

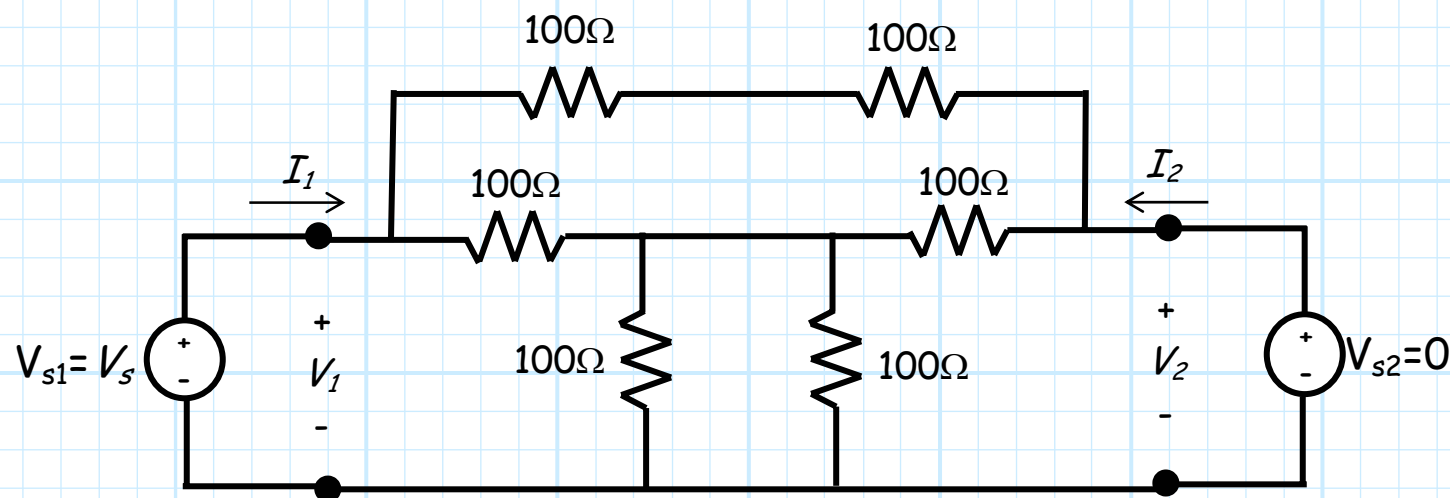
Odd/Even Mode Analysis

Q: *Although symmetric circuits appear to be plentiful in microwave engineering, it seems **unlikely** that we would often encounter symmetric **sources**. Do virtual shorts and opens typically ever occur?*

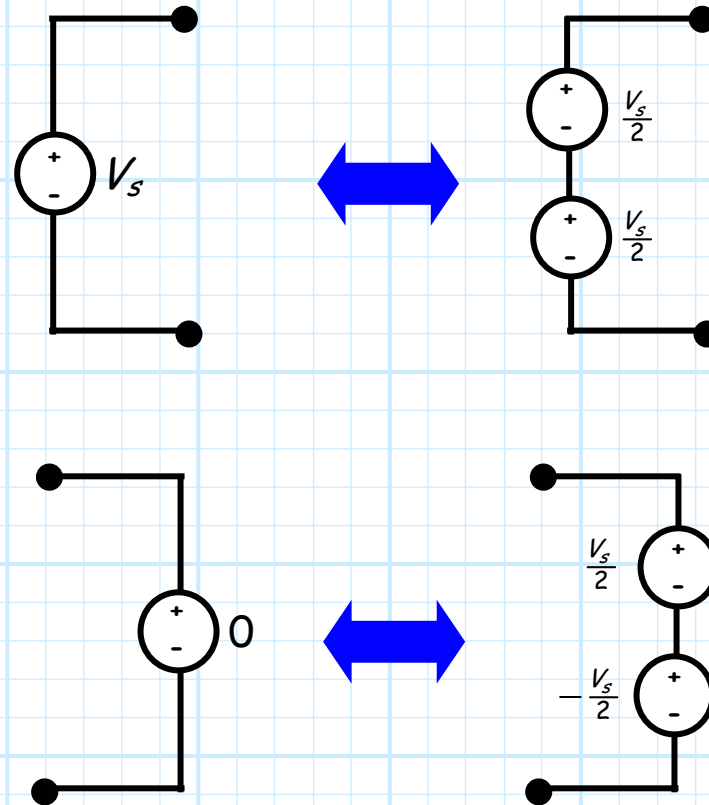
A: One word—**superposition!**

If the elements of our circuit are **independent** and **linear**, we can apply superposition to analyze **symmetric circuits** when **non-symmetric sources** are attached.

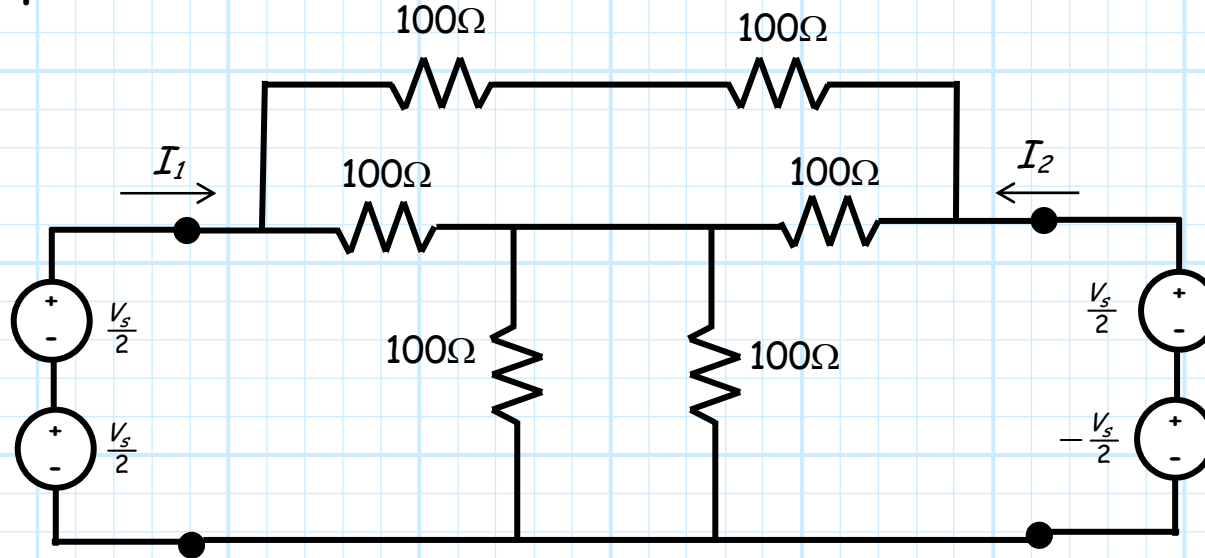
For example, say we wish to determine the **admittance matrix** of this circuit. We would place a **voltage source** at **port 1**, and a **short circuit** at **port 2**—a set of **asymmetric sources** if there ever was one!



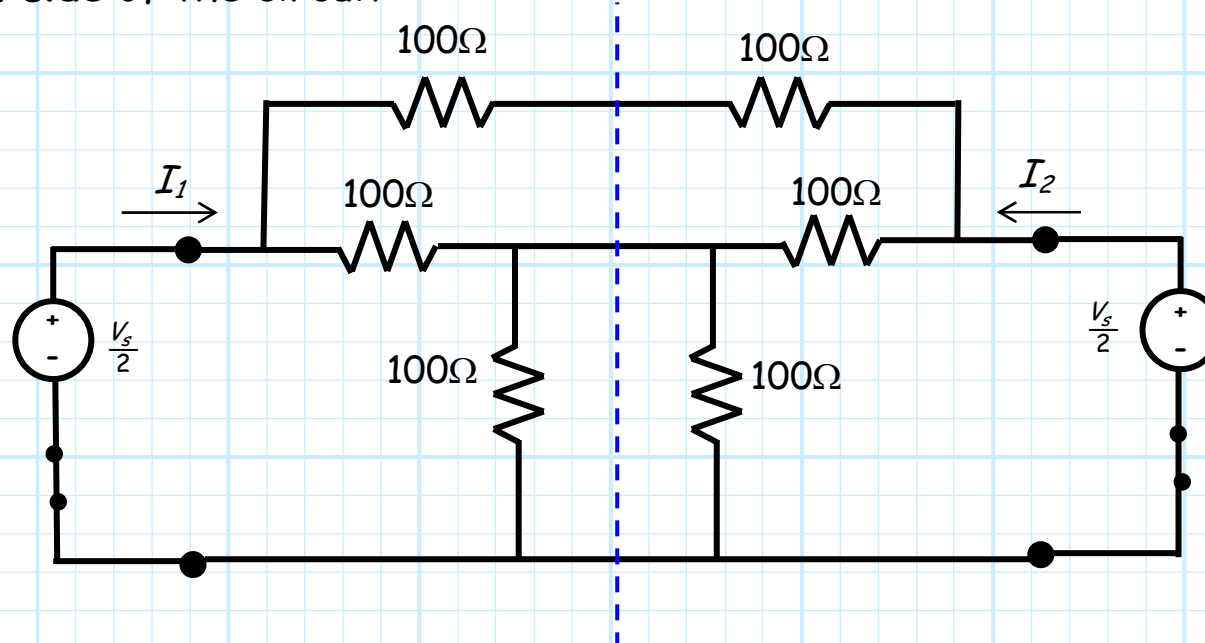
Here's the really **neat** part. We find that the source on port 1 can be model as **two equal** voltage sources in series, whereas the source at port 2 can be modeled as **two equal but opposite** sources in series.



Therefore an **equivalent** circuit is:



Now, the **above** circuit (due to the sources) is obviously **asymmetric**—no virtual ground, **nor** virtual short is present. But, let's say we **turn off** (i.e., set to $V=0$) the **bottom** source on **each side** of the circuit:

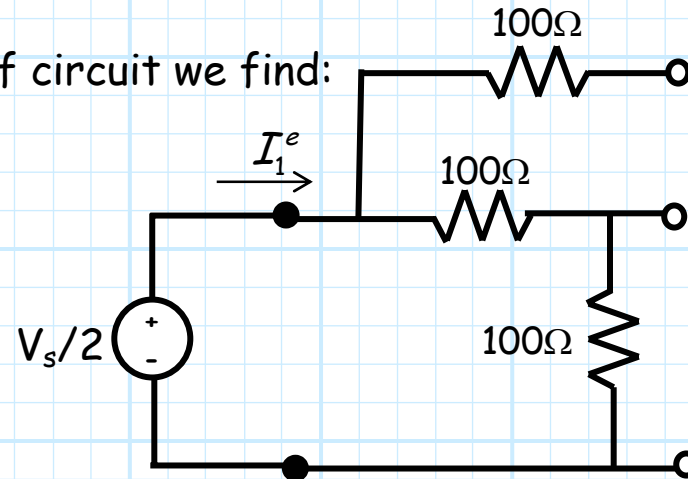


Our **symmetry** has been **restored!** The symmetry plane is a **virtual open**.

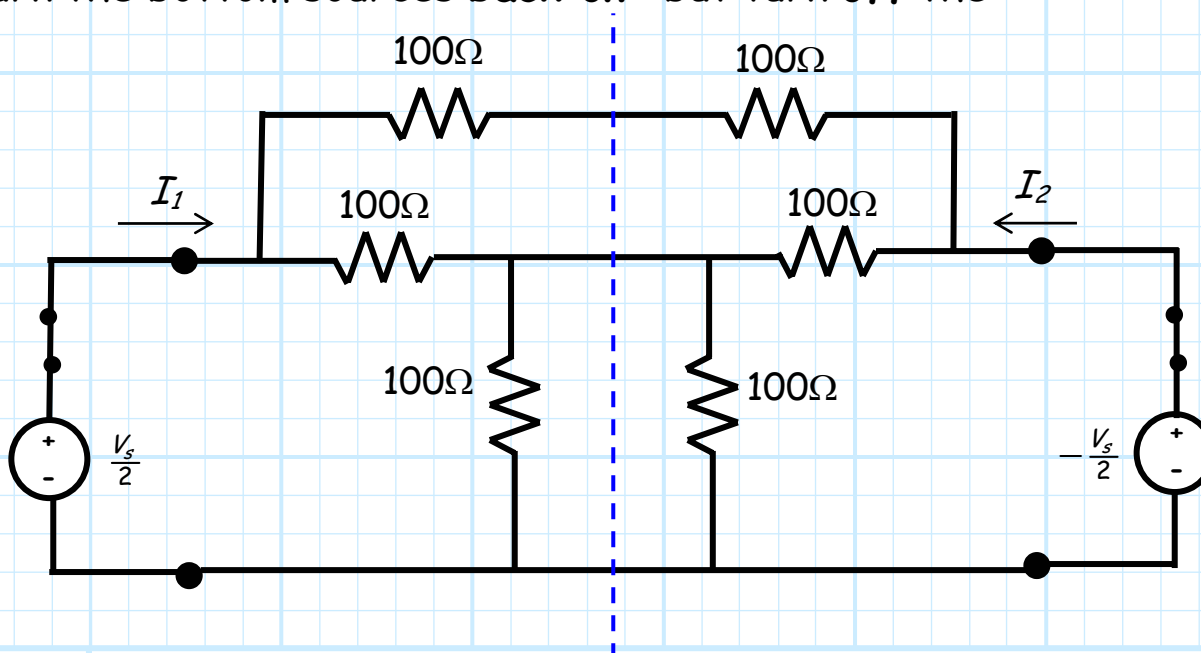
This circuit is referred to as its **even mode**, and analysis of it is known as the **even mode analysis**. The solutions are known as the even mode **currents** and **voltages!**

Evaluating the resulting **even mode** half circuit we find:

$$I_1^e = \frac{V_s}{2} \frac{1}{200} = \frac{V_s}{400} = I_2^e$$



Now, let's turn the bottom sources **back on**—but turn **off** the **top two!**

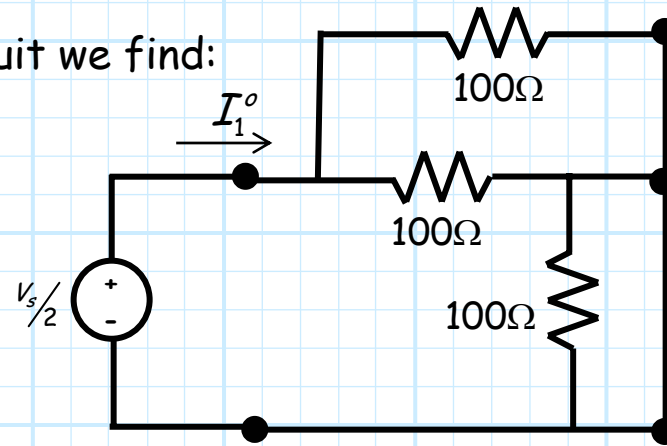


We now have a circuit with **odd symmetry**—the symmetry plane is a **virtual short!**

This circuit is referred to as its **odd mode**, and analysis of it is known as the **odd mode analysis**. The solutions are known as the odd mode **currents** and **voltages!**

Evaluating the resulting **odd mode** half circuit we find:

$$I_1^o = \frac{V_s}{2} \frac{1}{50} = \frac{V_s}{100} = -I_2^o$$

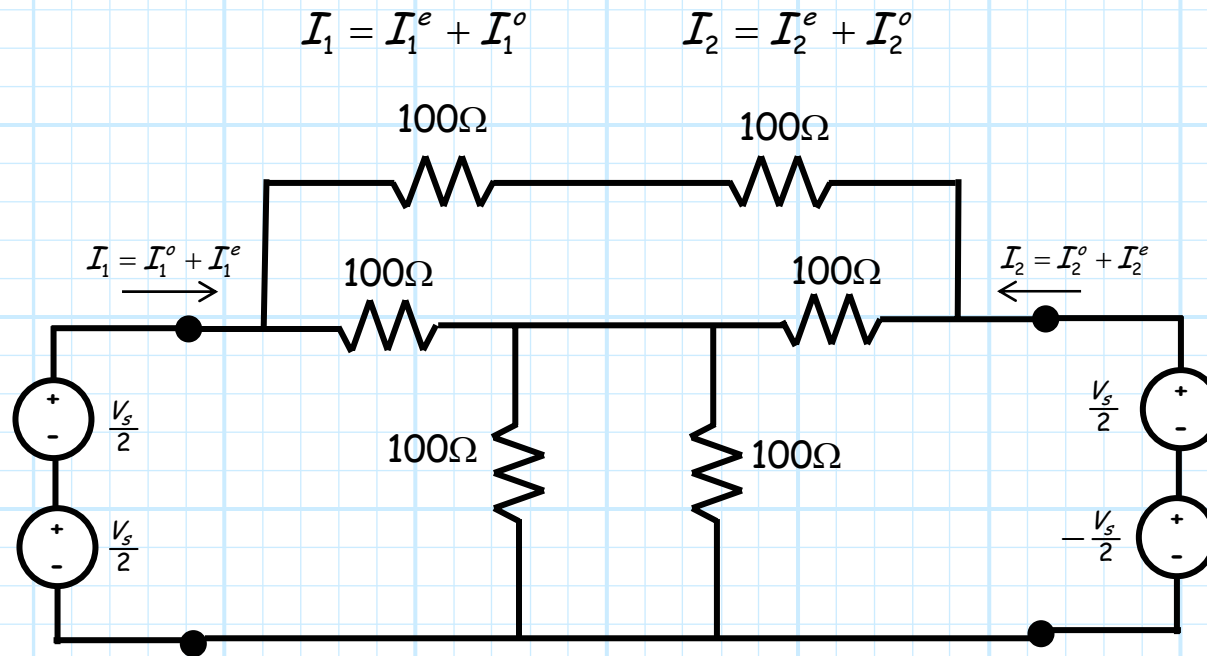


Q: *But what good is this "even mode" and "odd mode" analysis? After all, the source on port 1 is $V_{s1} = V_s$, and the source on port 2 is $V_{s2} = 0$. What are the currents I_1 and I_2 for these sources?*

A: Recall that these sources are the **sum** of the even and odd mode sources:

$$V_{s1} = V_s = \frac{V_s}{2} + \frac{V_s}{2} \quad V_{s2} = 0 = \frac{V_s}{2} - \frac{V_s}{2}$$

and thus—since all the devices in the circuit are **linear**—we know from superposition that the currents I_1 and I_2 are simply the **sum** of the **odd** and **even** mode currents!



Thus, **adding** the odd and even mode analysis results together:

$$\begin{aligned}
 I_1 &= I_1^e + I_1^o \\
 &= \frac{V_s}{400} + \frac{V_s}{100} \\
 &= \frac{V_s}{80}
 \end{aligned}$$

$$\begin{aligned}
 I_2 &= I_2^e + I_2^o \\
 &= \frac{V_s}{400} - \frac{V_s}{100} \\
 &= -\frac{3V_s}{400}
 \end{aligned}$$

And then the **admittance parameters** for this two port network is:

$$Y_{11} = \left. \frac{I_1}{V_{s1}} \right|_{V_{s2}=0} = \frac{V_s}{80} \frac{1}{V_s} = \frac{1}{80}$$

$$Y_{21} = \left. \frac{I_2}{V_{s1}} \right|_{V_{s2}=0} = -\frac{3V_s}{400} \frac{1}{V_s} = -\frac{3}{400}$$

And from the **symmetry** of the device we know:

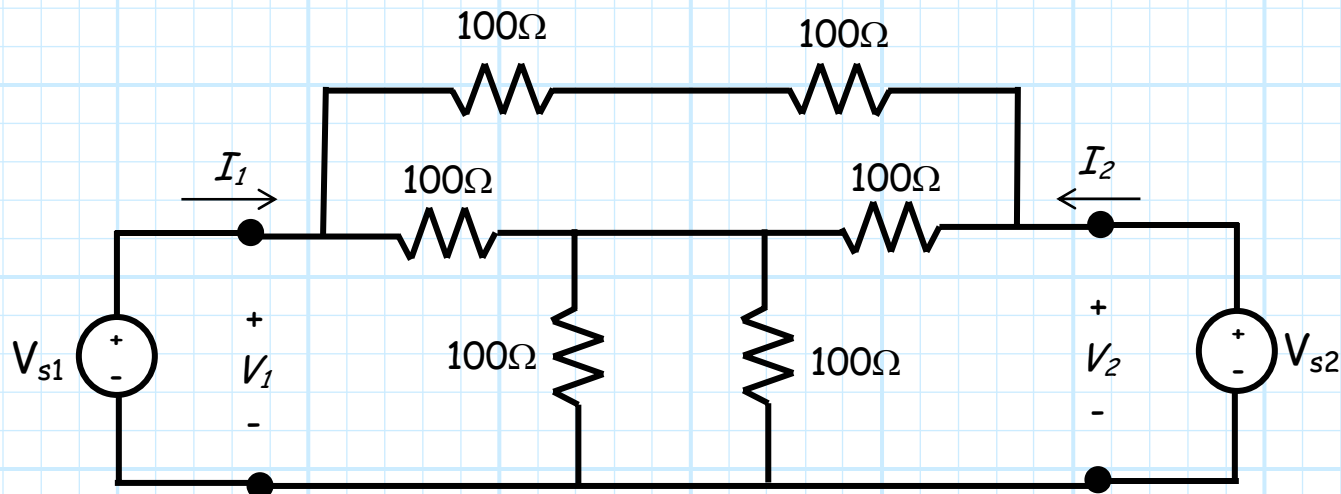
$$Y_{22} = Y_{11} = \frac{1}{80} \qquad Y_{12} = Y_{21} = \frac{-3}{400}$$

Thus, the full **admittance matrix** is:

$$\mathbf{y} = \begin{bmatrix} \frac{1}{80} & -\frac{3}{400} \\ -\frac{3}{400} & \frac{1}{80} \end{bmatrix}$$

Q: What happens if **both** sources are **non-zero**? Can we use symmetry then?

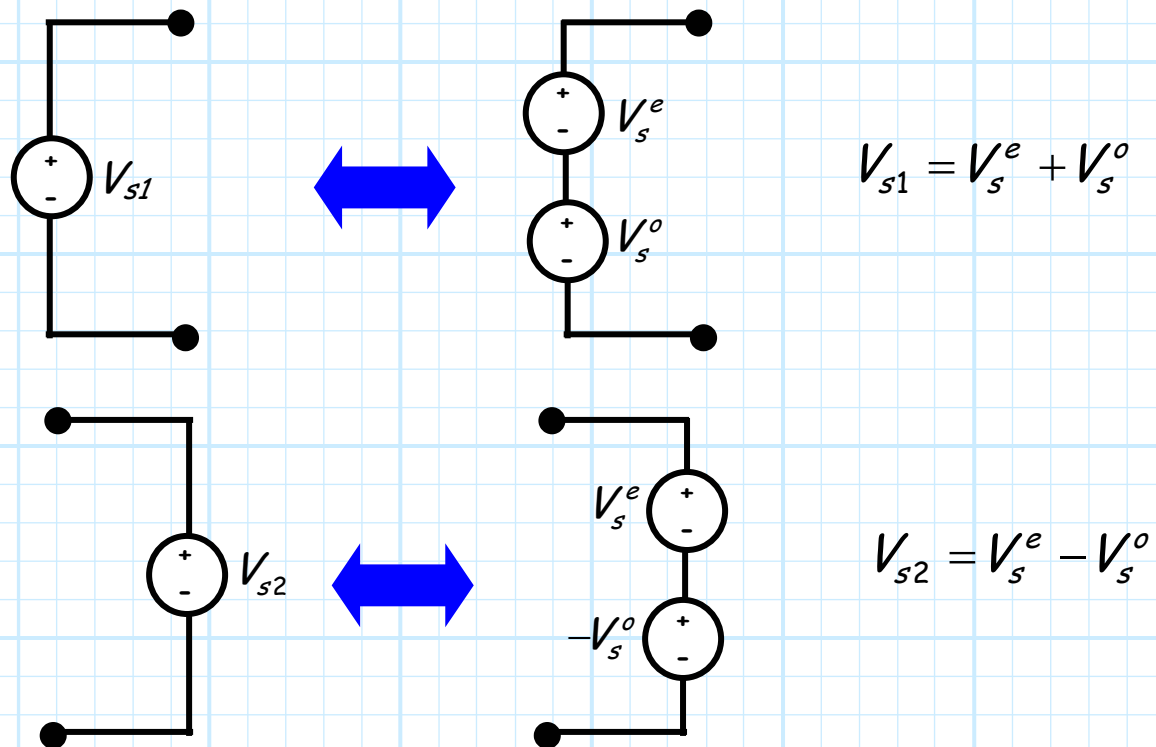
A: Absolutely! Consider the problem below, where **neither** source is equal to zero:



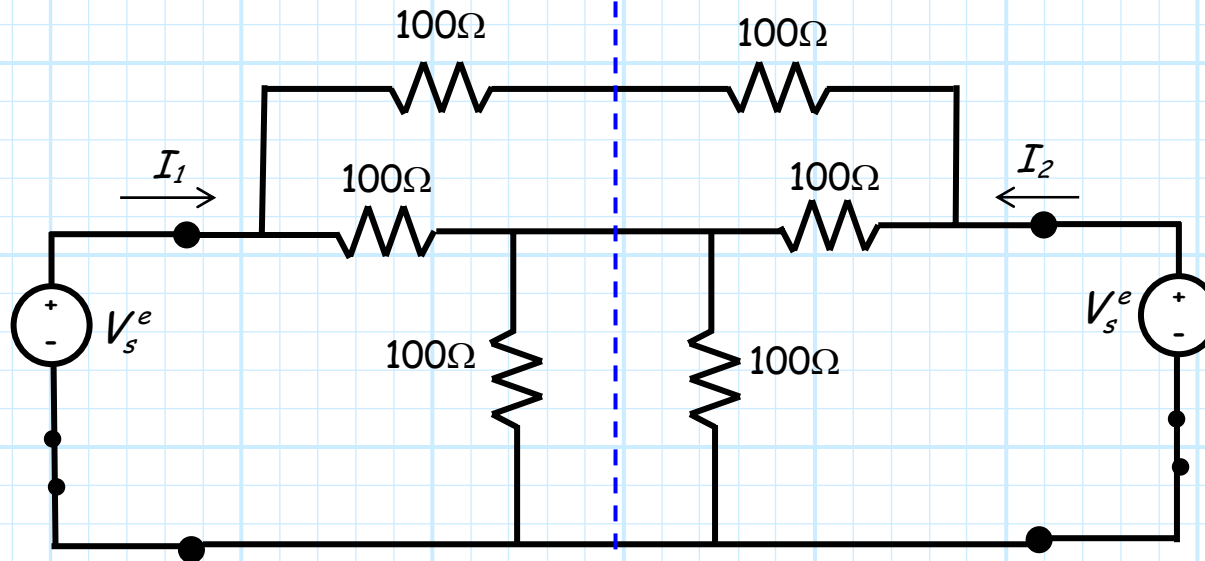
In this case we can define an even mode and an odd mode source as:

$$V_s^e = \frac{V_{s1} + V_{s2}}{2}$$

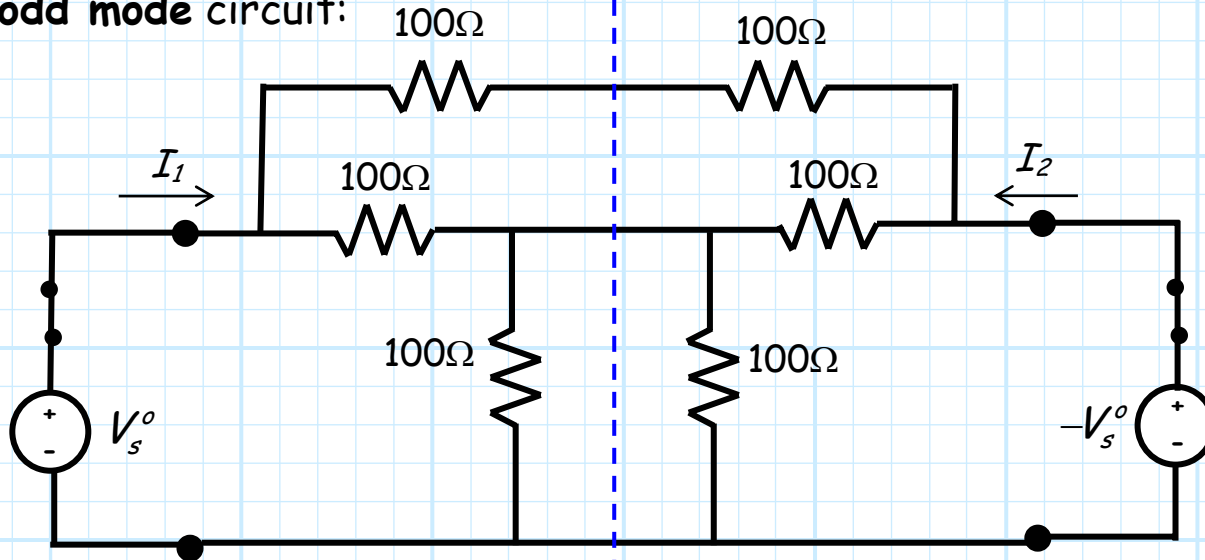
$$V_s^o = \frac{V_{s1} - V_{s2}}{2}$$



We then can analyze the **even mode** circuit:



And then the **odd mode** circuit:

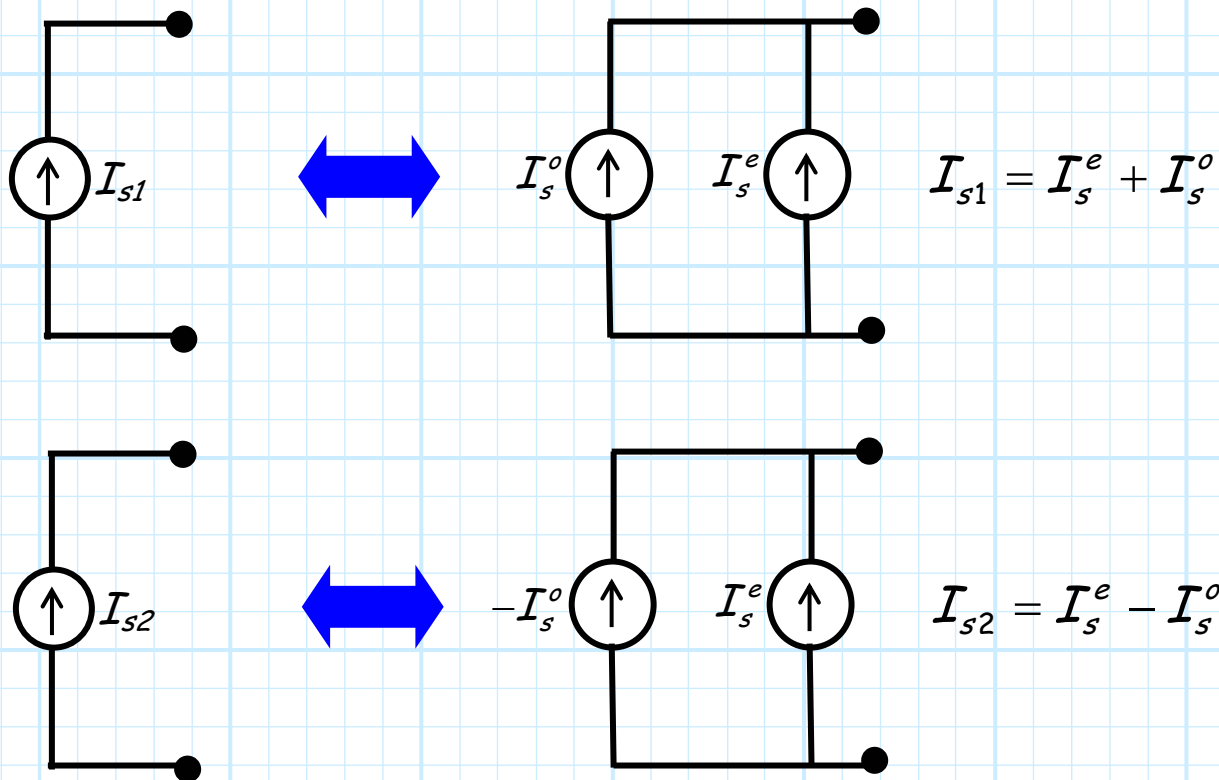


And then combine these results in a **linear superposition!**

Q: What about **current sources**? Can I likewise consider them to be a **sum of an odd mode source and an even mode source**?

A: Yes, but be **very** careful! The current of two source will add if they are placed in **parallel**—not in series! Therefore:

$$I_s^e = \frac{I_{s1} + I_{s2}}{2} \qquad I_s^o = \frac{I_{s1} - I_{s2}}{2}$$



One **final** word (I promise!) about circuit symmetry and even/odd mode analysis: **precisely** the **same** concept exists in **electronic circuit design!**

Specifically, the **differential** (odd) and **common** (even) mode analysis of bilaterally symmetric electronic circuits, such as **differential amplifiers!**



Hi! You might remember differential and common mode analysis from such classes as "EECS 412- Electronics II", or handouts such as "Differential Mode Small-Signal Analysis of BJT Differential Pairs"

