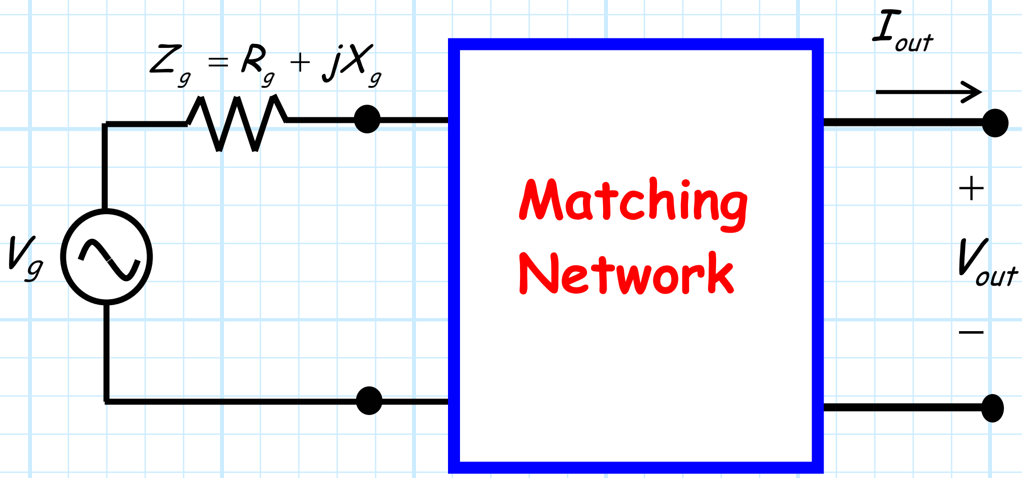
Thus, constructing a proper lossless matching network will lead to the **happy** condition where:

$$P_L = P_{in} = P_{avl}$$

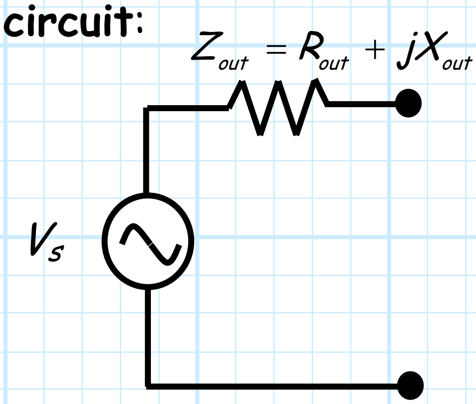
* Note that the design and construction of this lossless network will depend on **both** the value of source impedance Z_g and load impedance Z_L.

* However, the matching network does **not physically alter** the values of either of these two quantities—the source and load are left physically unchanged!

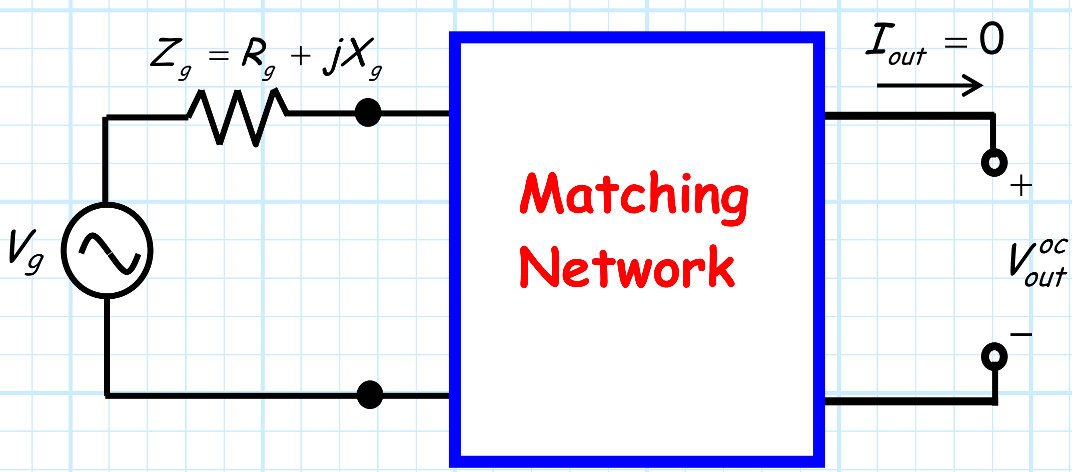
Now, let's consider the matching network from a **different perspective**. Instead of defining it in terms of its **input impedance** when attached the **load**, let's describe it in terms of its **output impedance** when attached to the **source**:



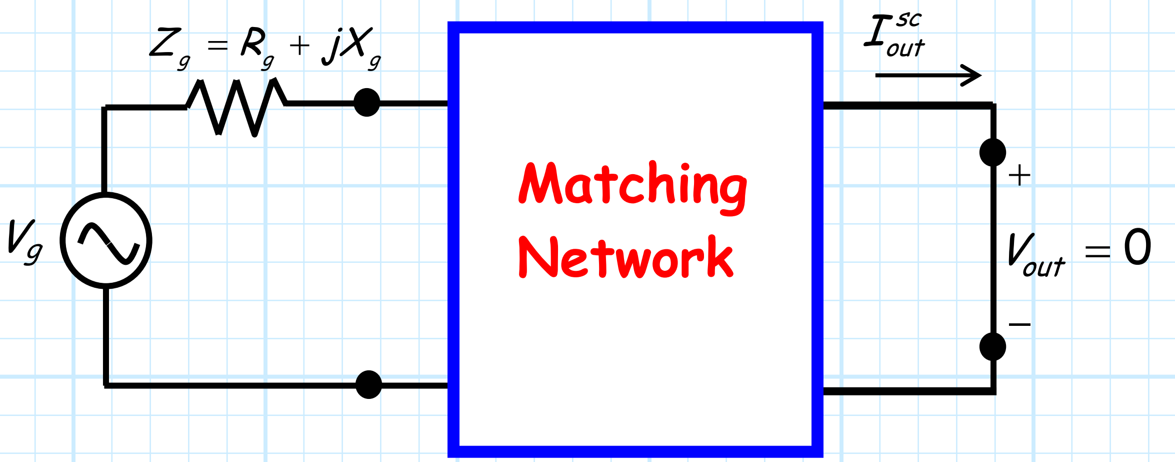
This "new" source (i.e., the original source with the matching network attached) can be expressed in terms of its **Thevenin's equivalent circuit**:



This equivalent circuit can be determined by first evaluating (or measuring) the **open-circuit** output **voltage** V_{out}^{oc} :



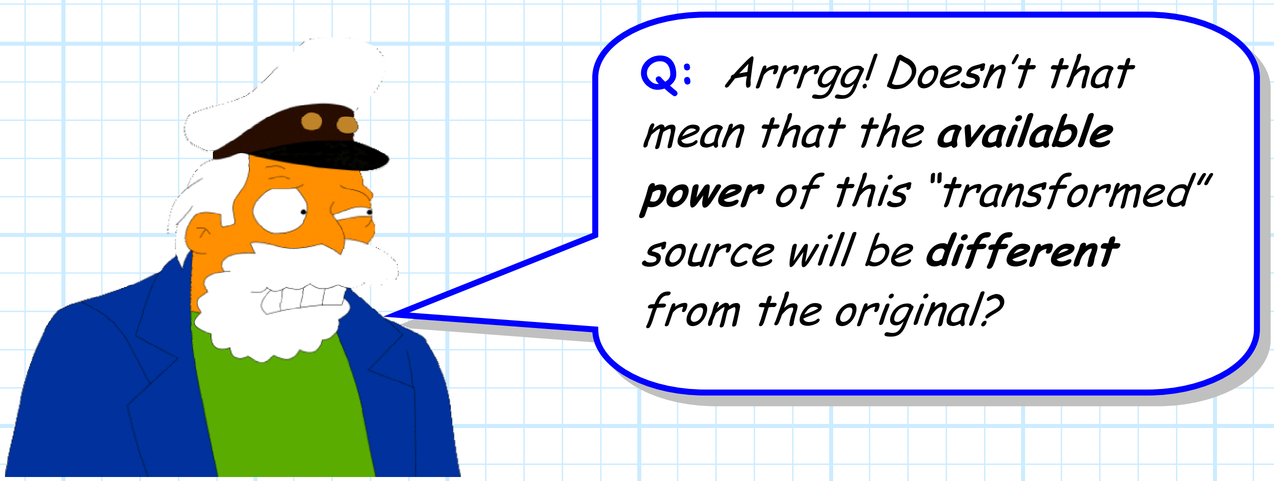
And likewise evaluating (or measuring) the **short-circuit** output **current** I_{out}^{sc}:



From these two values (V_{out}^{oc} and I_{out}^{sc}) we can determine the **Thevenin's equivalent source**:

$$V_s = V_{out}^{oc} \qquad\qquad Z_{out} = \frac{V_{out}^{oc}}{I_{out}^{oc}}$$

Note that in general that $V_s \neq V_g$ and $Z_{out} \neq Z_g$—the matching network "transforms" **both** the values of both the impedance **and** the voltage source.



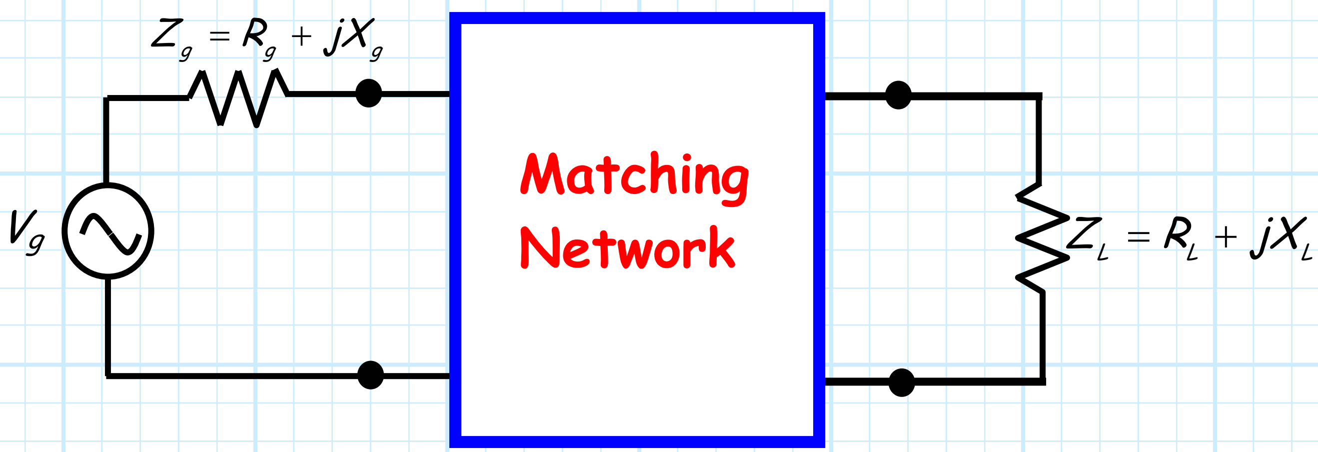
A: Nope. **If** the matching network is **lossless,** the available power of this equivalent source is **identical** to the available power of the original source—the lossless matching network does **not** alter the available power!

Now, for a **properly** designed, **lossless** matching network, it turns out that (as **you** might have expected!) the output impedance Z_{out} is equal to the **complex conjugate** of the load impedance. I.E.:

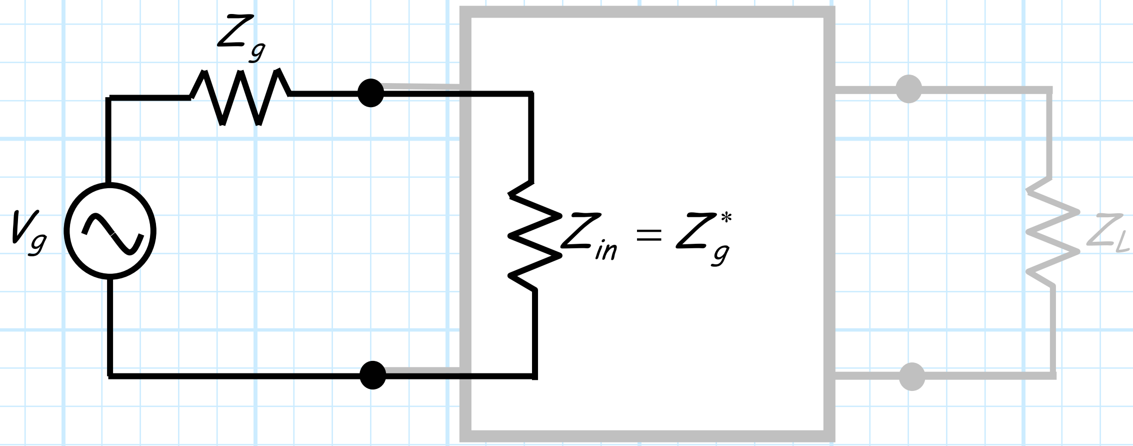
$$Z_{out} = Z_L^*$$

→ The source and load are again matched!

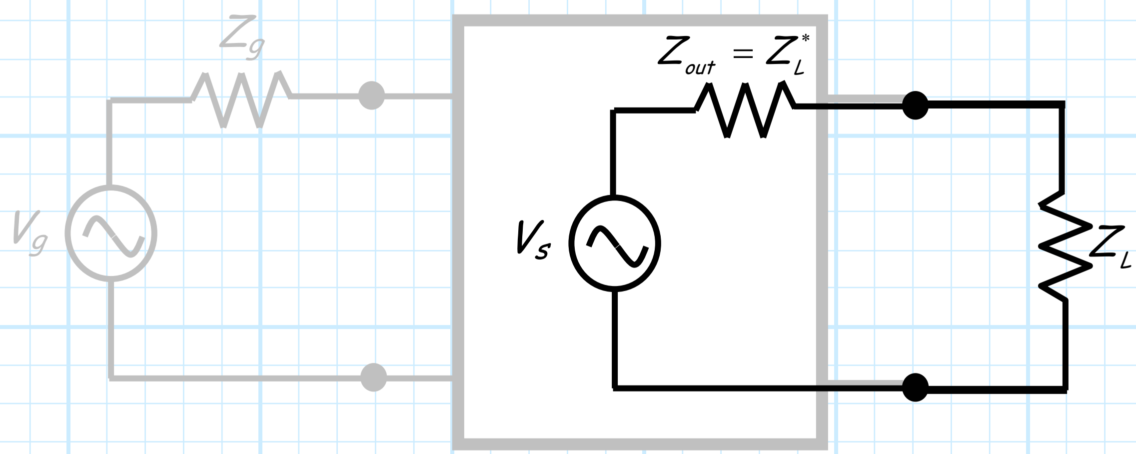
Thus, we can look at the matching network in two equivalent ways:



1. As a network attached to a load, one that "transforms" its impedance to Z_{in}—a value matched to the source impedance Z_g:



2. Or, as network attached to a source, one that "transforms" its impedance to Z_{out}—a value matched to the load impedance Z_L:



Either way, the source and load impedance are conjugate matched—**all** the available power is delivered to the load!