Self-Loop Rule

Now consider the equation:

$$b_1 = \alpha\, a_1 + \beta\, a_2 + \gamma\, b_1$$

A little dab of **algebra** allows us to determine the value of node b_1:

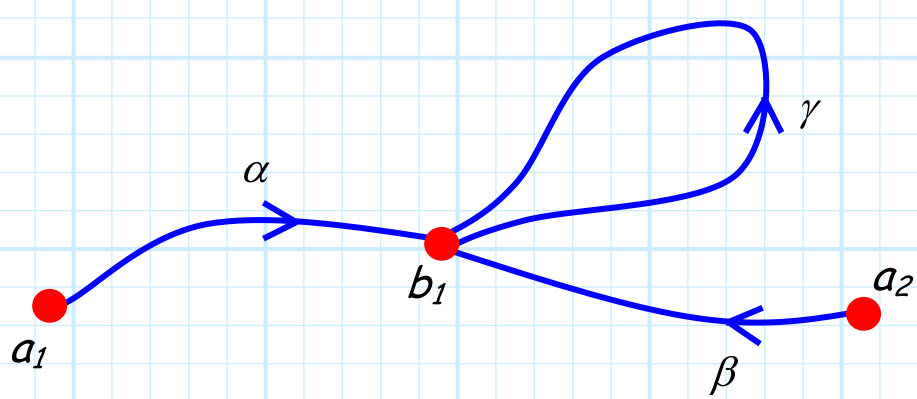
$$b_1 = \alpha\, a_1 + \beta\, a_2 + \gamma\, b_1$$

$$b_1 - \gamma\, b_1 = \alpha\, a_1 + \beta\, a_2$$

$$(1-\gamma)\, b_1 = \alpha\, a_1 + \beta\, a_2$$

$$b_1 = \frac{\alpha}{1-\gamma}\, a_1 + \frac{\beta}{1-\gamma}\, a_2$$

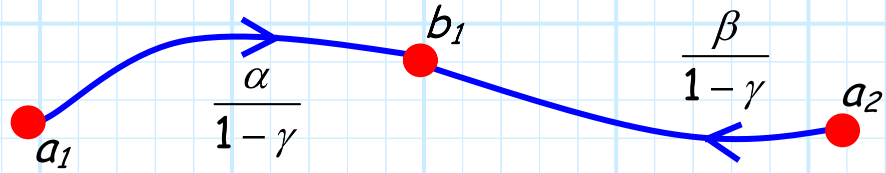
The signal flow graph of the **first** equation is:



$$b_1 = \alpha\, a_1 + \beta\, a_2 + \gamma\, b_1$$

While the signal flow graph of the **second** is:

$$b_1 = \frac{\alpha}{1-\gamma} a_1 + \frac{\beta}{1-\gamma} a_2$$



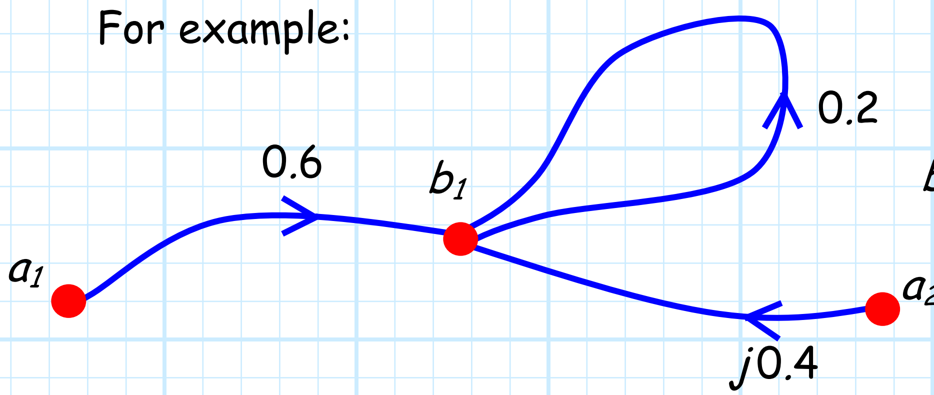
These two signal flow graphs are **equivalent**!

Note the self-loop has been **"removed"** in the second graph. Thus, we now have a method for removing self-loops. This method is **rule 3**.

Rule 3 - Self-Loop Rule

*A self-loop can be eliminate by multiplying **all** of the branches "**feeding**" the self-loop node by $1/(1-S_{sl})$, where S_{sl} is the value of the self loop branch.*

For example:

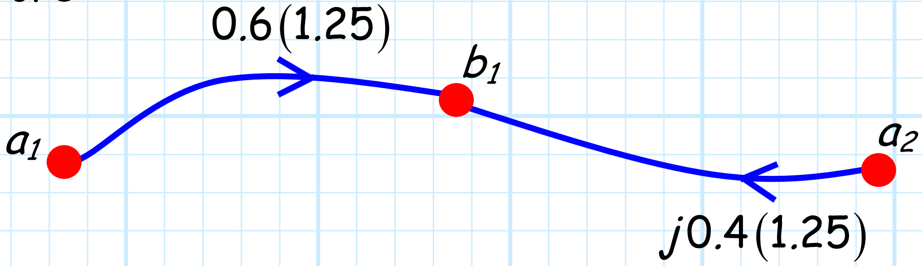


$$b_1 = 0.6\, a_1 + j0.4 a_2 + 0.2 b_1$$

can be simplified by **eliminating the self-loop**. We multiply **both** of the two branches **feeding** the self-loop node by:

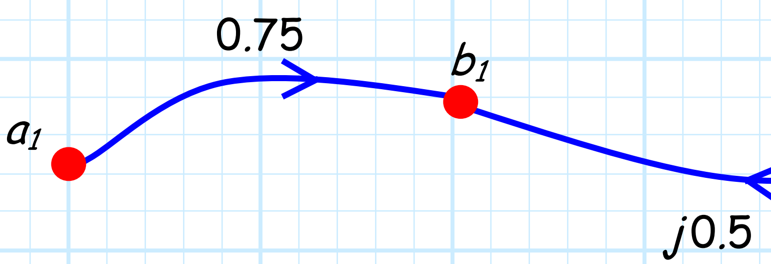
$$\frac{1}{1-\mathcal{S}_{sl}} = \frac{1}{1-0.2} = 1.25$$

Therefore:



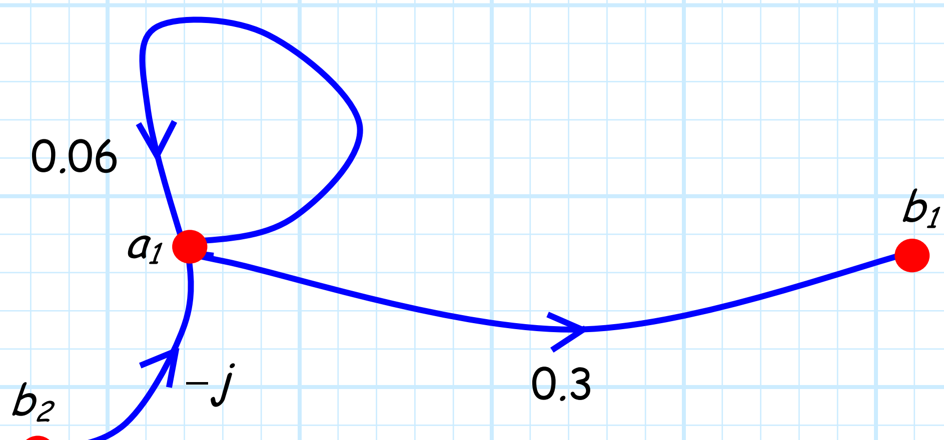
And thus:

$$b_1 = 0.75\, a_1 + j0.5\, a_2$$



Or **another example**:

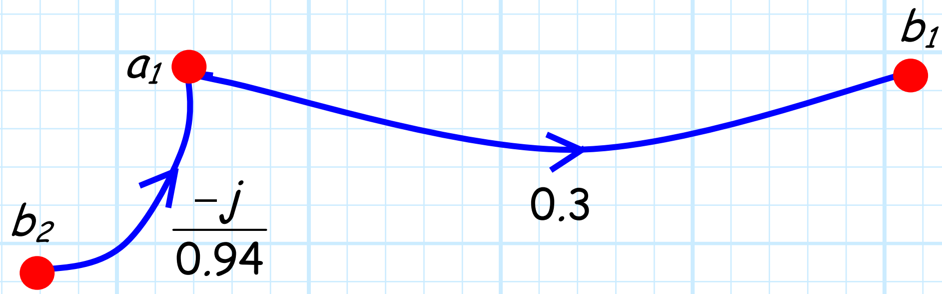
$$a_1 = 0.06\, a_1 - j\, b_2$$
$$b_1 = 0.3\, a_1$$



becomes after reduction using **rule 3**:

$$a_1 = \frac{-j}{0.94}\, b_2$$

$$b_1 = 0.3\, a_1$$



Q: *Wait a minute! I think you* ***forgot*** *something. Shouldn't you* ***also*** *divide the 0.3 branch value by* $1-0.06=0.94$ *??*

A: Nope! The 0.3 branch is **exiting** the self-loop node a_1. **Only** incoming branches (e.g., the $-j$ branch) to the self-loop node are modified by the self-loop rule!