Multi-Section Coupled Line Couplers

We can add **multiple** coupled lines in series to increase coupler bandwidth.

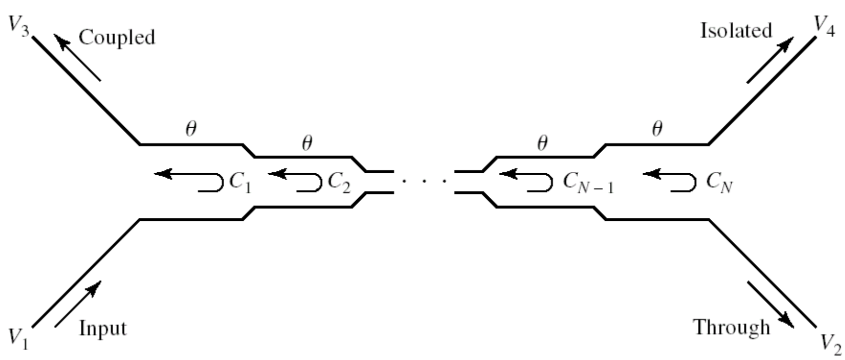


Figure 7.35 (p. 346)
An N-section coupled line

We typically design the coupler such that it is **symmetric**, i.e.:

$$C_1 = C_N, \; C_2 = C_{N-1}, \; C_3 = C_{N-2}, \text{ etc.}$$

where N is **odd**.

Q: *What is the coupling of this device as a function of **frequency**?*

A: To analyze this structure, we make an **approximation** similar to that of the theory of small reflections.

First, if c is **small** (i.e., less than 0.3), then we can make the approximation:

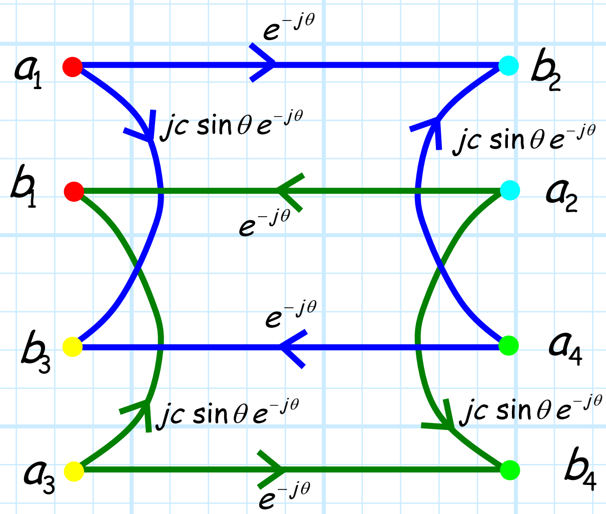
$$
\begin{aligned}
S_{31}(\theta) &= \frac{jc\tan\theta}{\sqrt{1-c^2}+j\tan\theta} \\
&\approx \frac{jc\tan\theta}{1+j\tan\theta} \\
&= jc\sin\theta\, e^{-j\theta}
\end{aligned}
$$

Likewise:

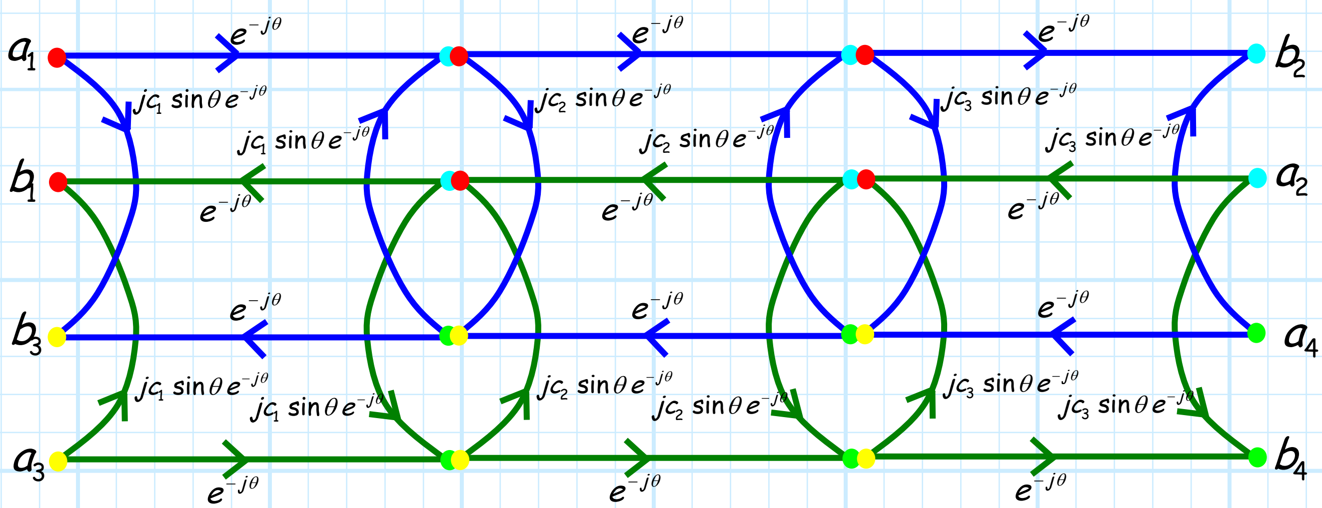
$$
\begin{aligned}
S_{21}(\theta) &= \frac{\sqrt{1-c^2}}{\sqrt{1-c^2}\cos\theta + j\sin\theta} \\
&\approx \frac{1}{\cos\theta + j\sin\theta} \\
&= e^{-j\theta}
\end{aligned}
$$

where of course $\theta = \beta\ell = \omega T$, and $T = \ell / v_p$.

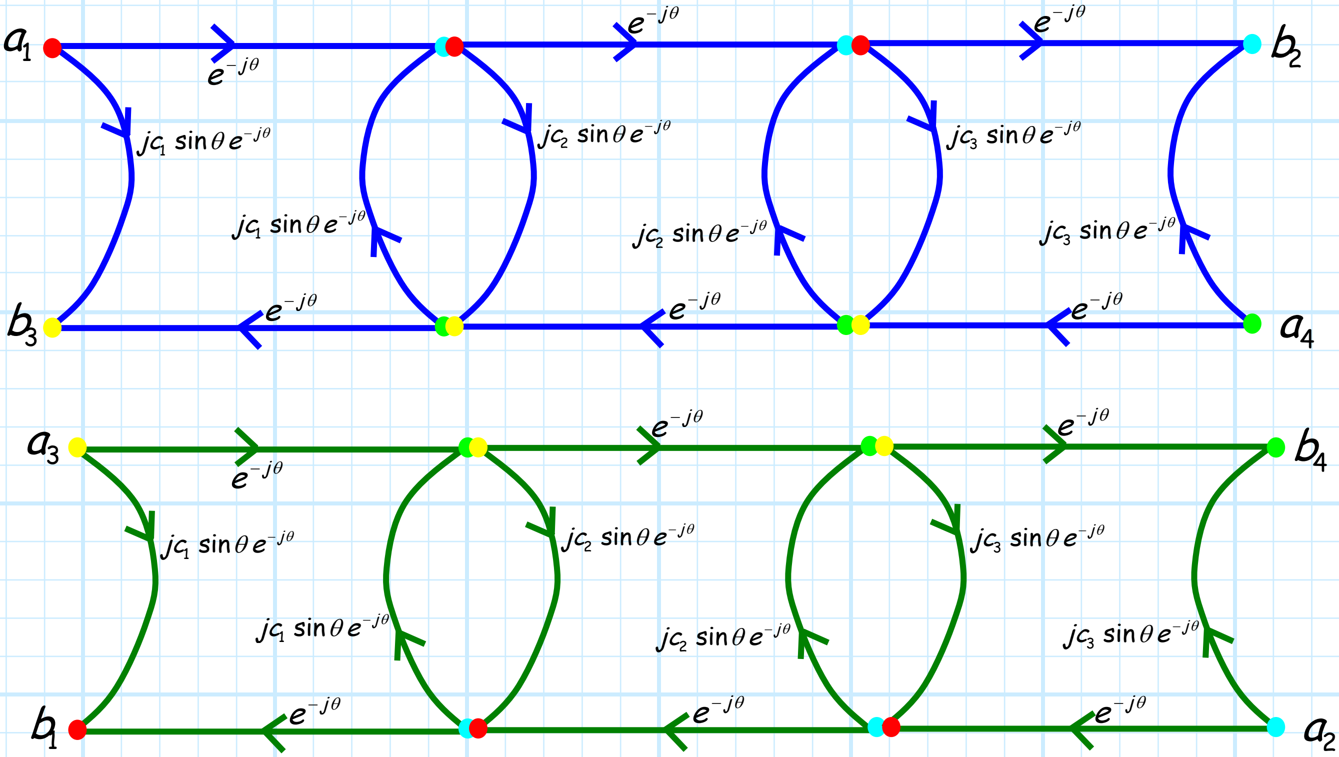
We can use these approximations to construct a **signal flow graph** of a **single**-section coupler:



Now, say we cascade **three** coupled line pairs, to form a **three section** coupled line coupler. The signal flow graph would thus be:

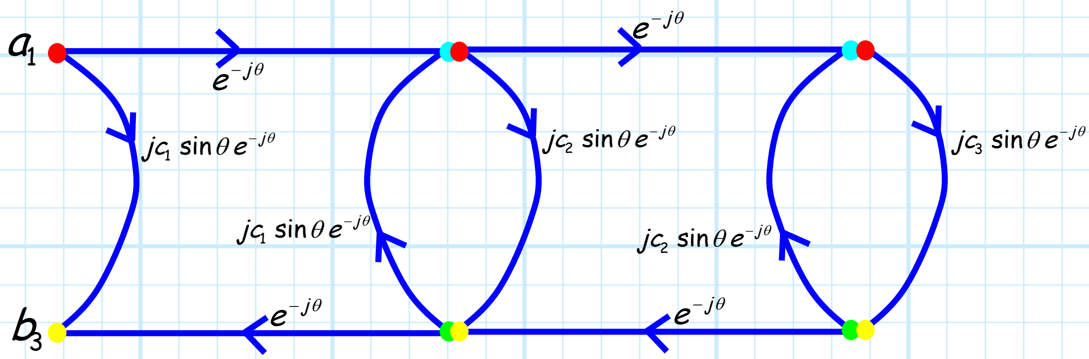


Note that this signal flow graph **decouples** into two separate and graphs (i.e., the blue graph and the green graph).



Note also that these two graphs are essentially **identical**, and emphasize the **symmetric** structure of the coupled-line coupler.

Now, we are interested in describing the **coupled output** (i.e., b_3) in terms of the incident wave (i.e., a_1). Assuming ports 2, 3 and 4 are **matched** (i.e., $a_2 = a_3 = a_4 = 0$), we can reduce the graph to simply:

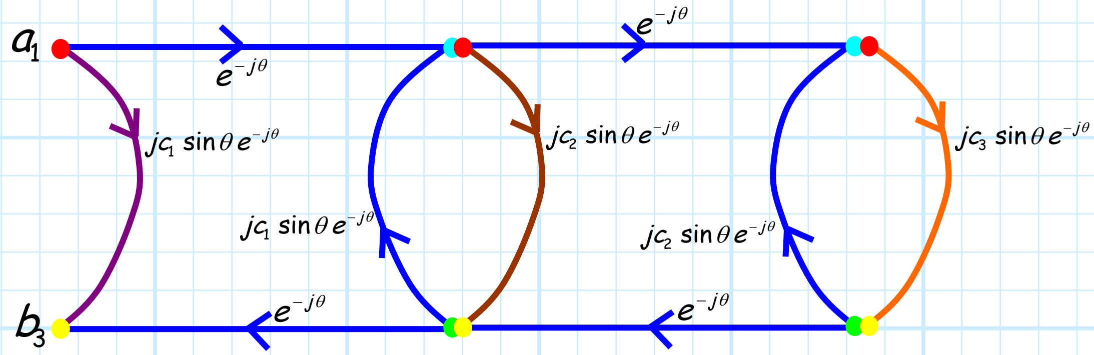


Now, we **could** reduce this signal flow graph even further—**or** we could truncate a **propagation series** by considering only the **direct paths**!

We of course used this idea to analyze multi-section matching networks, an approach dubbed the "theory of small reflections".

Essentially we are now applying a **"theory of small couplings"**. In other words, we consider only the propagation paths where **one coupling** is involved—the signal propagates **across** a coupled-line pair only **once**!

Note from the signal flow graph that there are **three** such mechanisms, corresponding to the coupling across each of the **three** separate coupled line pairs:



$$b_3 \approx \left(jc_1 \sin\theta\, e^{-j\theta} + e^{-j\theta} jc_2 \sin\theta\, e^{-j\theta} e^{-j\theta} + e^{-j2\theta} jc_3 \sin\theta\, e^{-j\theta} e^{-j2\theta} \right) a_1$$
$$= \left(jc_1 \sin\theta\, e^{-j\theta} + jc_2 \sin\theta\, e^{-j3\theta} + jc_3 \sin\theta\, e^{-j5\theta} \right) a_1$$

Note that **all other** terms of the infinite series would involve at least **three** couplings (i.e., three crossings), and thus these terms would be **exceeding** small (i.e., $c^3 \approx 0$).

Therefore, according to this **approximation**:

$$S_{31}(\theta) = \frac{V_3^-}{V_1^+}(\theta) = \frac{b_3}{a_1}(\theta) = jc_1 \sin\theta\, e^{-j\theta} + jc_2 \sin\theta\, e^{-j3\theta} + jc_3 \sin\theta\, e^{-j5\theta}$$

Moreover, for a **multi**-section coupler with N sections, we find:

$$S_{31}(\theta) = j\,c_1 \sin\theta\, e^{-j\theta} + j\,c_2 \sin\theta\, e^{-j3\theta} + j\,c_3 \sin\theta e^{-j5\theta} + \cdots + j\,c_N \sin\theta\, e^{-j(2N-1)\theta}$$

And for **symmetric** couplers with an **odd** value N, we find:

$$S_{31}(\theta) = j2\sin\theta\, e^{-jN\theta}\left[c_1\cos(N-1)\theta + c_2\cos(N-3)\theta + c_3\cos(N-5)\theta + \cdots + \tfrac{1}{2}c_M\right]$$

where $M = (N+1)/2$.

Thus, we find the coupling **magnitude** as a function of frequency is:

$$\begin{aligned}|c(\theta)| &= |S_{31}(\theta)| \\ &= c_1\, 2\sin\theta\cos(N-1)\theta + c_2\, 2\sin\theta\cos(N-3)\theta \\ &\quad + c_3\, 2\sin\theta\cos(N-5)\theta + \cdots + c_M\sin\theta\end{aligned}$$

And thus the **coupling in** ***dB*** is:

$$C(\theta) = -10\log_{10}|c(\theta)|^2$$

Now, our design goals are to **select** the coupling values $c_1, c_2, \cdots c_N$ such that:

1. The coupling value $C(\theta)$ is a specific, **desired** value at our design frequency.

2. The coupling **bandwidth** is as **large** as possible.

For the first condition, recall that the at the **design frequency**:

$$\theta = \beta \ell = \pi/2$$

I.E., the section lengths are a **quarter-wavelength** at our design frequency.

Thus, we find our **first** design equation:

$$\begin{aligned} |c(\theta)|\Big|_{\theta=\pi/2} &= c_1\, 2\cos(N-1)\pi/2 + c_2\, 2\cos(N-3)\pi/2 \\ &+ c_3\, 2\cos(N-5)\pi/2 + \cdots + c_M \end{aligned}$$

where we have used the fact that $\sin(\pi/2) = 1$.

Note the value $|c(\theta)|\Big|_{\theta=\pi/2}$ is set to the value necessary to achieve the **desired** coupling value. This equation thus provides **one** design constraint—we have **M-1** degrees of design freedom left to accomplish our **second** goal!

To **maximize bandwidth**, we typically impose the **maximally flat** condition:

$$\left.\frac{d^m |c(\theta)|}{d\theta^m}\right|_{\theta=\pi/2} = 0 \qquad m = 1,2,3\cdots$$

Be careful! Remember to perform the derivative **first**, and **then** evaluate the result at $\theta = \pi/2$.

You will find for a **symmetric** coupler, the **odd**-ordered derivatives (e.g., $d\left|c(\theta)\right|/d\theta$, $d^3\left|c(\theta)\right|/d\theta^3$, $d^5\left|c(\theta)\right|/d\theta^5$)are uniquely **zero**. In other words, they are zero-valued at $\theta=\pi/2$ **regardless** of the values of coupling coefficients $c_1, c_2, c_3, \cdots$!

As a result, these **odd**-order derivatives do **not** impose a maximally flat **design equation**—only the **even**-ordered derivatives do. **Keep taking** these derivatives until your design is **fully** constrained (i.e., the number of design equations **equals** the number of unknown coefficients $c_1, c_2, c_3, \cdots$).

One final note, you may find that this **trig** expression is helpful in **simplifying** your derivatives:

$$\sin\phi\cos\psi = \frac{1}{2}\sin(\phi+\psi)+\frac{1}{2}\sin(\phi-\psi)$$

For **example**, we find that:

$$\begin{aligned} 2\sin\theta\cos 2\theta &= \sin(\theta+2\theta)+\sin(\theta-2\theta) \\ &= \sin(3\theta)+\sin(-\theta) \\ &= \sin(3\theta)-\sin(\theta) \end{aligned}$$