Reciprocal and Lossless Networks

We can **classify** multi-port devices or networks as either **lossless** or lossy; **reciprocal** or non-reciprocal. Let's look at each classification individually:

Lossless

A **lossless** network or device is simply one that **cannot** absorb power. This does **not** mean that the delivered power at **every port** is zero; rather, it means the total power flowing **into** the **device** must equal the total power **exiting** the **device**.

A lossless device exhibits an impedance matrix with an interesting **property**. Perhaps not surprisingly, we find for a lossless device that the **elements** of its impedance matrix will be **purely reactive**:

$$Re\{Z_{mn}\} = 0 \qquad \text{for a } \textbf{lossless} \text{ device.}$$

If the device is lossy, then the elements of the impedance matrix must have **at least** one element with a real (i.e., resistive) component.

Moreover, we similarly find that if the elements of an **admittance** matrix are **all** purely imaginary (i.e., $Re\{Y_{mn}\} = 0$), then the device is lossless.

Reciprocal

Generally speaking, most **passive, linear** microwave components will turn out to be **reciprocal**—regardless of whether the designer **intended** it to be or not!

Reciprocity is basically a "natural" effect of using simple linear materials such as **dielectrics** and **conductors**. It results from a characteristic in **electromagnetics** called "reciprocity"—a characteristic that is difficult to **prevent**!

But reciprocity is a tremendously important characteristic, as it greatly **simplifies** an impedance or admittance matrix!

Specifically, we find that a reciprocal device will result in a **symmetric** impedance and admittance **matrix**, meaning that:

$$\boxed{Z_{mn} = Z_{nm} \qquad Y_{mn} = Y_{nm} \qquad \text{for reciprocal devices}}$$

For **example**, we find for a reciprocal device that $Z_{23} = Z_{32}$, and $Y_{21} = Y_{12}$.

Let's illustrate these concepts with **four examples:**

$$\mathcal{Z} = \begin{bmatrix} j2 & 0.1 & j3 \\ -j & -1 & 1 \\ 4 & -2 & 0.5 \end{bmatrix}$$

Neither lossless **nor** reciprocal.

$$\mathcal{Z} = \begin{bmatrix} j2 & j0.1 & j3 \\ -j & -j1 & j1 \\ j4 & -j2 & j0.5 \end{bmatrix}$$

Lossless, but not reciprocal.

$$\mathcal{Z} = \begin{bmatrix} j2 & -j & 4 \\ -j & -1 & -j2 \\ 4 & -j2 & j0.5 \end{bmatrix}$$

Reciprocal, but not lossless.

$$\mathcal{Z} = \begin{bmatrix} j2 & -j & j4 \\ -j & -j & -j2 \\ j4 & -j2 & j0.5 \end{bmatrix}$$

Both reciprocal **and** lossless.