Power Flow and Return Loss

We have discovered that two waves propagate along a transmission line, one in each direction ($V^+(z)$ and $V^-(z)$).

\[
\begin{align*}
I(z) &= \frac{V_0^+}{Z_0} \left[ e^{-j\beta z} - \Gamma_L e^{+j\beta z} \right] \\
V(z) &= V_0^+ \left[ e^{-j\beta z} + \Gamma_L e^{+j\beta z} \right]
\end{align*}
\]

The result is that electromagnetic energy flows along the transmission line at a given rate (i.e., power).

**Q:** How much power flows along a transmission line, and where does that power go?

**A:** We can answer that question by determining the power absorbed by the load!
The time average power absorbed by an impedance $Z_L$ is:

$$\rho_{abs} = \frac{1}{2} \text{Re}\{V_L I_L^*\}$$

$$= \frac{1}{2} \text{Re}\{V(z = 0) I(z = 0)^*\}$$

$$= \frac{1}{2Z_0} \text{Re}\left\{ \left( V_0^* \left[ e^{-j\beta_0} + \Gamma_L e^{+j\beta_0} \right] \right) \left( V_0^* \left[ e^{-j\beta_0} - \Gamma_L e^{+j\beta_0} \right] \right)^* \right\}$$

$$= \frac{|V_0^*|^2}{2Z_0} \text{Re}\left\{ 1 - \left( \Gamma_L^* - \Gamma_L \right) - |\Gamma_L|^2 \right\}$$

$$= \frac{|V_0^*|^2}{2Z_0} \left( 1 - |\Gamma_L|^2 \right)$$

The significance of this result can be seen by rewriting the expression as:

$$\rho_{abs} = \frac{|V_0^*|^2}{2Z_0} \left( 1 - |\Gamma_L|^2 \right)$$

$$= \frac{|V_0^*|^2}{2Z_0} - \frac{|V_0^*\Gamma_L|^2}{2Z_0}$$

$$= \frac{|V_0^*|^2}{2Z_0} - \frac{|V_0^*|^2}{2Z_0}$$

The two terms in above expression have a very definite physical meaning. The first term is the time-averaged power of the wave propagating along the transmission line toward the load.
We say that this wave is incident on the load:

\[ P_{inc} = P_+ = \frac{|V_0^+|^2}{2Z_0} \]

Likewise, the second term of the \( P_{abs} \) equation describes the power of the wave moving in the other direction (away from the load). We refer to this as the wave reflected from the load:

\[ P_{ref} = P_- = \frac{|V_0^-|^2}{2Z_0} = \frac{|\Gamma_L|^2 |V_0^+|^2}{2Z_0} = |\Gamma_L|^2 P_{inc} \]

Thus, the power absorbed by the load is simply:

\[ P_{abs} = P_{inc} - P_{ref} \]

or, rearranging, we find:

\[ P_{inc} = P_{abs} + P_{ref} \]

This equation is simply an expression of the conservation of energy!

It says that power flowing toward the load (\( P_{inc} \)) is either absorbed by the load (\( P_{abs} \)) or reflected back from the load (\( P_{ref} \)).
Note that if \( |\Gamma_L|^2 = 1 \), then \( P_{\text{inc}} = P_{\text{ref}} \), and therefore no power is absorbed by the load.

This of course makes sense!

The magnitude of the reflection coefficient (\( |\Gamma_L| \)) is equal to one only when the load impedance is purely reactive (i.e., purely imaginary).

Of course, a purely reactive element (e.g., capacitor or inductor) cannot absorb any power—all the power must be reflected!

**Return Loss**

The ratio of the reflected power to the incident power is known as return loss. Typically, return loss is expressed in dB:

\[
R.L. = -10 \log_{10} \left( \frac{P_{\text{ref}}}{P_{\text{inc}}} \right) = -10 \log_{10} |\Gamma_L|^2
\]
For example, if the return loss is $10\text{dB}$, then $10\%$ of the incident power is reflected at the load, with the remaining $90\%$ being absorbed by the load—we “lose” $10\%$ of the incident power.

Likewise, if the return loss is $30\text{dB}$, then $0.1\%$ of the incident power is reflected at the load, with the remaining $99.9\%$ being absorbed by the load—we “lose” $0.1\%$ of the incident power.

Thus, a larger numeric value for return loss actually indicates less lost power! An ideal return loss would be $\infty\text{dB}$, whereas a return loss of $0\text{dB}$ indicates that $|\Gamma_L|=1$—the load is reactive!