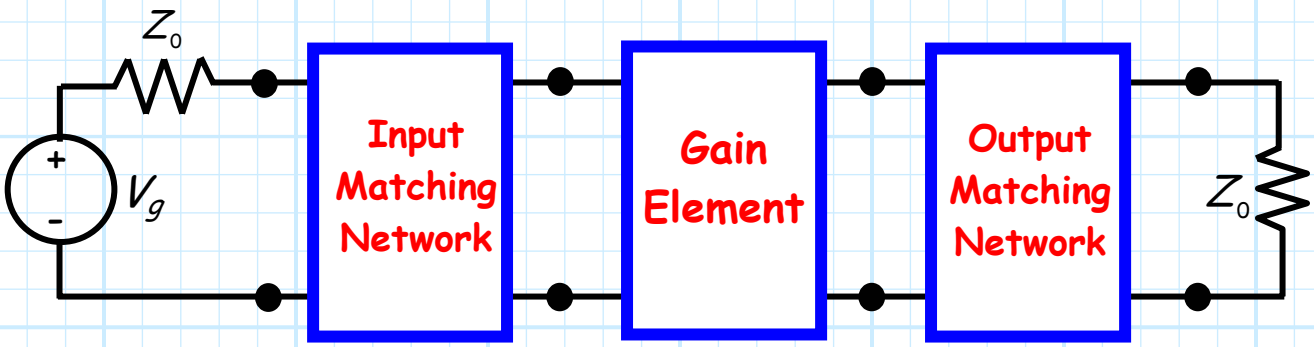
Maximum Gain Amplifiers



$$\Gamma_s = \Gamma_{in}^* \qquad \Gamma_{out} = \Gamma_L^*$$

Q: *So if we design our amplifier such that the source is **matched** to the input of the gain element, and the output of the gain element is **matched** to the load, what **is** the resulting gain?*

A: Recall the **transducer gain** of an amplifier is:

$$G_T = \frac{\left(1-|\Gamma_s|^2\right)|S_{21}|^2\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_s\Gamma_{in}|^2\,|1-\Gamma_L S_{22}|^2}$$

If the amplifier is a **unilateral** amplifier ($|S_{12}| \ll |S_{21}|$), where:

$$\Gamma_{in} = S_{11} \qquad \text{and} \qquad \Gamma_{out} = S_{22}$$

the transducer gain becomes:

$$G_{UT} = \frac{\left(1-\left|\Gamma_s\right|^2\right)\left|S_{21}\right|^2\left(1-\left|\Gamma_L\right|^2\right)}{\left|1-\Gamma_s S_{11}\right|^2\left|1-\Gamma_L S_{22}\right|^2}$$

Thus, inserting the **matched conditions** above, we find the transducer gain for the **matched** case is:

$$G_{T\,max} = \frac{1}{1-\left|\Gamma_s\right|^2}\left|S_{21}\right|^2\frac{1-\left|\Gamma_L\right|^2}{\left|1-\Gamma_L S_{22}\right|^2}$$

and the unilateral transducer gain for the **matched** case is:

$$G_{UT\,max} = \frac{1}{1-\left|\Gamma_s\right|^2}\left|S_{21}\right|^2\frac{1}{1-\left|\Gamma_L\right|^2}$$

$$= \frac{1}{1-\left|S_{11}\right|^2}\left|S_{21}\right|^2\frac{1}{1-\left|S_{22}\right|^2}$$

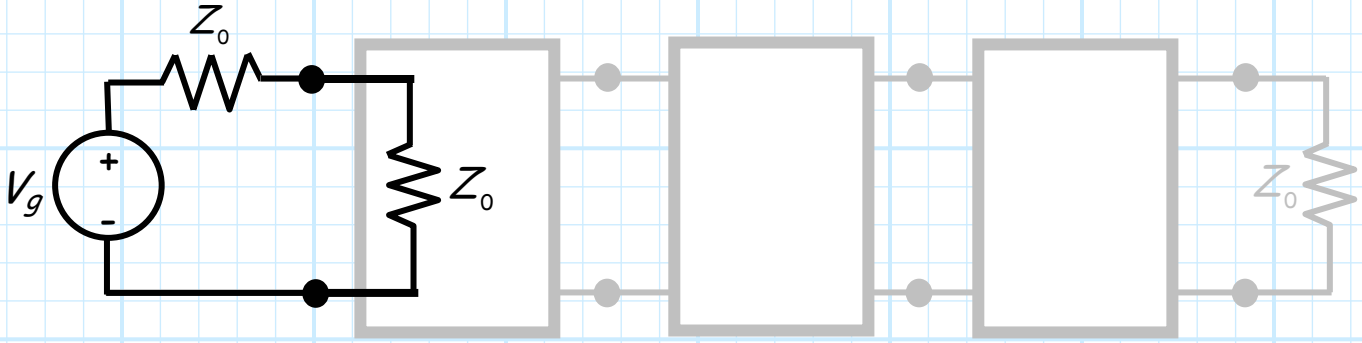
These of course are the **maximum** transducer gain possible, **given** a specific gain element, and a source and load impedance of Z_0.

Q: *What about the **scattering matrix** of the **amplifier**? Can we determine the scattering parameters of the resulting amplifier?*

A: We can certainly determine their **magnitude**!

First of all, remember that if a matching network establishes a match at its **output**, then a match is likewise present at its **input**.

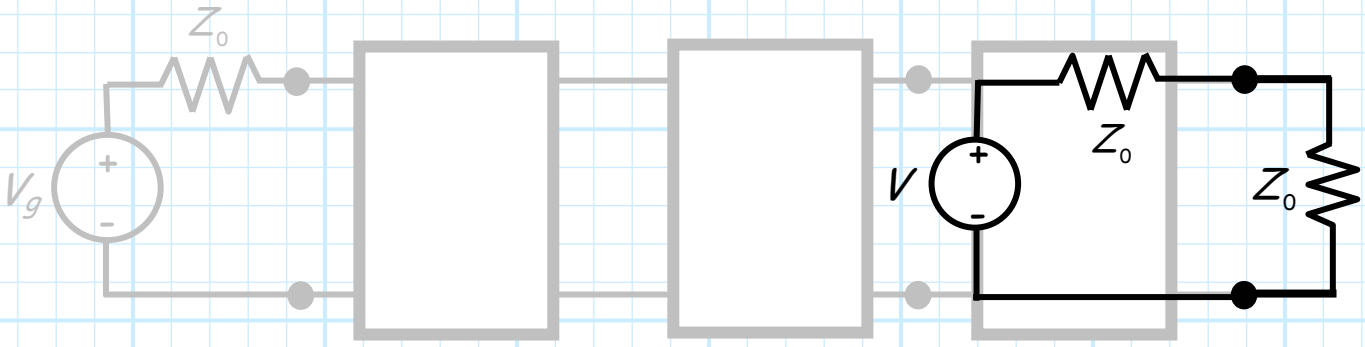
As a result, we know that the input impedance of the input matching network must be Z_0:



Meaning that the scattering parameter S_{11} of the matched amplifier is **zero**!

$$S_{11}^{amp} = 0$$

Likewise, the output impedance of the output matching network must be also be Z_0:



As a result, the scattering parameter S_{22} of the matched amplifier is also **zero**!

$$S_{22}^{amp} = 0$$

Now, since both ports of the amplifier are matched, we can determine that the magnitude of the **amplifier** scattering parameter S_{21} is simply the transducer gain $G_{T\,max}$.

$$\left|S_{21}^{amp}\right| = G_{T\,max} = \frac{1}{1-\left|\Gamma_s\right|^2}\left|S_{21}\right|^2 \frac{1-\left|\Gamma_L\right|^2}{\left|1-\Gamma_L S_{22}\right|^2}$$

Remember, the scattering parameters S_{12} and S_{21} in the expression above are those of the **gain element**.

From this result, we can likewise conclude that for the remaining scattering parameter:

$$\left|S_{12}^{amp}\right| = \frac{1}{1-\left|\Gamma_L\right|^2}\left|S_{12}\right|^2 \frac{1-\left|\Gamma_s\right|^2}{\left|1-\Gamma_s S_{11}\right|^2}$$

Note that **if** the gain element is **unilateral**, then so too will be the **amplifier**!