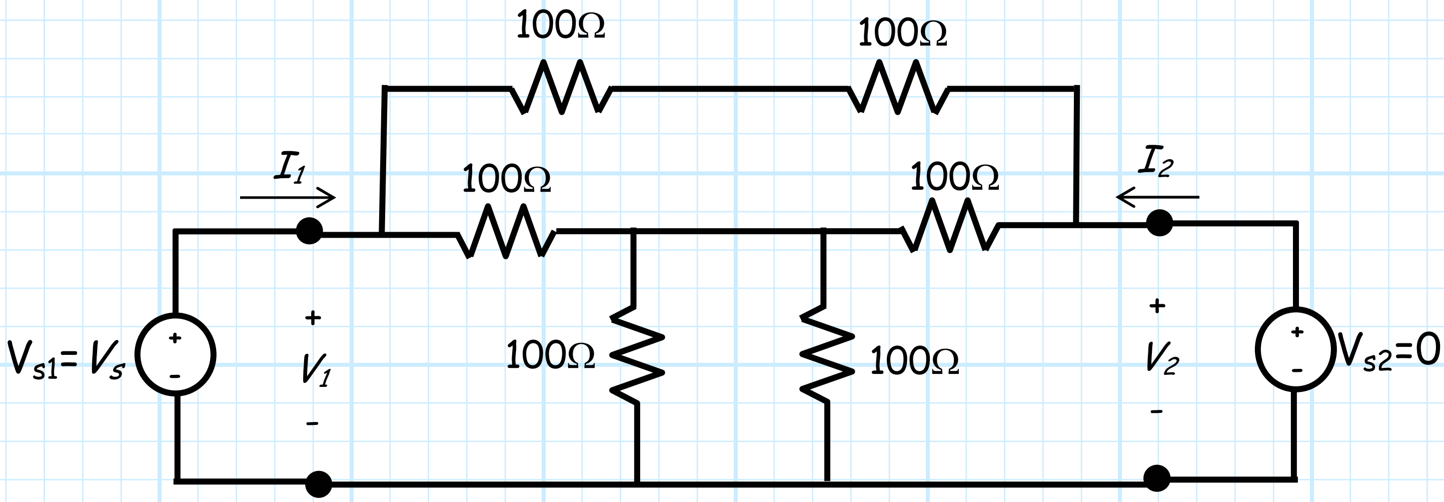
Odd/Even Mode Analysis

Q: *Although symmetric **circuits** appear to be plentiful in microwave engineering, it seems **unlikely** that we would often encounter symmetric **sources**. Do virtual shorts and opens typically ever occur?*

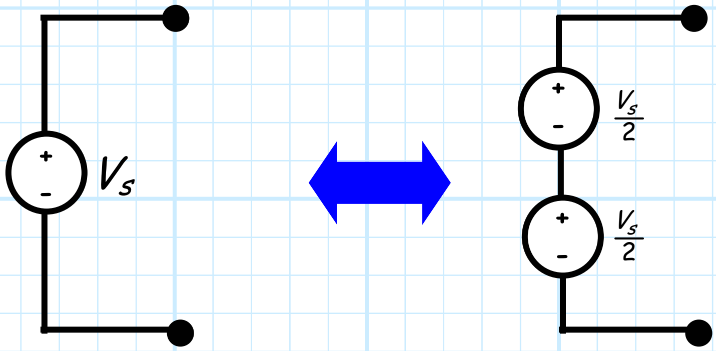
A: One word—**superposition**!

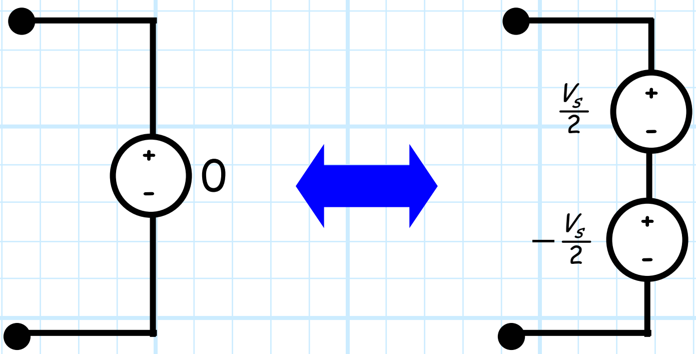
If the elements of our circuit are **independent** and **linear**, we can apply superposition to analyze **symmetric circuits** when **non-symmetric** sources are attached.

For example, say we wish to determine the **admittance matrix** of this circuit. We would place a **voltage source at port 1**, and a **short circuit at port 2**—a set of **asymmetric** sources if there ever was one!

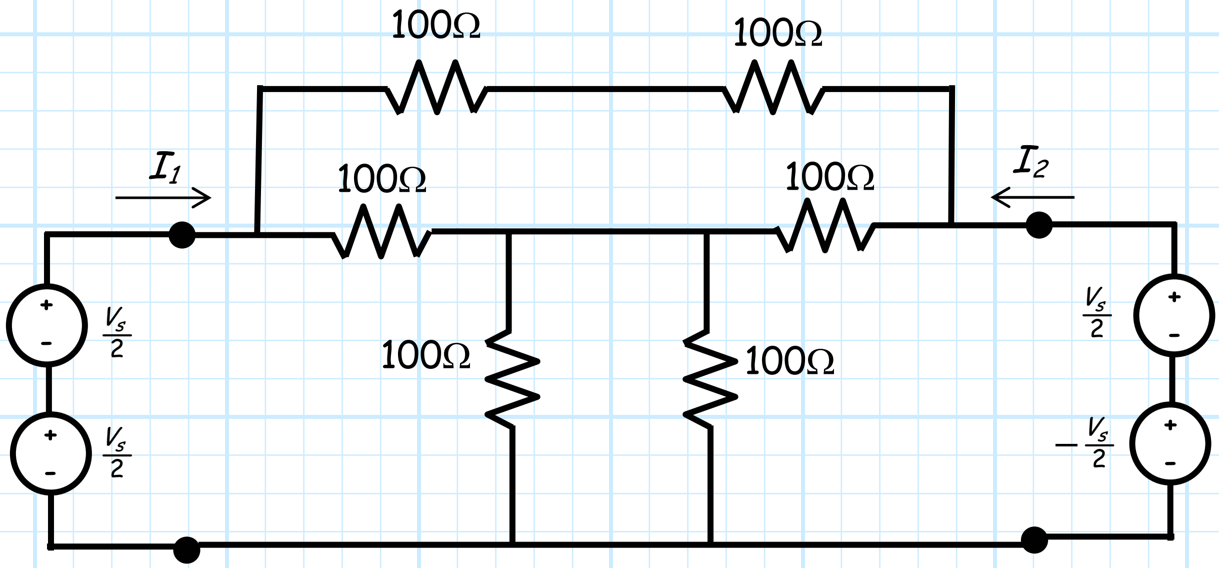


Here's the really **neat** part. We find that the source on port 1 can be model as **two equal** voltage sources in series, whereas the source at port 2 can be modeled as **two equal but opposite** sources in series.

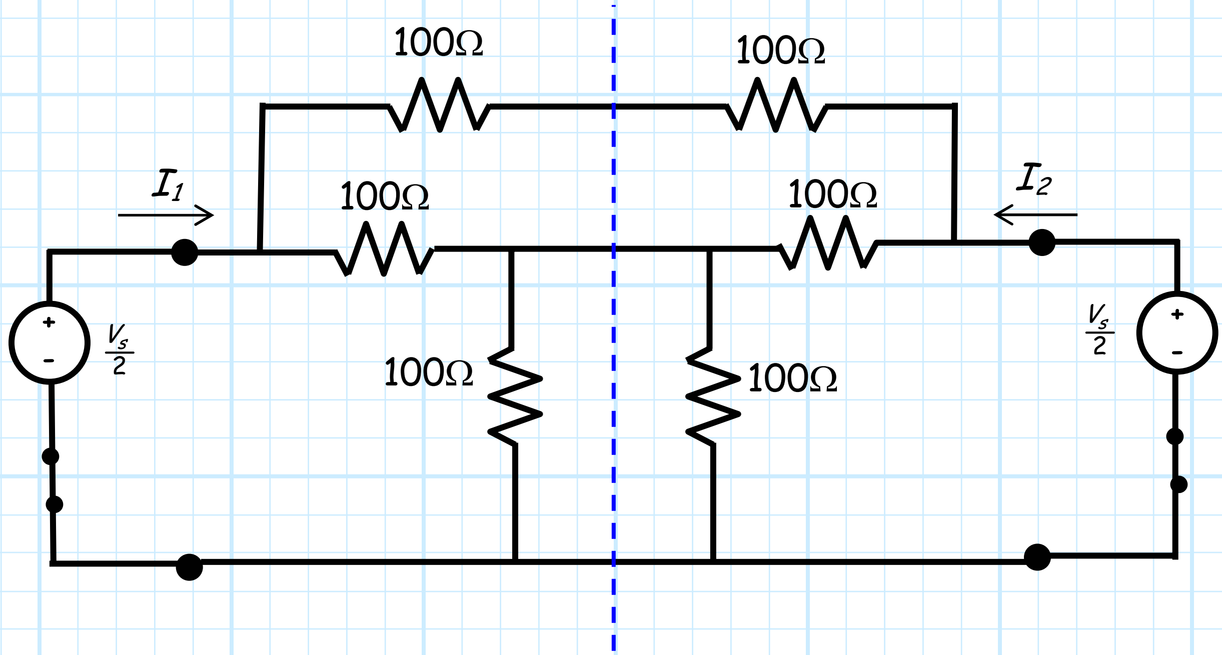




Therefore an **equivalent** circuit is:



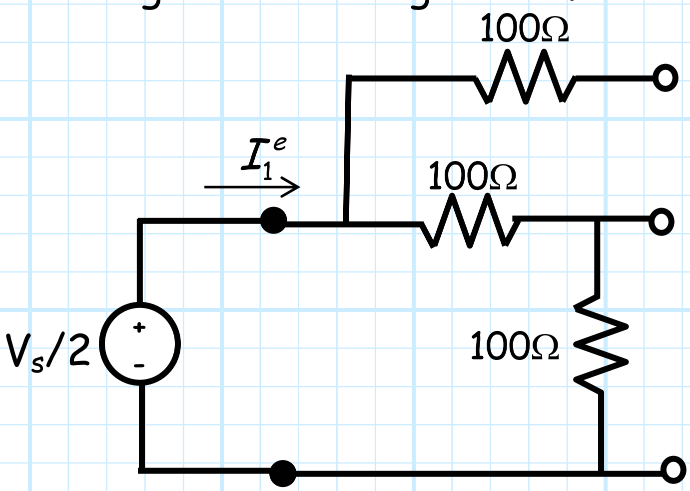
Now, the above circuit (due to the sources) is obviously **asymmetric**—**no** virtual ground, **nor** virtual short is present. But, let's say we **turn off** (i.e., set to V=0) the **bottom** source on **each side** of the circuit:



Our **symmetry** has been **restored**! The symmetry plane is a **virtual open**.

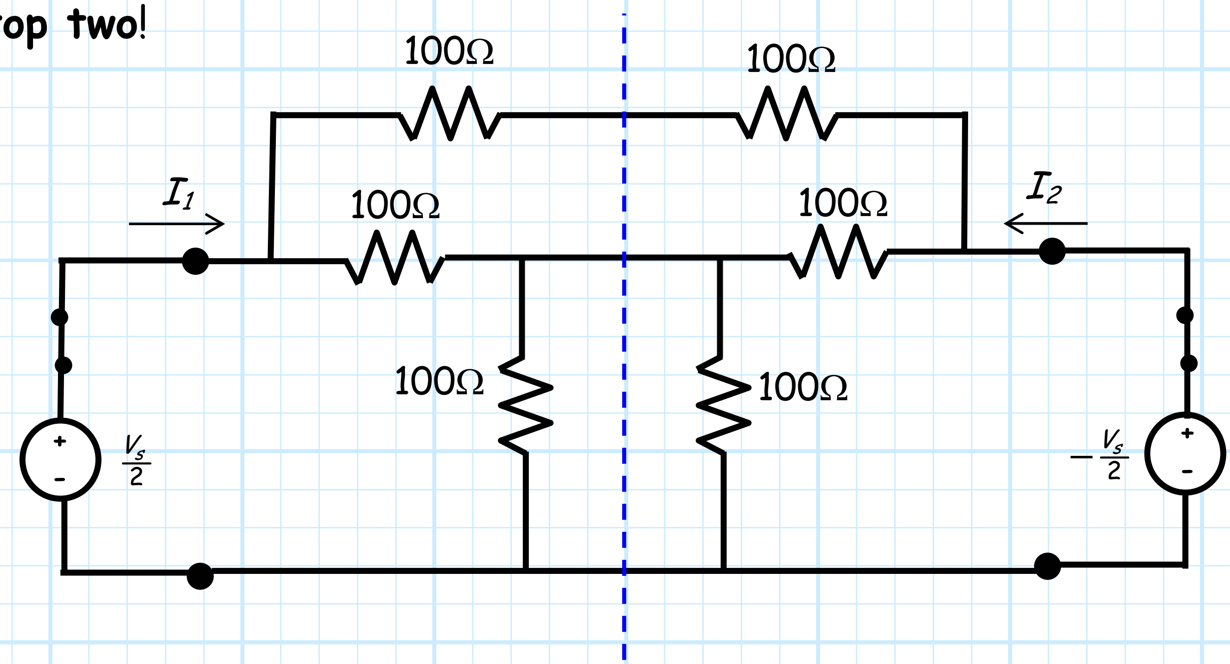
This circuit is referred to as its **even mode**, and analysis of it is known as the **even mode analysis**. The solutions are known as the even mode **currents** and **voltages**!

Evaluating the resulting **even mode** half circuit we find:



$$I_1^e = \frac{V_s}{2}\frac{1}{200} = \frac{V_s}{400} = I_2^e$$

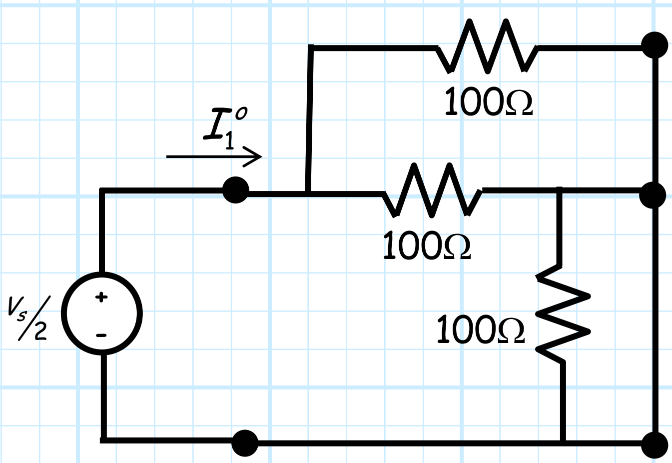
Now, let's turn the bottom sources **back on**—but turn **off** the **top two**!



We now have a circuit with **odd symmetry**—the symmetry plane is a **virtual short**!

This circuit is referred to as its **odd mode**, and analysis of it is known as the **odd mode analysis**. The solutions are known as the odd mode **currents** and **voltages**!

Evaluating the resulting **odd mode** half circuit we find:



$$I_1^o = \frac{V_s}{2}\frac{1}{50} = \frac{V_s}{100} = -I_2^o$$

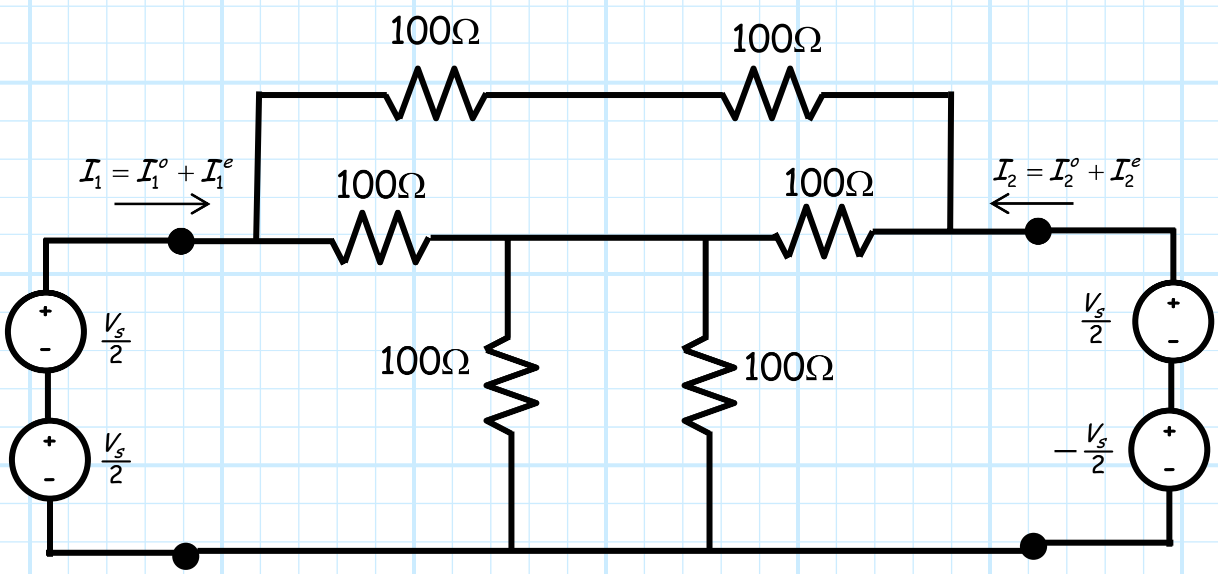
Q: *But what good is this "even mode" and "odd mode" analysis? After all, the source on port 1 is $V_{s1} = V_s$, and the source on port 2 is $V_{s2} = 0$. What are the currents I_1 and I_2 for* ***these*** *sources?*

A: Recall that these sources are the **sum** of the even and odd mode sources:

$$V_{s1} = V_s = \frac{V_s}{2} + \frac{V_s}{2} \qquad\qquad V_{s2} = 0 = \frac{V_s}{2} - \frac{V_s}{2}$$

and thus—since all the devices in the circuit are **linear**—we know from superposition that the currents I_1 and I_2 are simply the **sum** of the **odd** and **even** mode currents !

$$I_1 = I_1^e + I_1^o \qquad\qquad I_2 = I_2^e + I_2^o$$



Thus, **adding** the odd and even mode analysis results together:

$$I_1 = I_1^e + I_1^o = \frac{V_s}{400} + \frac{V_s}{100} = \frac{V_s}{80}$$

$$I_2 = I_2^e + I_2^o = \frac{V_s}{400} - \frac{V_s}{100} = -\frac{3\,V_s}{400}$$

And then the **admittance parameters** for this two port network is:

$$Y_{11} = \left.\frac{I_1}{V_{s1}}\right|_{V_{s2}=0} = \frac{V_s}{80}\frac{1}{V_s} = \frac{1}{80}$$

$$Y_{21} = \left.\frac{I_2}{V_{s1}}\right|_{V_{s2}=0} = -\frac{3\,V_s}{400}\frac{1}{V_s} = \frac{-3}{400}$$

And from the **symmetry** of the device we know:

$$Y_{22} = Y_{11} = \frac{1}{80}$$

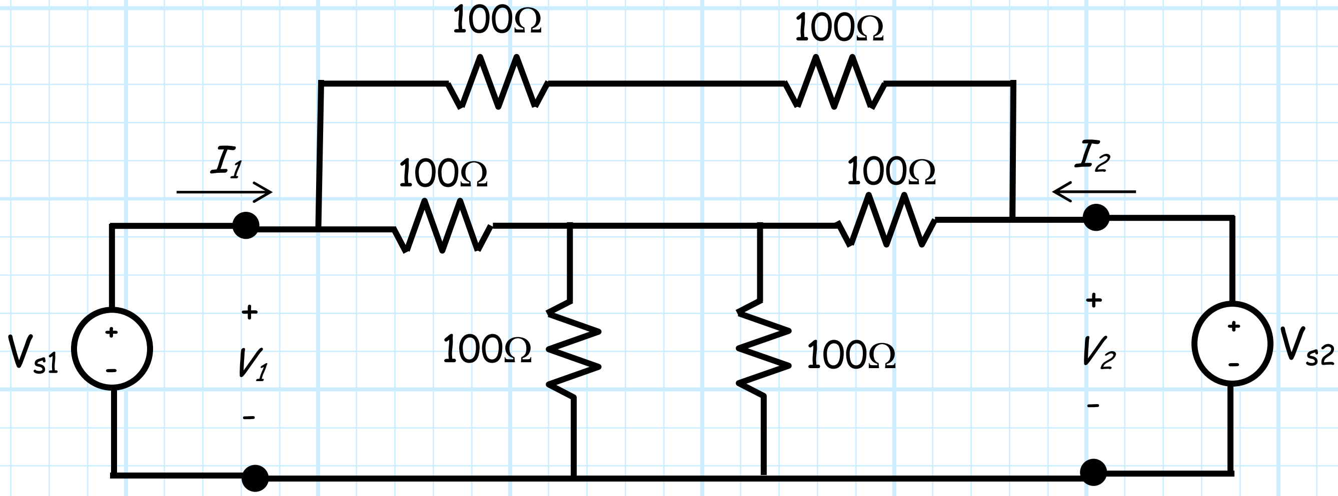
$$Y_{12} = Y_{21} = \frac{-3}{400}$$

Thus, the full **admittance matrix** is:

$$\mathcal{Y} = \begin{bmatrix} 1/80 & -3/400 \\ -3/400 & 1/80 \end{bmatrix}$$

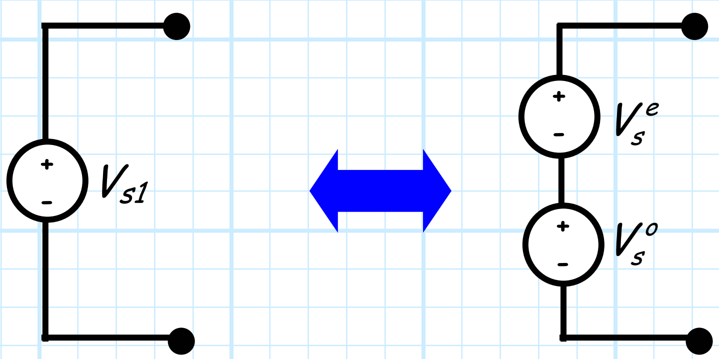
Q: *What happens if both sources are non-zero? Can we use symmetry then?*

A: Absolutely! Consider the problem below, where neither source is equal to zero:

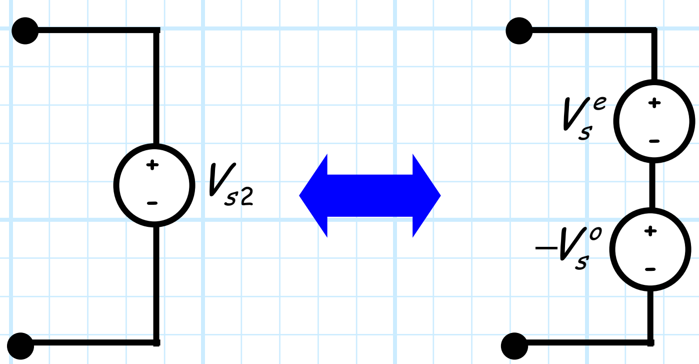


In this case we can define an even mode and an odd mode source as:

$$V_s^e = \frac{V_{s1} + V_{s2}}{2} \qquad\qquad V_s^o = \frac{V_{s1} - V_{s2}}{2}$$

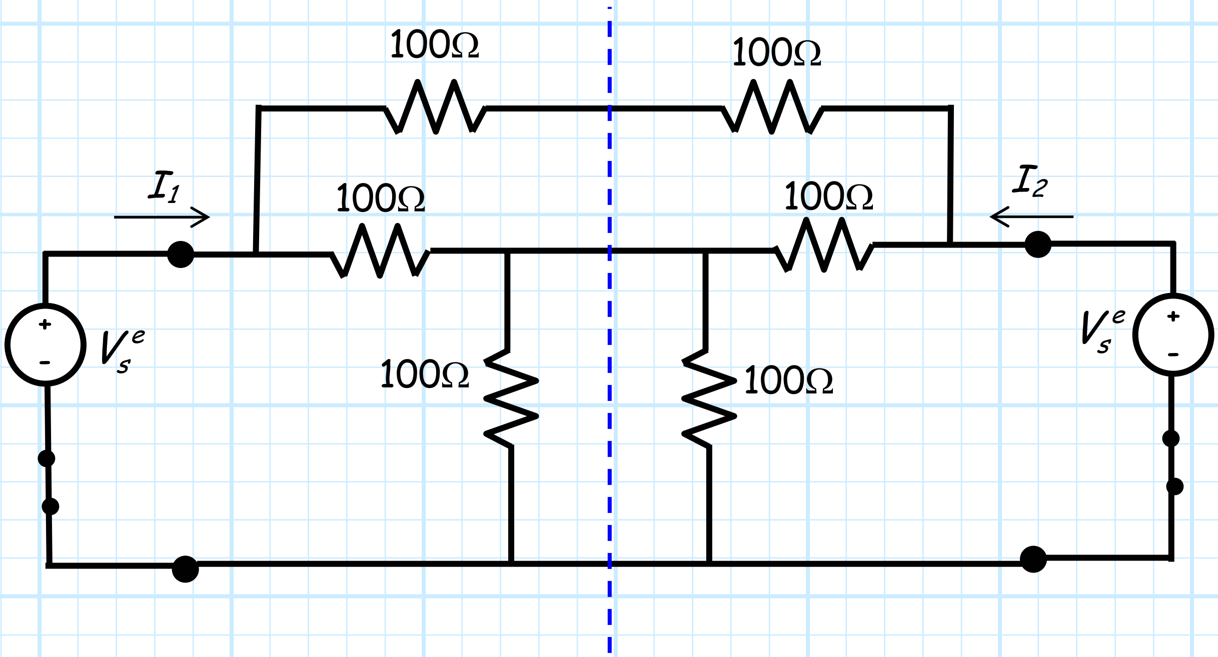


$$V_{s1} = V_s^e + V_s^o$$

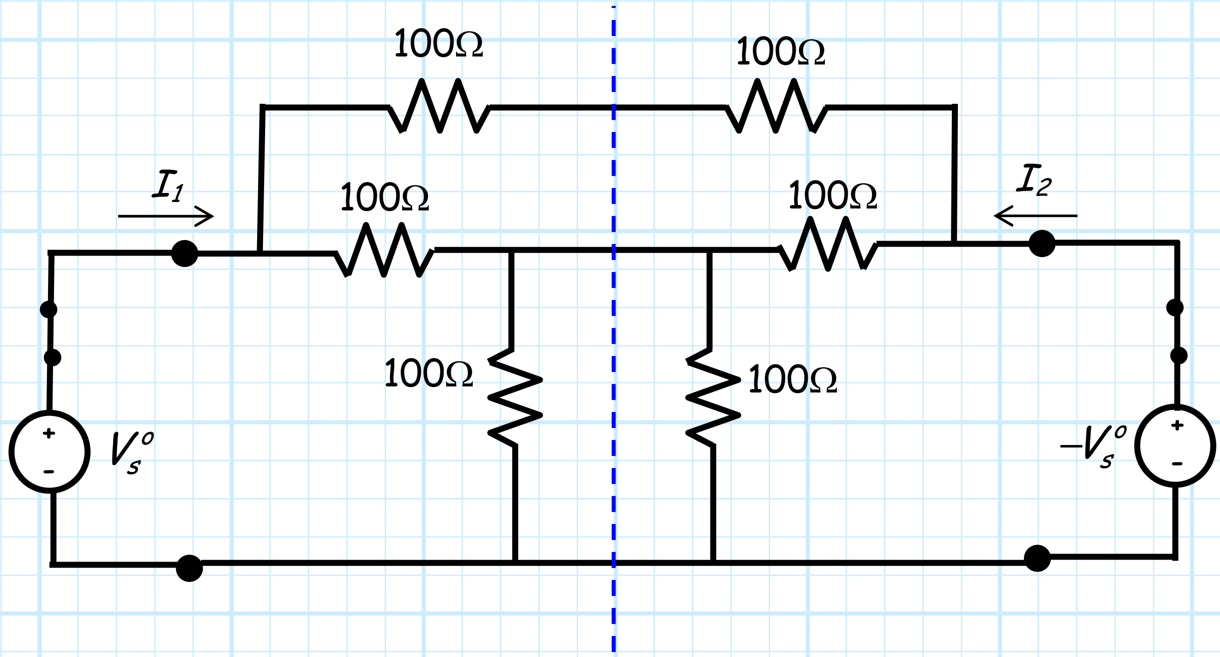


$$V_{s2} = V_s^e - V_s^o$$

We then can analyze the **even mode** circuit:



And then the **odd mode** circuit:



And then combine these results in a **linear superposition**!

One **final** word (I promise!) about circuit symmetry and even/odd mode analysis: **precisely** the **same** concept exits in **electronic circuit** design!

Specifically, the **differential** (odd) and **common** (even) **mode** analysis of bilaterally symmetric electronic circuits, such as **differential amplifiers**!

*Hi! You might remember differential and common mode analysis from such classes as "**EECS 412**- Electronics II", or handouts such as "**Differential Mode** Small-Signal Analysis of **BJT Differential Pairs**"*