Parallel Rule

Consider the complex equation:

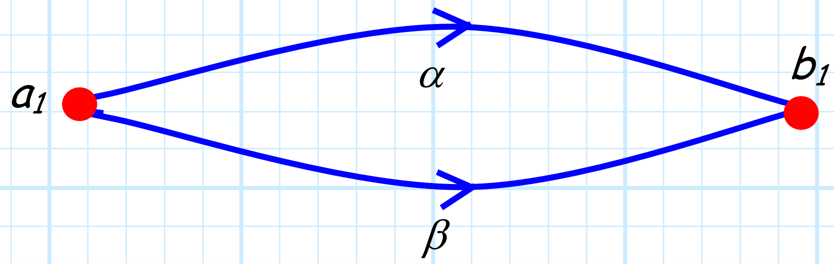
$$b_1 = \alpha\, a_1 + \beta\, a_1$$

where α and β are **arbitrary** complex constants. Using the **distributive property**, the equation can equivalently be expressed as:

$$b_1 = (\alpha + \beta)\, a_1$$

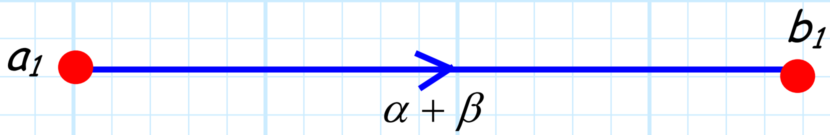
Now let's express these two equations as **signal flow graphs**!

The first is:



$$b_1 = \alpha\, a_1 + \beta\, a_1$$

With the second:



$$b_1 = (\alpha + \beta)\, a_1$$

Q: *Hey wait! If the two equations are* ***equivalent****, shouldn't the two resulting signal flow graphs* ***likewise*** *be equivalent?*

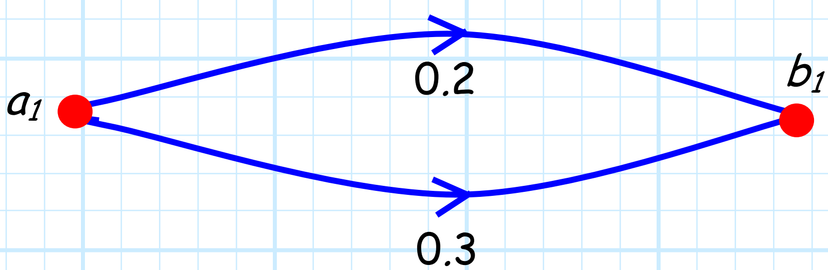
A: Absolutely! The two signal flow graphs are indeed **equivalent**.

This leads us to our **second** signal flow graph reduction rule:

Rule 2 - Parallel Rule

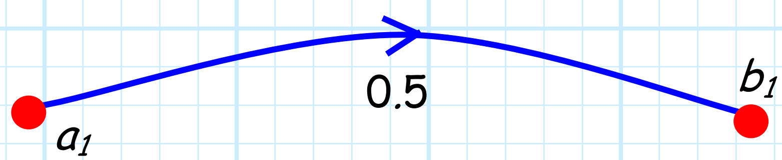
If two nodes are connected by parallel branches—and the branches have the ***same direction****—the branches can be combined into a single branch, with a value equal to the* ***sum*** *of each two original branches.*

For example, the graph:



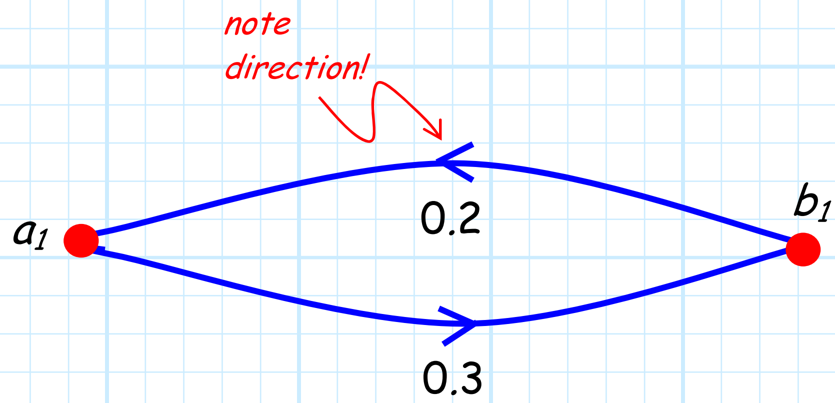
$$b_1 = 0.3\, a_1 + 0.2\, a_1$$

Can be reduced to:

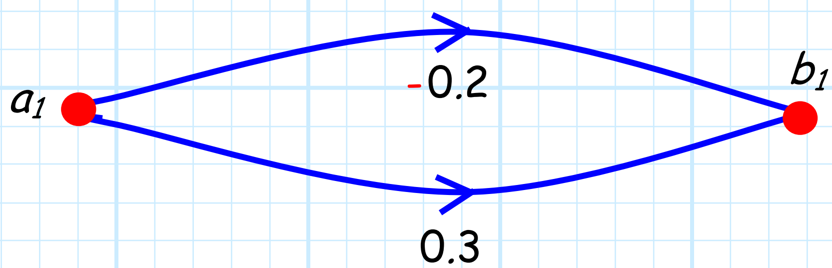


$$\begin{aligned} b_1 &= (0.3 + 0.2)\, a_1 \\ &= 0.5\, a_1 \end{aligned}$$

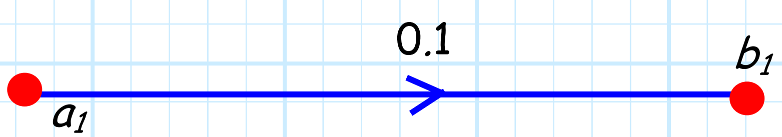
Q: *What about **this** signal flow graph?*



***Can** I rewrite this as:*



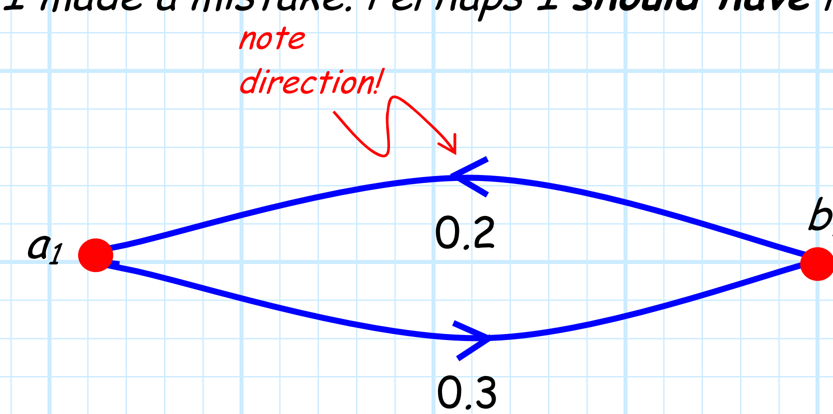
so that (since 0.3-0.2=0.1):



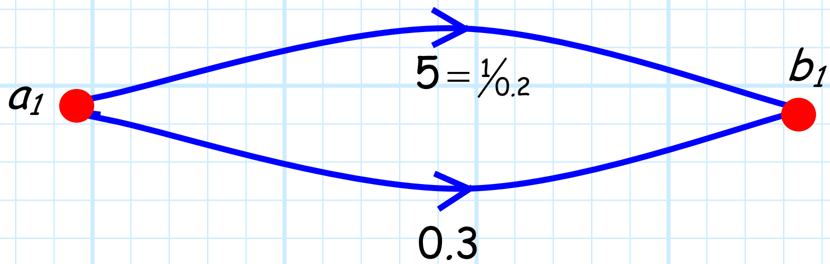
???

A: Absolutely not! **NEVER DO THIS!!**

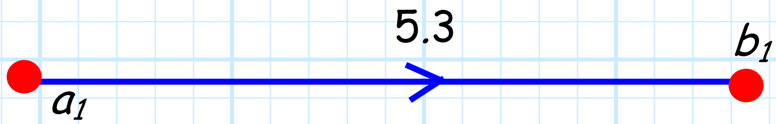
Q: *Maybe I made a mistake. Perhaps I **should have** rewritten:*



as this:



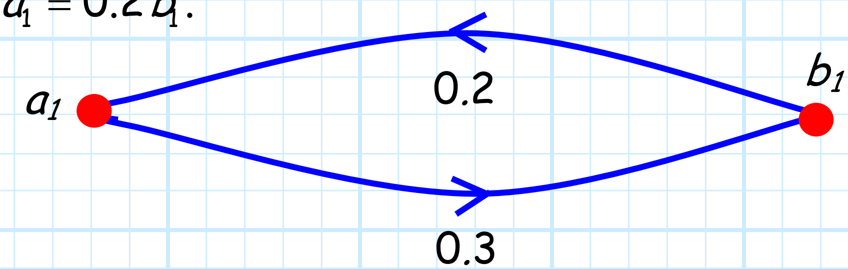
so that (since 5.0+0.3=5.3):



???

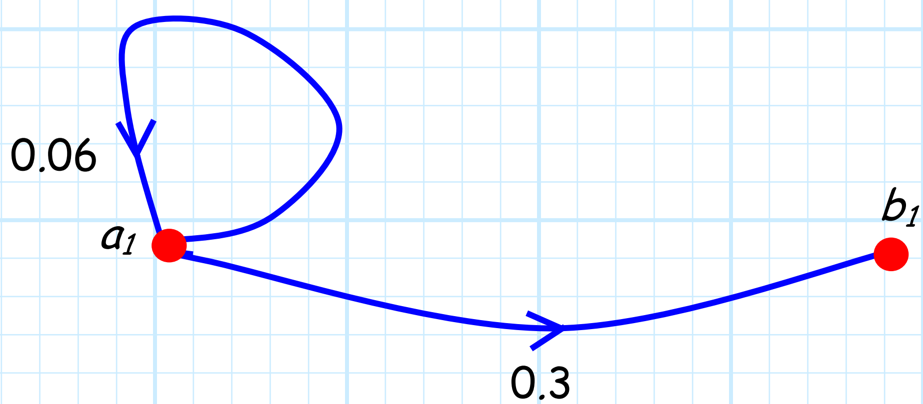
A: Absolutely not! **NEVER DO THIS EITHER!!**

From the signal flow graph below, we can **only** conclude that $b_1 = 0.3\,a_1$ and $a_1 = 0.2\,b_1$.



Using the **series rule** (or little bit of algebra), we can conclude that an **equivalent** signal flow graph to this is:

$$a_1 = 0.06\,a_1$$
$$b_1 = 0.3\,a_1$$



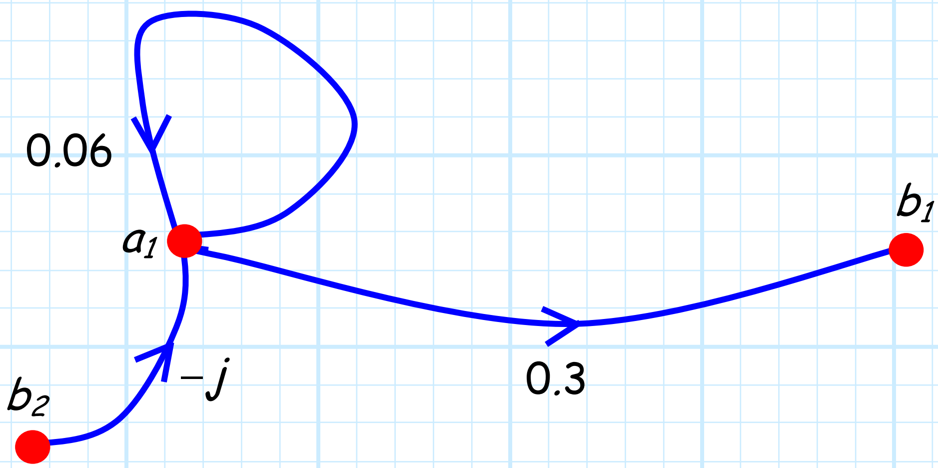
Q: *Yikes! What kind of* ***goofy*** *branch begins and ends at the same node?*

A: Branches that begin and end at the same node are called **self-loops**.

Q: *Do these self-loops actually **appear** in signal flow graphs?*

A: **Yes**, but the self-loop node will **always** have at least **one other incoming branch**. For example:

$$a_1 = 0.06\,a_1 - j\,b_2$$
$$b_1 = 0.3\,a_1$$



Q: *But how do we **reduce** a signal flow graph containing a **self-loop**?*

A: See **rule 3** !