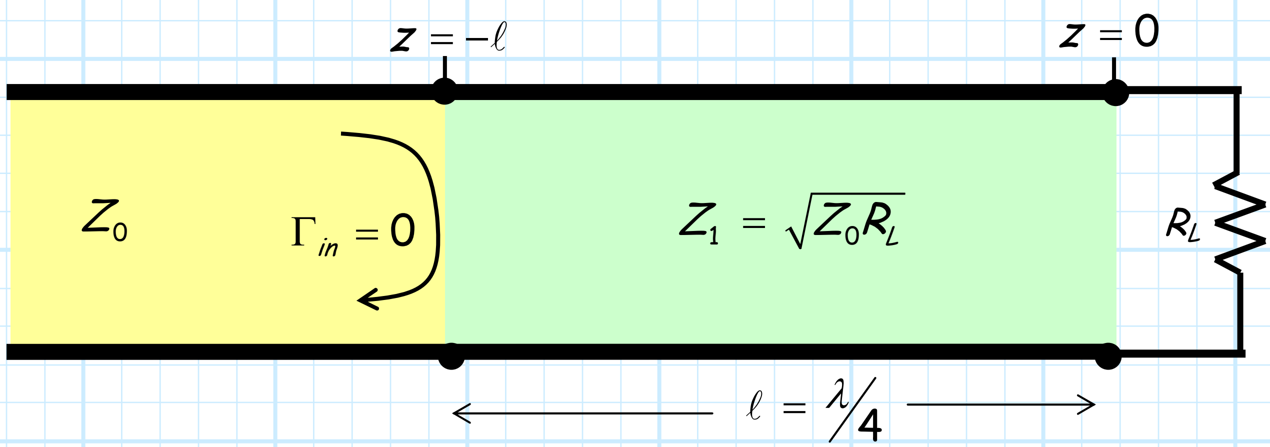
Multiple Reflection Viewpoint

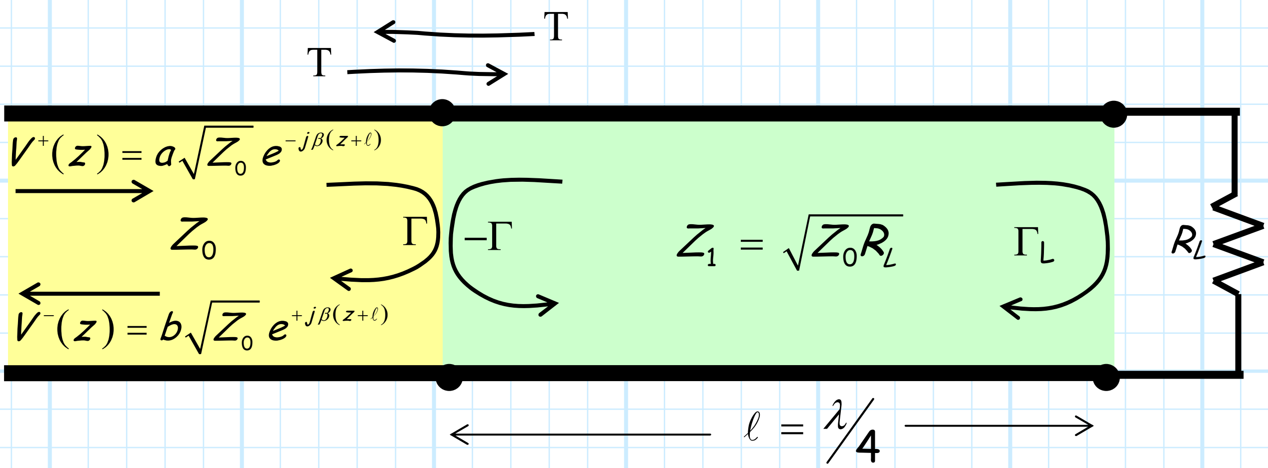
The **quarter-wave** transformer brings up an interesting question in μ-wave engineering.



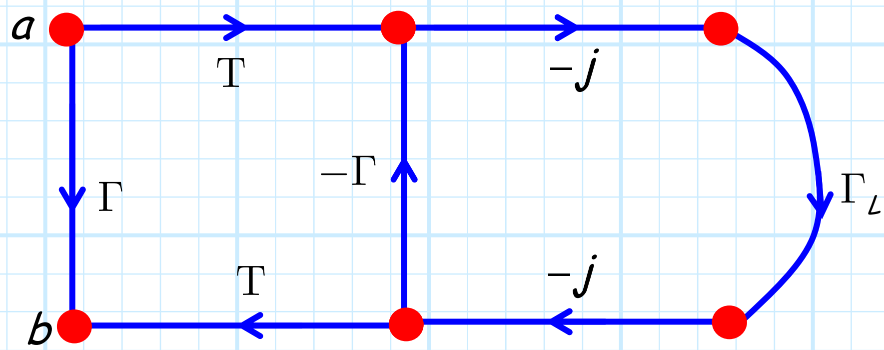
Q: ***Why*** *is there* ***no*** *reflection at $z = -\ell$? It appears that the line is* ***mismatched*** *at both $z = 0$ and $z = -\ell$.*

A: In fact there **are** reflections at these mismatched interfaces—an **infinite** number of them!

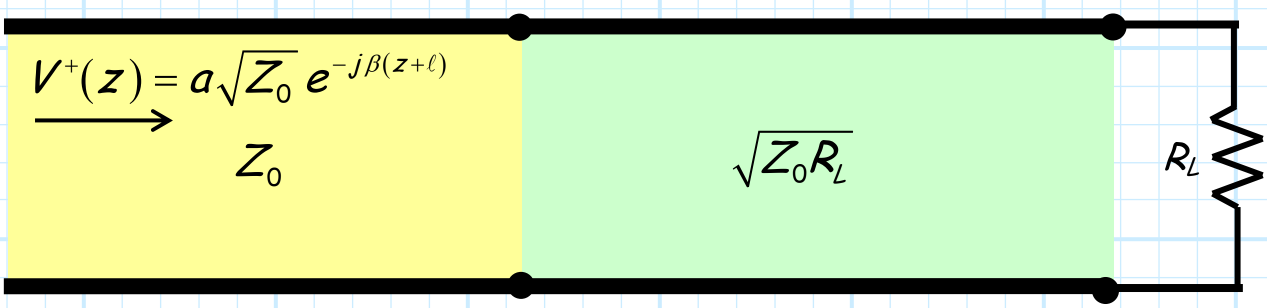
We can use our **signal flow graph** to determine the propagation series, once we determine all the **propagation paths** through the quarter-wave transformer.



$$b = a \sum_{n=1}^{\infty} p_n$$

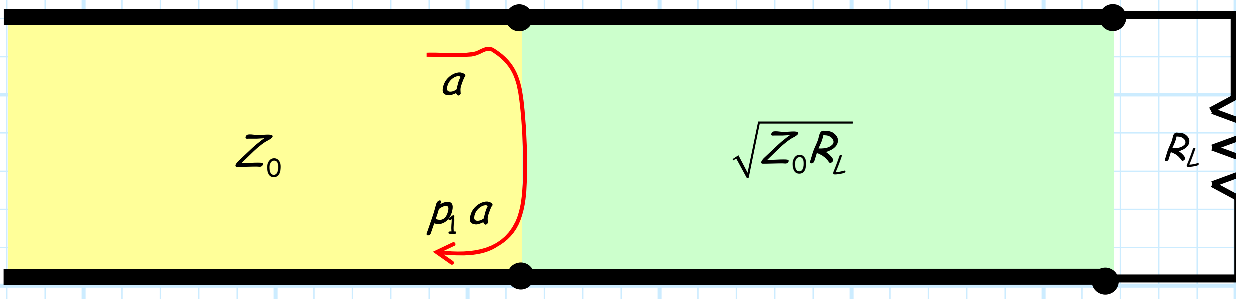


Now let's try to interpret what **physically** happens when the **incident** voltage wave:

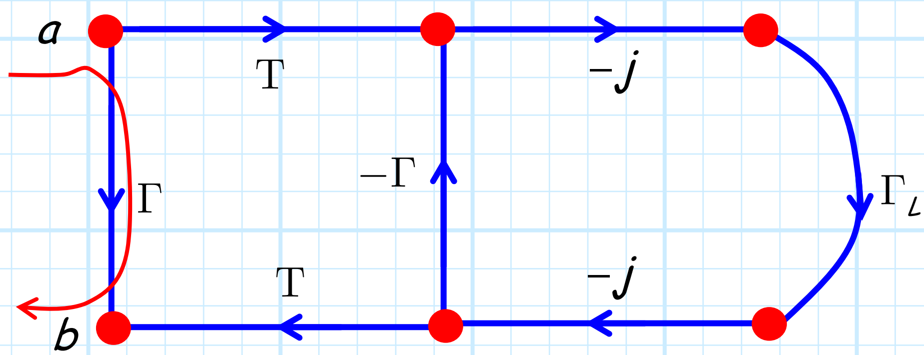


reaches the interface at $z = -\ell$. We find that there are **two forward paths** through the quarter-wave transformer signal flow graph.

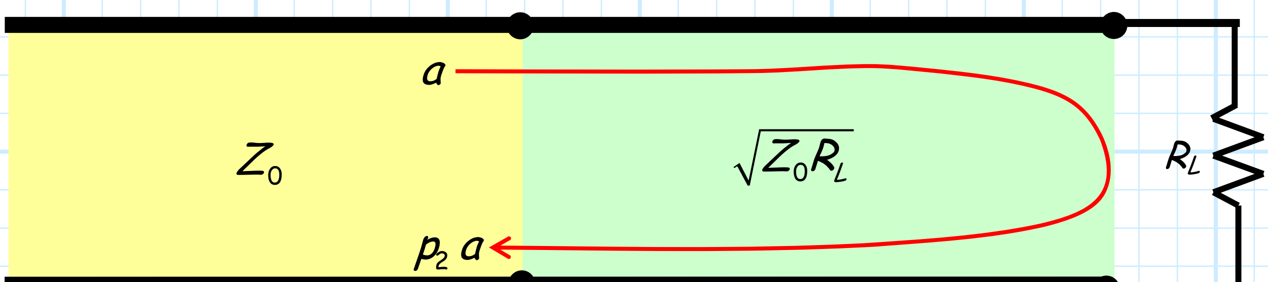
Path 1. At $z=-\ell$, the characteristic impedance of the transmission line changes from Z_0 to Z_1. This mismatch creates a **reflected** wave, with complex amplitude $p_1 a_1$:



So, $p_1 = \Gamma$.



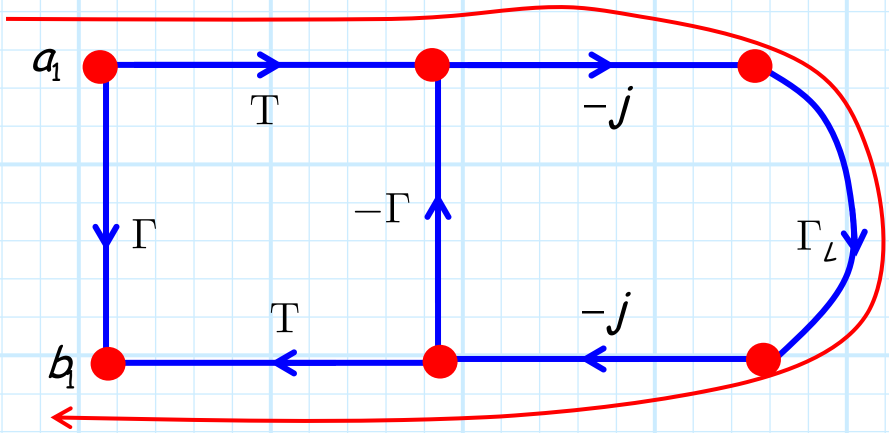
Path 2. However, a **portion** of the incident wave is transmitted (T) across the interface at $z=-\ell$, this wave travels a distance of $\beta\ell = 90^\circ$ to the load at $z=0$, where a portion of it is reflected (Γ_L). This wave travels back $\beta\ell = 90^\circ$ to the interface at $z=-\ell$, where a portion is again transmitted (T) across into the Z_0 transmission line—**another** reflected wave !



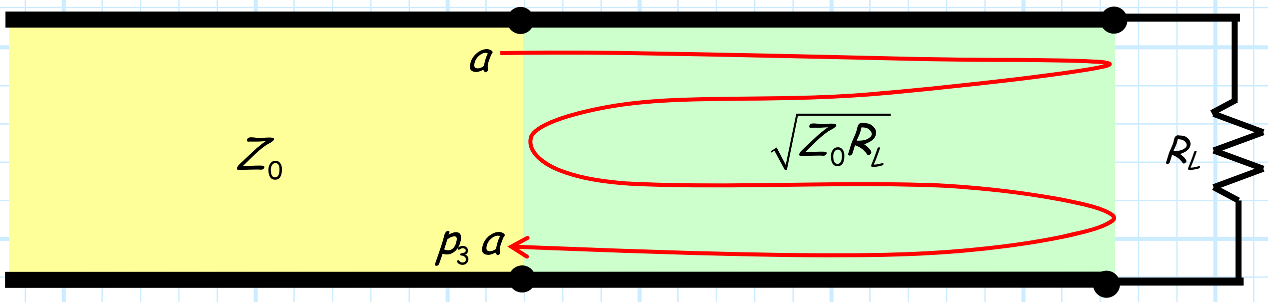
So the **second direct path** is

$$p_2 = \mathrm{T}\ e^{-j90^\circ}\,\Gamma_L\,e^{-j90^\circ}\,\mathrm{T} = -\mathrm{T}^2\,\Gamma_L$$

note that traveling $2\beta\ell = 180^\circ$ has produced a **minus** sign in the result.

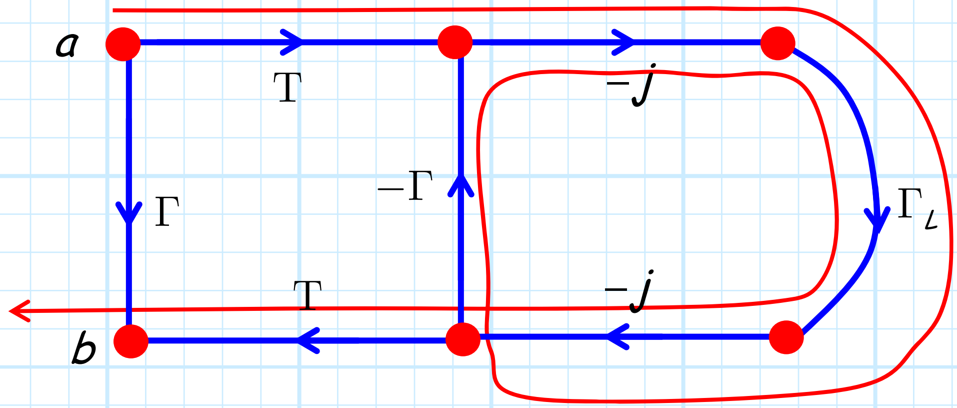


Path 3. However, a **portion** of this **second** wave is also **reflected** (Γ) back into the Z_1 transmission line at $z = -\ell$, where it again travels to $\beta\ell = 90^\circ$ the load, is partially reflected (Γ_L), travels $\beta\ell = 90^\circ$ back to $z = -\ell$, and is partially transmitted into Z_0 (T)—our **third** reflected wave!



where:

$$\begin{aligned} p_3 &= \mathrm{T}\,e^{-j90^\circ}\,\Gamma_L\,e^{-j90^\circ}\,(-\Gamma)\,e^{-j90^\circ}\,\Gamma_L\,e^{-j90^\circ}\,\mathrm{T} \\ &= -\mathrm{T}^2\,(\Gamma_L)^2\,\Gamma \end{aligned}$$



Note that path 3 is **not** a direct path!

Path *n*. We can see that this "bouncing" back and forth can go on **forever**, with each trip launching a **new** reflected wave into the Z_0 transmission line.

Note however, that the **power** associated with each successive reflected wave is **smaller** than the previous, and so eventually, the power associated with the reflected waves will **diminish** to insignificance!

Q: *But, why then is* $\Gamma = 0$ *?*

A: Each reflected wave is a **coherent** wave. That is, they all oscillate at same frequency ω; the reflected waves differ only in terms of their **magnitude** and **phase**.

Therefore, to determine the **total** reflected wave, we must perform a **coherent summation** of each reflected wave—this summation of course results in our **propagation series**, a series that must converge for passive devices.

$$b = a \sum_{n=1}^{\infty} p_n$$

It can be shown that the infinite propagation series for **this** quarter-wavelength structure **converges** to the closed-form expression:

$$\frac{b}{a} = \sum_{n=1}^{\infty} p_n = \frac{\Gamma - \Gamma^2\,\Gamma_L - \mathrm{T}^2\,\Gamma_L}{1-\Gamma^2}$$

Thus, the **input** reflection coefficient is:

$$\Gamma_{in} = \frac{b}{a} = \frac{\Gamma - \Gamma^2\,\Gamma_L - \mathrm{T}^2\,\Gamma_L}{1-\Gamma^2}$$

Using our definitions, it can likewise be shown that the **numerator** of the above expression is:

$$\Gamma - \Gamma^2\,\Gamma_L - \mathrm{T}^2\,\Gamma_L = \frac{2\left(Z_1^2 - Z_0\,R_L\right)}{\left(Z_1+Z_0\right)\left(R_L+Z_1\right)}$$

It is evident that the numerator (and therefore Γ) will be **zero if**:

$$Z_1^2 - Z_0\,R_L = 0 \qquad \Rightarrow \qquad Z_1 = \sqrt{Z_0\,R_L}$$

Just as we expected!

Physically, this results insures that all the reflected waves add coherently together to produce a **zero value**!

Note **all** of our transmission line analysis has been **steady-state** analysis. We assume our signals are **sinusoidal**, of the form $\exp(j\omega t)$. Note this signal exists for **all time** t—the signal is assumed to have been "on" **forever**, and assumed to continue on forever.

In other words, in steady-state analysis, **all** the multiple reflections have long since occurred, and thus have reached a steady state—the reflected wave is **zero**!