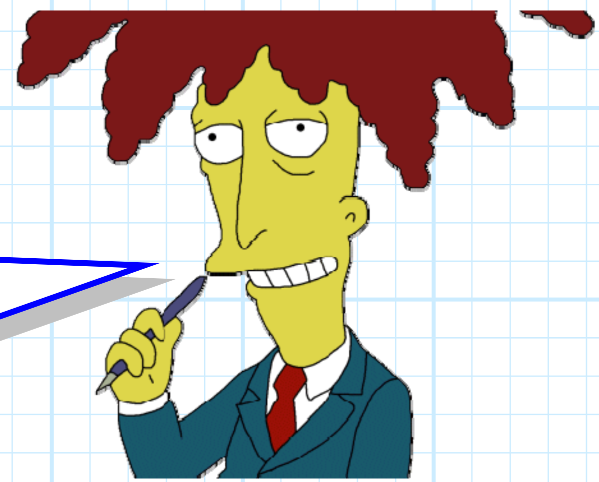
The Reflection Coefficient

So, we know that the transmission line **voltage** $V(z)$ and the transmission line **current** $I(z)$ can be related by the **line impedance** $Z(z)$:

$$V(z) = Z(z)\, I(z) \qquad \text{or equivalently} \qquad I(z) = \frac{V(z)}{Z(z)}$$

Q: *Piece of cake! I fully understand the concepts of **voltage**, **current** and **impedance** from my **circuits** classes.*

*Let's move on to something more important (or, at the very least, more **interesting**).*



Expressing the "activity" on a transmission line in terms of **voltage**, **current** and **impedance** is of course **perfectly** valid.

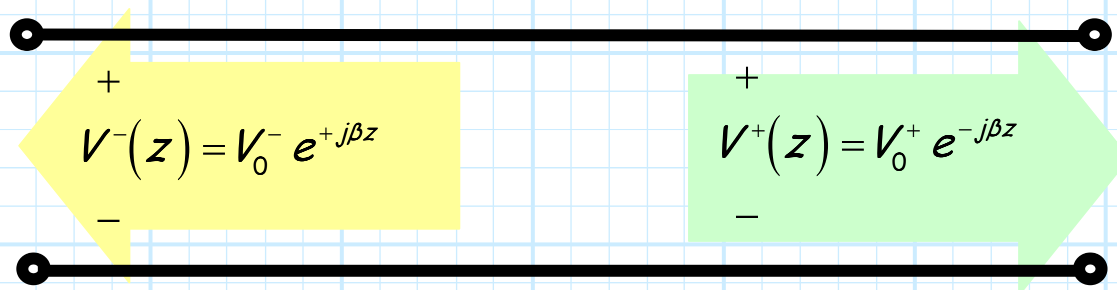
→ However, there is an **alternative** (and much simpler!) way to describe transmission line activity !!!!

Wave Functions $V^+(z)$ and $V^-(z)$ Describe All!

Look closely at the expressions for **voltage**, **current**, and **impedance**:

$$V(z) = V^+(z) + V^-(z) \qquad I(z) = \frac{V^+(z) - V^-(z)}{Z_0} \qquad Z(z) = Z_0\left(\frac{V^+(z) + V^-(z)}{V^+(z) - V^-(z)}\right)$$

It is evident that we can **alternatively** express all "activity" on the transmission line in terms of the two transmission line **waves** $V^+(z)$ and $V^-(z)$.



Q: *I know* $V(z)$ *and* $I(z)$ *are* ***related*** *by* ***line impedance*** $Z(z)$:

$$Z(z) = \frac{V(z)}{I(z)}$$

But how are $V^+(z)$ *and* $V^-(z)$ *related?*

The Reflection Coefficient Function

A: Similar to line impedance, we can define a new parameter—the **reflection coefficient** $\Gamma(z)$—as the **ratio** of the two quantities:

$$\Gamma(z) \doteq \frac{V^-(z)}{V^+(z)} \quad \Rightarrow \quad V^-(z) = \Gamma(z)\, V^+(z)$$

More **specifically**, we can express $\Gamma(z)$ as:

$$\Gamma(z) = \frac{V_0^- \, e^{+j\beta z}}{V_0^+ \, e^{-j\beta z}} = \frac{V_0^-}{V_0^+} \, e^{+j2\beta z}$$

Note then, the value of the reflection coefficient at ***z* = 0** is:

$$\Gamma(z=0) = \frac{V^-(z=0)}{V_0^+(z=0)} \, e^{+j2\beta(0)} = \frac{V_0^-}{V_0^+}$$

The Value Γ_0

We define this value as Γ_0, where:

$$\Gamma_0 \doteq \Gamma(z=0) = \frac{V_0^-}{V_0^+}$$

Note then that we can **alternatively** write $\Gamma(z)$ as:

$$\Gamma(z) = \Gamma_0 \; e^{+j2\beta z}$$

So we have **two different,** but equivalent ways, to describe transmission line activity!

We **can** use (total) **voltage** and **current**, related by **line impedance**:

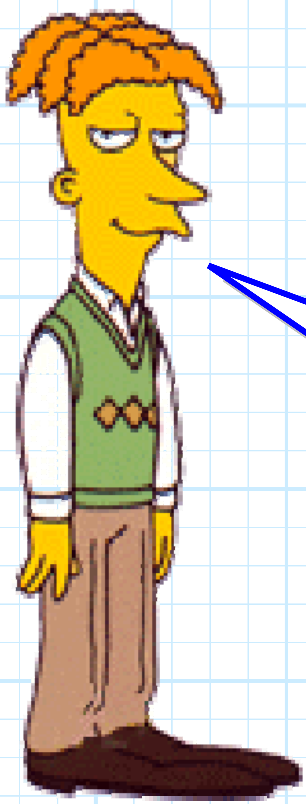
$$Z(z) = \frac{V(z)}{I(z)} \quad \therefore \quad V(z) = Z(z)\, I(z)$$

Or, …

The Wave Description of Transmission Line Activity

…….we can use the two propagating **voltage waves**, related by the **reflection coefficient**:

$$\Gamma(z) = \frac{V^{-}(z)}{V^{+}(z)} \quad \therefore \quad V^{-}(z) = \Gamma(z)\, V^{+}(z)$$



These are **equivalent** relationships—we can use **either** when describing a transmission line.

*Based on your **circuits** experience, you might well be **tempted** to always use the **first** relationship.*

*However, we will find it useful (as well as simple) indeed to describe activity on a transmission line in terms of the **second** relationship—in terms of the **two** propagating transmission line **waves**!*